MATH 4053: INTRODUCTION TO ALGEBRAIC TOPOLOGY

Homework 7
Due: 03/26/20 by 5pm

(1) Hatcher 2.1.17

(2) Hatcher 2.1.22

(3) Hatcher 2.1.28

(4) Hatcher 2.1.29

The following problem might be helpful to do before Hatcher 2.1.28.

5. Let Σ_g be a surface with genus $g \geq 1$. Let $S\Sigma_g$ be the suspension of Σ_g, that is, $S\Sigma_g = \Sigma \times [0,1]/\{\Sigma \times 0 \coprod \Sigma \times 1\}$. Compute the local homology groups $H_*(S\Sigma_g, S\Sigma_g \backslash x)$ for all $x \in S\Sigma_g$. Conclude that $S\Sigma_g$ is not locally homeomorphic to $\mathbb{R}^n$ for any n.

6. (Extra credit)

Since S^k is the union of two k-simplices Δ_1, Δ_2, we can view an arbitrary map $f : S^k \to X$ as an n-chain $f|_{\Delta_1} - f|_{\Delta_2}$. This is always a cycle (since $f|_{\partial_i \Delta_1} = f|_{\partial_i \Delta_2}$), and hence defines an element of $H_k(M)$. Let $X = S^n$. Using the long exact sequence for the pairs (D^m, S^{m-1}), we proved that $H_k(S^n) = 0$ if $k \neq 0, n$. Make this explicit. Namely, if $k \neq 0, n$, describe an explicit $k+1$ chain $\sum a_i \sigma_i$ so that $\partial(\sum a_i \sigma_i) = f|_{\Delta_1} - f|_{\Delta_2}$.

Describe this $k+1$ chain as a map $g : Q^{k+1} \to S^n$ from a singular $k+1$ manifold Q^{k+1} (with singularities of dimension at most $k-1$) so that $\partial Q = S^k$ and $g|_{\partial Q} = f$. (see the discussion on p. 109 of Hatcher).