Heuristic Algorithm for 3D Modelling of a Railway Track Route

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Abstract: A heuristic algorithm for 3-dimensional modelling of the railway route has been developed. In determining the geometric parameters of the route, cost optimization was used, for which an asymmetric bell-shaped function was introduced, having a minimum at the zero-work line and maximums in bridge and tunnel construction. A gradual optimization technique is used as the main heuristic method, which consists of a sequential solution of the optimization problem from the simplest cases to the most complex ones. The increase in complexity occurs by dividing the segments of the alignment by a point whose coordinates undergo a variation in three dimensions until the optimal cost is obtained. After iteration, the alignment is modified according to current codes for railways, tunnels and bridges. The algorithm was tested on synthesized digital elevation models using a modified diamond-square algorithm. The experimental investigation consisted in variation of scaling factor of altitude matrix values. It was shown that the use of the developed algorithm leads to finding a railway track route that differs in cost from the global optimum by not more than 5-15% on average. The computational complexity of the constructed algorithm has a linear-logarithmic dependence on the trajectory length.

1 INTRODUCTION

The design of the railway alignment takes place at the strategic and tactical level. The strategic level includes the tasks of implementing global economic and social trends: it needs to understand which settlements and logistics points should be involved in order to optimise freight and passenger traffic flows. Problems at the tactical level are those that arise after a strategic decision has been taken: it is assumed that the choice of transport network focal points has already been made and is not negotiable. This level includes the specific design of future railways, bridges and tunnels, taking into account topography, hydrology, geology and climate.

This study addresses the task of automatically constructing a cost-optimal track in plan and profile based on elevation information for a preliminary economic justification of a future detailed design. Focusing on relief is explained by the fact that even with geometrically insignificant changes in the route to be laid, there is a significant increase in construction costs, resources used and road operation due to the non-linearity of the cost function (Ghoreishi, 2019). The possibility of adequate functioning of algorithms in mountainous terrain is of particular importance here, because the economic result in stressed sections is several times greater than the effect in free sections (Struchenkov, 2021).

To obtain information on relief and further application of traditional cameral tracing, it is possible to use digitized maps, e.g., topographic maps. The basic elements of tracing are its projection on a horizontal plane (plan) and a vertical section along the projected line (longitudinal profile). When tracing, the requirements of railway infrastructure codes, railways, bridges and tunnels must be complied with (Bushuev, 2019; Skutin, 2019). The competing directions along which the alignment is constructed are often chosen intuitively, based on the past experience of the designers, which can lead to different designers proposing different possible solutions for the trajectory, which does not meet the basic requirement of solving the optimisation problem.

Manual tracing uses modern CAD, which often allows the requirements on the nature of the trajectory to be met, but does not take into account cost

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optimisation. Among domestic software products performing some automation functions it is necessary to mention Credo, RVPlan, Korwin, Robur, among foreign ones – Civil 3D, GeoniCS, MXRAIL. Nevertheless, the issue of building a software system where it would be possible to select different optimality criteria and the development would be fully automated after the task of technical specification and input of geoinformation data remains almost unexplored. Currently, the operator is left with such problems as calculating construction and operating costs, maximum rail speed, fuel and resources consumption, and train travel times along the section. In some CAD, for example, Invest, the economic calculations of the main elements of the railway track are partially automated interactively. In CAD Aquila (Bykov, 2017), the length of calculated sections is limited to 15-25 km, which makes it possible to perform economic justification only for short sections of track.

The basic problem of the decision of a task of automatic designing is not only a high degree of a variability of a choice of geometrical sections (straight lines, circular and transitive curves) and their parameters, but also the quantity of these sections, depending on mountainous terrain. All this leads to the fact that the computational complexity of the potential algorithm will be:

$$T(N) = O(2^N) = O\left(2^{\frac{AB}{R\varepsilon}}\right) \tag{1}$$

where N is the number of track elements, AB is point-to-point distance between A and B, Rε – the minimum size of a single trajectory section.

Nevertheless, such a variational approach is also used in current research (Prokop'eva, 2017; Kholodov, 2019; Sidorova, 2020), with a manual method being implemented in automatic mode, often separately in plan and separately in profile. However, there are also emerging studies using new approaches to trajectory generation, such as iterative approach (Pu, 2021), fuzzy hierarchy analysis (Singh, 2019), genetic (Kang, 2020; Li, 2017) and evolutionary algorithms (Polyanskiy, 2021), and swarm intelligence (Ghoreishi, 2019). The main idea of these methods is to treat the railroad track as a set of critical points, which allows implementing various approximate algorithms, without being strongly influenced by the variability of the geometric element selection. At the same time, for example in (Sushma, 2020), it remains possible to design the whole road network in parallel instead of a single trajectory.

2 MATERIALS AND METHODS

Real data for the algorithm under development can be obtained from aerial photographs (Roshchin, 2021) or directly from topographic maps (Dmitriev, 2019). In this case, the data are digital elevation models (DEM). The first method in itself is time consuming, although it saves later on planning tracing, while the second method can automatically obtain the relief data, but only in existing maps. Therefore, this study for completeness was carried out on generated DEMs using the diamond-square fractal algorithm (Smelik, 2014), an example of which is shown in Fig. 1. It is worth noting that the use of DEMs in the form of polynomials of high powers from two variables even with a long selection of coefficients does not give a real representation of the earth's surface, although in this case the problem of finding the optimal trajectory is reduced to the solution of simple functional equations.

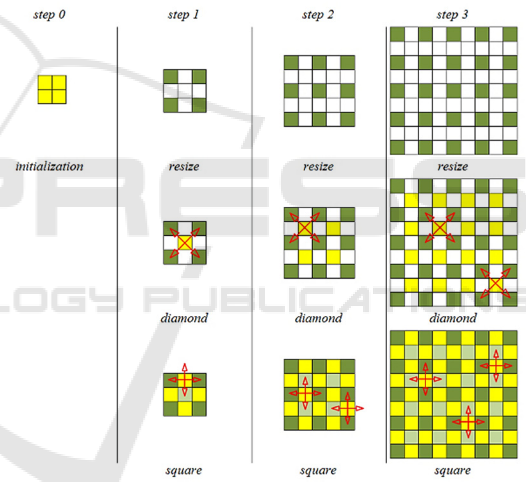

Figure 1: The first few steps of the diamond-square algorithm.

The matrix 2×2 is initialised with zero values before starting the algorithm. Then three actions are performed for n times in sequence. First, the matrix grid is scaled from the order of $(2^{n-1}+1)$ to the order (2^n+1) by adding null rows and columns between the existing ones.

In step diamond for every four neighbouring elements forming a square 3×3, the middle element is initialised with the height value using the formula:

$$z(i,j) = 0.25 * (z(i-1,j-1) + z(i-1,j+1) + \\ + z(i+1,j-1) + z(i+1,j+1)) + R \cdot p^n, \tag{2}$$

where (i, j) define the coordinates of the point under study, z(i, j) is the height value, R∈[-1, 1] is a random variable with a continuous uniform distribution, $p \in (0, 1)$ is a relief parameter, n is a step iteration number.

The greater the parameter p, the more uneven the terrain is. For the algorithm to work, a parameter equal to the Golden Ratio was chosen, resulting in good plausibility.

In step square a similar operation is performed, but for every four neighbouring elements forming a square rotated by 45 degrees. This may involve referring to elements outside the existing matrix, in which case zero is taken as the missing value. Then the height value for the middle element is calculated by the formula:

$$z(i,j) = 0.25 * (z(i-1,j) + z(i,j-1) + \\ + z(i,j+1) + z(i+1,j)) + R \cdot p^n. \quad (3)$$

In general, the elevation values in the matrix can take values of different meanings that are unrelated to the real values, so the final matrix after n steps can be transformed by performing linear operations, median filtering and normalization. Elevations and troughs with random characteristics were added to the final elevation map to increase plausibility. An example of the resulting digital map together with the contour lines is shown in Figure 2.

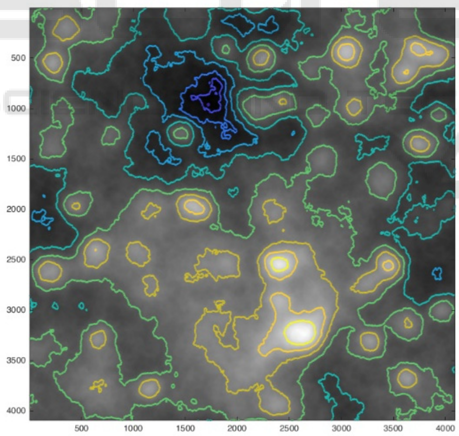

Figure 2: Example of a digital map obtained; brightness corresponds to height.

The total construction cost can be calculated using the formula for the economic justification of the proposed track trajectory:

$$Cost(L) = \int_L c(x(t), y(t), z(t) - z_{DEM}(x(t), y(t)))dl, \quad (4)$$

where L is a curve in three-dimensional space, x(t), y(t), z(t) define its parametric representation using Cartesian coordinates, $t \in [0, 1]$ is a parameter for which zero corresponds to the start of the trajectory, one to the end, c(x, y, Δz) is a unit cost function, z_{DEM} is an elevation values from a digital elevation model.

Although in the general case it is the integral that must be calculated, since the map is a discrete square matrix, it is possible to go from calculating the integral to the sum:

$$Cost(L) = \sum_{k=1}^{n} c(x_k, y_k, z_k - z_{DEM}(x_k, y_k)) \cdot L_k. \quad (5)$$

We construct a mathematical model of the unit cost function based on the following conditions:

a) at Δz = 0 it takes the value c0, which corresponds to the line of zero work;

b) at $\Delta^{+}_{crit} > \Delta z > 0$, it is quadratic around zero, which corresponds to an increase in the cross-sectional area of the required embankments;

c) at $\Delta z > \Delta^{+}_{crit}$, it takes the value $M_{+\infty} >> c0$, which corresponds to the construction of a bridge with a cost that does not depend on its height;

d) at $\Delta^{-}_{crit} < \Delta z < 0$, it has a quadratic character around zero (but with a different coefficient), which corresponds to an increase in the cross-sectional area of the required excavation;

e) at $\Delta z < \Delta^{-}_{crit}$, it takes the value $M_{-\infty} >> c0$, which corresponds to the construction of a tunnel that is not dependent on the depth of the tunnel;

f) the function should have a smooth transition between b-c and d-e to avoid complicating the design with separate bridge and tunnel sections at this stage.

All presented parameters can vary considerably for different soils, the study assumes that they are equal for the given site. Point c may also not be fulfilled at certain bridge designs. The study assumes that all railway alignments are of the same type.

Considering these points and the conditions for the specific cost function at a point, a mathematical model of the asymmetric bell-shaped function has been derived:

$$c(x,y,\Delta z) = \theta(\Delta z) * \left(\frac{c_0}{2} + \frac{M_{+\infty} \cdot \Delta^2 z}{\sigma^2_{+\infty} + \Delta^2 z}\right) + \\ + \theta(-\Delta z) * \left(\frac{c_0}{2} + \frac{M_{-\infty} \cdot \Delta^2 z}{\sigma^2_{-\infty} + \Delta^2 z}\right), \quad (6)$$

where θ(x) is the Heaviside function, c0 defines track cost along the zero-work line, $M_{+\infty}$, $M_{-\infty}$ are the costs for bridge and tunnel construction respectively, $\sigma_{+\infty}$, $\sigma_{-\infty}$ are the parameters defining the critical value Δz, where the strategy is changed to building a bridge or tunnel. Although the Heaviside function θ(x) has a discontinuity at a point x = 0, the specific cost function c will be smooth, since its derivative has no discontinuity at a given point.

Thus, to solve the trajectory optimization problem between two points A and B, it is necessary to find the functions x(t), y(t), z(t) satisfying the following conditions:

$$\begin{gathered} Cost(L_{AB}) \to min, \\ x(0) = x_A, y(0) = y_A, z(0) = z_A, \\ x(1) = x_B, y(1) = y_B, z(1) = z_B. \end{gathered} \tag{7}$$

In addition, the restrictions expressed in the codes of practice should be complied with, e.g., for the railways of 1520 mm gauge (PC 119.13330.2017), tunnels (PC 122.13330.2012) and bridges (PC 46.13330.2012). The main features which are most affected by topography and which must be taken into account when tracing a railway are the maximum track gradient, the minimum curvature radius in the plan and the minimum gradient in tunnels.

Any discrete elevation matrix can be converted to analytical form by interpolation, e.g., by Lagrange polynomials. At that, the functions with the number of parameters equal to the number of DEM elements will be analysed, which will lead to high computational complexity. For example, the DEM used below has the number of elements of the order of 65 million. It is worth noting that this method does not ensure that the railway requirements are met. If local interpolation is used, for example bicubic interpolation, it will not be possible to apply variational calculus methods to the entire map set.

Another approach is to represent the three-dimensional space as a uniform graph with N levels of discretization along each of the axes. Thus, the number of vertices in this graph will be N^3, and the number of edges depends on the connectivity. For example, a Von Neumann neighbourhood connects only cells that have a common side, while a Moore neighbourhood also connects common vertices. Applying graph theory and graph traversal methods, such as Dijkstra's algorithm, it is possible to obtain a path of minimum length. The computational complexity of such an algorithm will be $O(N^6)$, which is already large enough for a number of sampling levels on the order of 1000, corresponding to a small DEM of 1MB size. The algorithm A* generally uses an exponential number of points relative to the path length, which is also not practical in this case.

Nevertheless, it is not necessary to obtain a strictly optimal solution if large computational power is needed. For example, if a trajectory that differs by only a few percent from the optimal cost in a sufficient amount of time is obtained, this would also be an acceptable result.

One class of algorithms that have the property of quickly finding a solution, albeit not the most optimal one, but a good one, is heuristics. One of the heuristic methods is the technique of gradual optimization. Then a difficult optimization problem is solved first for a much-simplified problem, gradually increasing the complexity until the complexity equals the initial one. The intermediate results should be less and less different with each step: when this change stops or becomes less than some threshold, the heuristic algorithm stops.

At step zero, the trajectory is an AB segment. To implement the iterative steps of the algorithm, we should note the existence of invariants for passing any trajectory from A to B, including the optimal trajectory. If we need to connect two points in three-dimensional space with a curve, then it should in any case intersect some set of planes. One of such planes is the plane perpendicular to the given segment and passing through its middle. Assuming that points A and B are not above each other, which is adequate for the problem, then such a plane is also the plane perpendicular to the horizontal plane and to the segment AB in the plane plan. Correspondingly, we will select a new point exactly in this plane, using the parametric definition of the planes:

$$\begin{gathered} x_C = \frac{x_A + x_B}{2} + \frac{y_B - y_A}{2} \cdot R_1, \\ y_C = \frac{y_A + y_B}{2} - \frac{x_B - x_A}{2} \cdot R_1, \\ z_C = \frac{z_{high} + z_{low}}{2} + \frac{z_{high} - z_{low}}{2} \cdot R_2, \end{gathered} \tag{8}$$

where R_1 and R_2 are random variables having a continuous uniform distribution and specifying the variation, z_{high} and z_{low} are the boundary heights, which can be iteratively calculated as follows:

$$\begin{aligned} & z_{high} \\ & = min(z_A + i_p \cdot AC, z_B + i_p \cdot BC, max(z_A, z_B)), \\ & z_{low} \\ & = max(z_A - i_p \cdot AC, z_B - i_p \cdot BC, min(z_A, z_B)), \end{aligned} \tag{9}$$

where i_p – the value of the maximum gradient. First, the heights at points A and B are selected as boundaries, then the boundary values are recalculated with the new value z_C. At larger distances, the boundary values resulting from the maximum gradient are chosen more often, and at small distances, the elevation values at points A and B remain.

To select the correct variation, it is necessary to calculate the value of the alignment along the ACB polygon, which can be done in $O(L_{ACB})$ operations. It can be assumed that the cost calculation for each broken line is approximately the same and its time complexity belongs to the class O(L). In the next iterations the heuristic algorithm is repeated, but already for new pairs of points AC and CB, generating new two points, then 4 segments are investigated, in the next step 8 and so on.

We assume a difference in the number of variations of points at each separate step: let the j-th iteration of the algorithm be v_j variations, then the complexity of the algorithm after the nth iteration will belong to the class $O\left(\sum_{j=1}^{n} v_j \cdot 2^{j-1} \cdot L\right)$. Thanks to the formulas, any point on the map can be chosen as the midpoint. This naturally reduces both the size of the segments in question (at least as much as $\sqrt{2}$), as well as the area over which variation can occur (by at least a factor of 2). In order to maintain the density of variations per unit area, the number of variations can be taken as $v_j = v/2^{j-1}$, which will simplify the computational complexity to a class of $O(v \cdot n \cdot L)$. Taking into account the discrete nature of the resulting trajectory, it can be noted that the length of the optimal trajectory is equal to the number of points on it. The number of iterations before reaching segments of length one is asymptotically equal to $O(lg\, L)$. As a result, the complexity class of the algorithm presented is $O(L \cdot lg\, L)$, which allows to use elevation maps of practically any size and accuracy, as the constructed algorithm will work in linear-logarithmic time from DEM diameter.

The developed algorithm naturally uses the nature of three-dimensional space, implementing modification of points in parallel along all axes, which can also be used to speed up calculations on several processor cores. In addition, the possibility of increasing the number of points to the order of O(L) is assumed, that on the one hand increases the used memory, but allows with more confidence to find the global optimum, than a limited number of points, as in the algorithms of collective intelligence. All points obtained in each subsequent step are based on the problem already solved for a smaller number of points.

To analyse the bridge and tunnel constraints, we study the resulting profile. If there will be two elements of the same type next to each other, e.g., two tunnels with a small crossing, they should be merged into one. At correcting the elevation values at the element boundaries, the slope in these areas may exceed the allowable slope, so it is necessary to normalize it, which will lead to a system error. In this case the global optimum may be lost, but the number of such special sections is not that large relative to the total length of the alignment, which is accounted for by bridges and tunnels.

Next, it is necessary to correct the sections with small curvature radii by replacing them with curves of larger radius. To do this, a straightening operation is performed for each of the three critical points:

$$x_i := \frac{1}{2}\left(x_i + \frac{x_{i+1} + x_{i-1}}{2}\right),$$
$$y_i := \frac{1}{2}\left(y_i + \frac{y_{i+1} + y_{i-1}}{2}\right). \quad (10)$$

Since the plan and profile points have been corrected, the optimization algorithm should be run again, but without adding new points. In any case these corrections do not take more than O(L) time, so they do not affect the total computational complexity.

3 RESULTS

The pseudo planetary relief generation and the heuristic automatic tracing algorithm have been implemented as software in the MatLab package. The quality of the generated data can be adjusted by specifying the number of steps of the diamond-square algorithm and its parameters. Each DEM element was matched to a 1 m^2 square. Modifications to the algorithm increased the confidence of the generated terrain.

As a computational experiment, the task of modelling a railroad track in plan and profile from point A to B was set for the elevation map presented above, the values in which are scaled with factor m for the relief degree task from near plain, to mountains. This experimental study raises the question, for which type of terrain would it be most difficult to construct an optimal trajectory? The following parameters were chosen: i_p = 10‰, i_{min} = 3‰, R_{min} = 500 m. The cost model parameters are: c0 = 1, $M_{+\infty}$ = 10, $\sigma_{+\infty}$ = 10 m, $M_{-\infty}$ = 20, $\sigma_{-\infty}$ = 20 m. The results for the different coefficients m are shown in Figure 3.

The lowest possible cost arises when there is no relief between points A and B of the smaller steering slope, when the optimum trajectory and is a segment AB running along the line of zero work. This value is unattainable because some relief always exists. In addition, the study also aims at solving the problem of designing a railway track in difficult terrain conditions. Nevertheless, this value is very convenient because it is possible to count the effectiveness of the constructed path optimisation in units of $Cost_{min}$. To investigate the stability of the

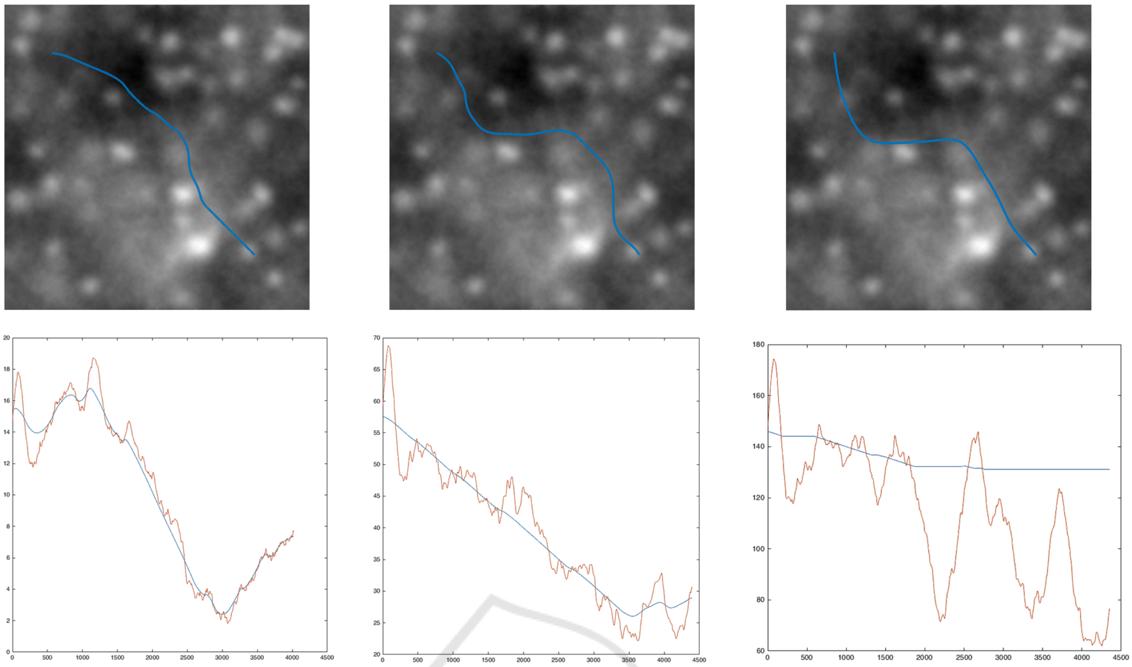

Figure 3: Model of the railway line in plan (above) and longitudinal profile (below) as a function of depending on the scaling factor, from left to right: m = 30; m = 120; m = 300.

algorithm and the quality of its performance, the algorithm was run 50 times on average for each value of the scaling factor. The calculated stability indices are shown in the Table 1.

Table 1: Stability indices of the developed heuristic algorithm depending on the scaling factor m.

m	Average value *Cost*	Standard deviation *Cost*	Optimal value *Cost*
30	1.17	0.07	1.11
60	1.32	0.09	1.26
90	1.59	0.11	1.51
120	1.80	0.32	1.53
150	3.03	0.65	2.57
300	13.05	1.57	12.06

4 DISCUSSION AND CONCLUSION

It can be concluded that the developed and implemented heuristic algorithm can be used to carry out an initial economic justification of the chosen direction, trajectory and route elements. The exception is considered to be the areas with very high mountainous terrain, or if it is necessary to solve the problem for two points that are quite close in plan but have quite a big difference in elevation, which leads to insufficient use of the existing DEM to implement a more complex bypass trajectory. Reducing the cost of constructing the railway alignment will not only lead to a reduction in labour costs, but also in the resources used, including during operation. Further research can focus on optimizing the algorithm to reduce computational complexity, adding constraints on the geometric parameters of the alignment, and using parallel computing. It is possible to create a more stable algorithm, the results of which will not be so strongly affected by the variations arising from the use of random variables.

Analysis of statistical data shows an increase in the average value of the cost with an increase in mountainous terrain, at that the optimum value grows more slowly than the algorithm average. First of all, this means that there is a much greater amount of variation with increasing terrain topography, causing the algorithm to drift further away from finding the global optimum. At further growth of the scaling factor, the average value of the cost again approaches the optimum, as it becomes easier to make the choice at sufficiently high elevations, which illustrates the effect when rigid nonlinearities are given by mathematical formulas that are simpler to analyse.

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