Multi-Level Grey Risky Multi-Attribute Decision-Making Method Based on Dynamic Regret Theory

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Abstract: There are some complex decision-making problems with incomplete information and multiple choices in the uncertain environment, in which the decision-makers are more vulnerable to the influence of their emotional psychology. Thus a new method based on dynamic regret theory is introduced in this paper for solving the multi-level risky multi-attribute decision-making problem with general grey numbers (GGNs). First, normalize GGNs and calculate the probability of the combined state to construct the combined decision matrix. Then, the dynamic regret theory is integrated into the grey multi-level decision-making problem by constructing grey dynamic regret function and grey dynamic perceived utility function. Finally, the regret equilibrium set is established and the best scheme is selected according to the comprehensive grey perceived utility value. The model proposed in this paper can solve the multi-level grey risky decision-making problem with the consideration of the regret emotion. It not only complements and perfects the multi-level decision-making method, but also widens the research scope and space of dynamic regret theory.

1 INTRODUCTION

In real life, there are many uncertain decision-making problems, that is, decision-making involves fuzziness, randomness, grey and so on. For example, decision makers may encounter the interval grey number whose true value is unknown but the range of the value is known (Liu and Lin, 2006). The interval grey number is the basic concept in grey system theory (Deng, 1982) which is an effective tool to process the systems with some known information and some unknown information. Decision information is grey in this circumstance. In addition, the attribute values in some decision-making problems may be stochastic variables which will change with the different state of nature. The decision-maker can predict various possible natural states, or quantify this randomness by setting the probability distribution, but the decision-maker cannot get the real state in the future. Decision information is random and risky. This decision-making problem is risky multi-attribute decision-making(RMADM). Due to the limitations of human cognition and the increasing complexity of practical problems, decision problems are often both grey and stochastic, which is called grey risky decision-making. It can objectively describe more uncertain decision-making problems and has a broad application background. Luo and Liu (2004) have constructed a risky decision-making model based on interval grey number and provided the grey risky decision-making method.

However, there are still practical decision-making conditions where interval grey numbers cannot accurately or fully describe information. With the continuous in-depth development of research, a kind of number, called a general grey number (GGN), which is in union of open or closed grey intervals, is introduced by Liu, Fang and Yang (2012). They think the general grey number can better describe the uncertainty. Suppose the market share of the products of a company is estimated in three ways. The interval grey number [0.67, 0.82] is less accurate than the general grey number $[0.67,0.7] \cup [0.73,0.75] \cup [0.77,0.82]$ in describing the market share. At present, little research has been done on risky multi-attribute decision-making with general grey numbers (GGNs). Qian et al. (2019) studied the risky multi-attribute decision-making based on the general grey number with kernel and greyness and integrated the regret theory into the problem. Zhou et al. (2017a) proposed the stochastic multi-criteria decision-

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making method with the extended grey number, which was introduced by Yang (2007). When dealing with grey number operation, Zhou et al. (2017a) adopted the idea and method of interval numbers to a certain extent, and could not catch the essential characteristics of grey numbers. As a matter of fact, the basic concepts such as the kernel, the degree of greyness, and the field of general grey numbers have been proposed by Liu et al. (2012), which can construct the reduced form of a general grey number and provide important information of the general grey number.

In addition, how the psychological behaviour of the decision-maker affects the actual decision-making process is also a hot spot of current research. A large number of practice and research show that in general, it is difficult for decision-makers to fully obtain accurate decision-making information. They are also affected by subjective factors such as their own emotions, psychological behaviour and experience intuition, thus making the decision-making process show the characteristics of irrational or bounded rationality. That is, decision makers do not always pursue the maximum utility, but make satisfactory decisions according to their own psychological and emotional behaviour and limited experience cognition. In the grey risky decision-making problem, the future environment is uncertain, and the information value is not completely accurate, which is grey. This dual uncertainty makes the decision-making more complex. Therefore, decision-makers are more vulnerable to the influence of emotional psychology in the stages of information acquisition, cognitive judgment and overall evaluation, thus showing a variety of irrational behaviour characteristics. Researchers hope to find a better way to explain this personal behaviour decision-making, which makes the classical expected utility theory continue to be expanded and improved. Among them, the most representative research results are the prospect theory proposed by Kahneman and Tversky (1979) and the regret theory independently proposed by Loomes and Sugden (1982) and Bell (1982). At present, many researchers apply prospect theory to grey risk decision-making, and rich results have been achieved. As the emotion that you want to avoid most in the decision-making process, regret has also attracted much attention in the field of decision-making research. Regret theory has gone through a process of continuous improvement. At first, it was applied to pair by pair selection, and then it was extended to a limited number of alternative objects. Finally, Quiggin (1994) extended it to the case of general selection sets. So regret theory can be applied to the case of any infinite alternative actions. Zeelenberg (1999) believed that anticipated regret could encourage decision makers to avoid risks and seek risks at the same time, thus affecting the decision-making results. Connolly (2002) put forward the decision judgment theory of regret. Humphrey(2004) introduced the feedback information of the abandoned scheme into regret theory and conducted empirical research. Bleichrodt (2010) gave the formula for measuring utility function and regret function. Chorus (2010) proposed a random regret model on basis of regret theory and applied it to the travel traffic choice problem.

Thanks to containing fewer parameters and simpler calculation steps, regret theory is more and more applied to risk-based multi-attribute decision-making(RMADM)problems, and has already become a hot spot in the field of decision-making research. To solve the RMADM problem in which the probabilities of states and the attribute values are both interval numbers, Zhang et al. (2013) proposed a decision analysis method based on regret theory by calculating the sum of utility and regret value of each alternative. Zhang et al. (2014) studied group decision-making method based on regret theory under multi-dimensional preference information of pair-wise alternatives； Liang et al. (2015) introduced the method of stochastic multi-attribute decision making with 2-tuple aspirations considering regret behaviour. Although regret theory has already become a hot spot in the field of decision-making, the research about grey risky decision-making with regret psychology is still in its infancy. Qian et al. (2017) established grey stochastic multi-criteria decision-making model; Guo et al. (2015) constructed multi-objective grey target model based on interval grey number; Zhou et al. (2017b) presented a solution to the grey random multi-criteria problem with extended grey numbers based on regret theory and TOPSIS method; Qian et al. (2019) introduced the grey extended EDAs method based on the general grey numbers, and then combined the regret theory to construct a new grey risky multi-attribute decision-making model.

It is noteworthy that the actual decision-making is quite complex. Except for uncertain fuzzy information or grey information, it also involves multi-stage and multi-level decision-making, which is dynamic decision-making. Different from the static decision-making background, decision-makers will face multiple choices in the dynamic decision-making process. Their later decision-making is not only determined by the expected consideration, but also affected by the early decision-making. Therefore, dynamic decision-making is more vulnerable to the subjective cognition and psychological behaviour of decision-makers. For example, the decision maker will try his best to reduce the regret caused by previous decisions. The regret caused by decisions in each stage will not "disappear" in the final evaluation stage.

Daniel and Rebecca (2005) introduced regret theory into multi-stage decision-making, proposed dynamic regret theory, and put forward specific behaviour prediction derived from regret equilibrium. The main idea is that in the dynamic decision-making process, the decision-maker will use the later choice to strategically reduce the overall risk he faces, that is, the decision maker's final regret is determined by the accumulation of the results of a series of decision-making actions, and the regret generated in each stage of decision-making will affect the final benefit. For multi-stage and multi-level decision-making analysis, dynamic regret theory can reflect the regret avoidance psychology of decision-makers in each stage, which is closer to reality and has broad application prospects. Cao(2013) applied the dynamic regret theory to the selection of different types of venture capital projects in multiple industries. However, few attempts have been conducted on applying dynamic regret to risky multi-attribute decision-making, especially in uncertain grey environment.

As a result, this paper proposes a grey risky multi-attribute decision-making model based on dynamic regret theory, in which the information value is taken in the form of general grey numbers, and the decision maker's regret avoidance is considered in multi-level dynamic risky multi-attribute decision-making.

The rest o this paper is organized as follows: Section 2 reviews basic concepts and methods such as general grey numbers and dynamic regret theory. In section3, an approach for risky multi-attribute decision-making method with general grey numbers is proposed based on dynamic regret theory. Section 4 illustrates a numerical example to show the feasibility and the validity. In section5, conclusions are discussed and drawn.

2 BASIC CONCEPTS

2.1 General Grey Number

Definition 1. *(Liu et al., 2012)*

Let $g^{\pm} \in \Re$ be an unknown real number within a union set of closed or open grey intervals:

$$g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i], \quad i = 1, 2, \cdots, n$$

n is an integer and $0 < n < \infty$, $\underline{a_i}, \overline{a}_i \in \Re$ and $\overline{a}_{i-1} \le \underline{a_i} \le \overline{a}_i \le \underline{a}_{i+1}$, for any interval $\otimes_i \in [\underline{a_i}, \overline{a}_i] \subset \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i]$, then $g^{\pm}$ is called a general grey number.

Definition 2. *(Liu et al., 2012)*

$\hat{g}_{(g^o)}$ is named the simplified form of the general grey number, if $\hat{g}$ is the "kernel" of a general grey number $g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i]$ and g^0 is the degree of greyness of the general grey number. Here,

$$\hat{g} = \frac{1}{n} \sum_{i=1}^{n} \hat{a}_i \tag{1}$$

is called the "kernel" of the general grey number.

$$g^o(g^{\pm}) = \frac{1}{\hat{g}} \sum_{i=1}^{n} \frac{\hat{a}_i \, \mu(\otimes_i)}{\mu(\Omega)} \tag{2}$$

is called the degree of greyness. Ω is the background which makes the general number $g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i]$ come into being and μ is the measure of Ω.

Proposition1. *(Liu et al., 2012)*

For a general grey number $g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i]$, in the event that all the $\hat{a}_i$ and $\mu(\otimes_i)$ are known, its simplified form $\hat{g}_{(g^o)}$ is one-one correspondence with the general grey number $g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a_i}, \overline{a}_i]$.

2.2 Dynamic Regret Theory

Daniel and Rebecca (2005) put forward dynamic regret theory on the basis of regret theory, which introduced regret theory into multi-stage decision-making, and realized the effective combination of regret theory and dynamic decision-making. Different from regret theory, dynamic regret theory emphasizes that in the process of dynamic decision-making, the final regret of the decision-maker is determined by the accumulation of a series of decision-making actions. The later decision-making is influenced not only by the expected consideration, but also by the early decision-making. The decision-maker will reduce the final regret caused by the previous decision as much as possible, that is, the regret caused by the decision-making in each stage will affect the final income.

Daniel and Rebecca (2005) also described and analyzed the idea of dynamic regret decision through a decision problem. There are two parent schemes A and B, in which A contains two sub schemes a_A and b_A , and B also contains two sub schemes a_B and b_B , as shown in Figure 1.

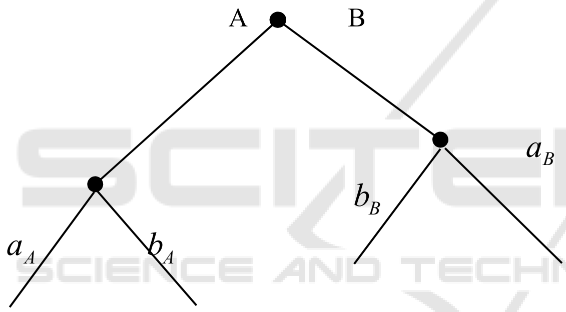

Figure 1: Dynamic Regret Decision Tree

In the first stage, the decision-maker makes the choice between the two parent schemes A and B. If he chooses A, he can only choose a_A or b_A in the second stage, and the regrets in both stages should be considered in the final evaluation. For example, if A is selected for the first stage and a_A is selected for the second stage, then comparing the utility of a_A and b_A will produce the regret value of the second stage. Then fix the selection a_A of the second stage, and compare the maximum utility of a_A and the two sub schemes a_B and b_B in B to produce the regret value of the first stage. Finally, the two regret values are weighted to obtain the comprehensive regret value of the final choice.

Daniel and Rebecca (2005) combined regret theory with dynamic decision-making to better solve the dynamic decision-making problem considering decision-makers' regret. The basic idea is that the comprehensive perceived utility value of the decision-making scheme is made up of the utility value of the scheme itself and the total regret value. The total regret value also consists of two parts. One is the regret value generated by comparing the utility value with the sub schemes of different parent schemes at the first stage; the other is the regret value generated by comparing the utility value with the sub scheme under the same parent scheme at the second stage.

$$U(x) = v(x) - \theta_1 R(\Delta_1 x) - \theta_2 R(\Delta_2 x) \quad (3)$$

$\theta_1(>0)$, $\theta_2(>0)$ respectively represent the degree of regret of the decision-maker for the first stage and the second stage.

Daniel and Rebecca (2005) believe that the behavior of the decision maker at the second stage is actually the result of his own game, constituting a psychological Nash equilibrium . Thus the set of regret equilibrium is put forward. Therefore, considering the decision-making behavior at the first stage, a parent scheme is selected to maximize the utility, so as to form a regret equilibrium solution.

3 RMADM APPROACH WITH GENERAL GREY NUMBERS BASED ON DYNAMIC REGRET THEORY

3.1 Problem Description

Consider a two-stage grey risky multi-attribute decision-making problem. Assume $A = \{A_1, \cdots, A_i, \cdots, A_n\}$is the set of schemes of the first stage. In each scheme A_i, there are q_i sub schemes selected at the second stage. $A_i^* = \{A_{i1}, \cdots, A_{i,i(k)}, \cdots, A_{iq_i}\}$is the set of the sub schemes of A_i . The set of the attributes is $C = \{C_1, \cdots, C_j, \cdots, C_m\}$. And $\omega = (\omega_1, \cdots, \omega_j, \cdots, \omega_m)$ is the weighted vector of the attributes, where $\sum_{j=1}^{m} \omega_j = 1, \omega_j \geq 0, j = 1, 2, \cdots, m$. Suppose the market faces l states, and the set of states is $S = \{S^{(1)}, \cdots, S^{(t)}, \cdots, S^{(l)}\}$. Assume the information value of the alternative $A_{i,i(k)}$ relative to the attribute C_j in the state $S^{(t)}$ is expressed by the general grey number $a_{i,i(k),j}^{(t)}(\otimes)$, $i=1,\cdots,n$, $j=1,\cdots,m$, $i(k)=k=1,\cdots,q_i$, $t=1,\cdots,l$ .Thus the grey decision matrix of all the sub schemes of A_i under the state $S^{(t)}$ at the second stage is obtained as $A_i^{(t)}(\otimes)=\left(a_{i,i(k),j}^{(t)}(\otimes)\right)_{q_i\times m}$. The problem to be solved now is how to choose the final best scheme under the consideration of the psychological behavior of the decision maker's regret avoidance.

3.2 Combined Processing of Natural States

For multi-stage and multi-level decision-making problems, the selection scheme in the first stage usually involves different countries, regions or different industries, so the probability of market situation is also different. Assuming that the market states faced by each scheme in the first stage are independent of each other, the combined market state can be obtained. The market state probabilities of $A_1,\cdots,A_i,\cdots,A_n$ are shown in Table1, and the probability of A_i under the state $S^{(t)}$ is $p_i^{(t)}$.

Table 1: The Probability of the State

	$S^{(1)}$	……	$S^{(t)}$	……	$S^{(l)}$
A_1	$p_1^{(1)}$	……	$p_1^{(t)}$	……	$p_1^{(l)}$
……			……		
A_i	$p_i^{(1)}$	……	$p_i^{(t)}$	……	$p_i^{(l)}$
……			……		
A_n	$p_n^{(1)}$	……	$p_n^{(t)}$	……	$p_n^{(l)}$

Obviously, there are l^n combined states. Assume the set of combined states is $S^*=\{S_1,\cdots,S_h,\cdots,S_{l^n}\}$, and the corresponding probability vector is $\vec{P}=(P_1,\cdots,P_h,\cdots,P_{l^n})$, where P_h is the probability of occurrence of S_h.

$$P_h=\prod_{i=1}^{n}\left(\sum_{t=1}^{l}\theta_i^{(t)}p_i^{(t)}\right) \tag{4}$$

$$h=1,\cdots,l^m$$

$$\theta_i^{(t)}=\begin{cases}1, & \textit{the state of } A_i \textit{ is } S_t\\ 0, & \textit{the state of } A_i \textit{ is not } S_t.\end{cases}$$

According to the combination state, the sub schemes in the second stage are matrix combined. Suppose the states of $A_1,\cdots,A_i,\cdots,A_n$ are $S^{(h_1)},\cdots,S^{(h_i)},\cdots,S^{(h_n)}$ respectively, where $h_i=1,\cdots,l$.

The decision matrix generated by all the sub schemes of A_i under the state $S^{(t)}$ is $A_i^{(h_i)}(\otimes)=\left(a_{i,i(k),j}^{(h_i)}(\otimes)\right)_{q_i\times m}$, where $i(k)=k=1,\cdots,q_i$, $j=1,\cdots,m$. Thus the combined decision matrix is obtained as

$$Y^{(h)}(\otimes)=\begin{pmatrix}A_1^{(h_1)}(\otimes)\\ \vdots\\ A_i^{(h_i)}(\otimes)\\ \vdots\\ A_n^{(h_n)}(\otimes)\end{pmatrix}_{(q_i+\cdots+q_n)\times m}$$
$$=\left(y_{Ij}^{(h)}(\otimes)\right)_{(q_i+\cdots+q_n)\times m} \tag{5}$$

3.3 Modeling Principle and Method

As different attributes have different dimensions, the data will be standardized first to facilitate the calculation.

The grey upper bound effect measurement is adopted as the conversion formula for the attribute of benefit type:

$$z_{Ij}^{(h)}(\otimes)=\frac{y_{Ij}^{(h)}(\otimes)}{\max_I\{y_{Ij}^{(h)}(\otimes)\}} \tag{6}$$

The grey lower bound effect measurement is adopted as the conversion formula for the attribute of cost type:

$$z_{Ij}^{(h)}(\otimes)=\frac{\min_I\{y_{Ij}^{(h)}(\otimes)\}}{y_{Ij}^{(h)}(\otimes)} \tag{7}$$

So the normalized combinatorial decision matrix is obtained as follows:

$$Z^{(h)}(\otimes)=\left(z_{Ij}^{(h)}(\otimes)\right)_{(q_1+\cdots+q_n)\times m} \tag{8}$$

The attributes are weighted to obtain a new decision matrix as follows:

$$X^{(h)}(\otimes)=\left(x_I^{(h)}(\otimes)\right)_{(q_1+\cdots+q_n)\times 1},$$

$$x_I^{(h)}(\otimes)=\sum_{j=1}^{m}\omega_j z_{Ij}^{(h)}(\otimes) \tag{9}$$

So the comprehensive decision matrix in the combined state can be expressed as below

$$X(\otimes)=\left(X^{(1)}(\otimes),\cdots,X^{(h)}(\otimes),\cdots X^{(l^n)}(\otimes)\right) = (x_{IJ}(\otimes))_{(q_1+\cdots+q_n)\times l^n} \tag{10}$$

Then the regret aversion of the decision maker is considered by the formula (3).

The power function

$$v(x)=x^{\alpha}(0<\alpha<1) \tag{11}$$

is used as the utility function of the attribute according to Tversky and Kahneman (1992). α is named as the risk aversion coefficient. The smaller the value of α is, the greater the risk aversion degree of the decision-maker is. On basis of the comprehensive decision matrix, the grey comprehensive perceived utility value of scheme I under the state S_J can be obtained:

$$U_I=\sum_{J=1}^{l^n}P_J[x_{IJ}(\otimes)^{\alpha}-\theta_1 R(\Delta_1(\otimes))-\theta_2 R(\Delta_2(\otimes))] \tag{12}$$

According to Carlos and Elke(2008),

$$R(\Delta)=\begin{cases}\lambda^{\Delta}-1,\Delta\geq 0\\ 0,\Delta<0\end{cases}, \tag{13}$$

$1.15\leq\lambda\leq 1.35$

is taken as the regret function.

$$\Delta_1(\otimes)=\max_{I'\in\Omega_1}X_{I'J}(\otimes)-X_{IJ}(\otimes) \tag{14}$$

$$\Delta_2(\otimes)=\max_{I'\in\Omega_2}X_{I'J}(\otimes)-X_{IJ}(\otimes) \tag{15}$$

$\Omega_2=\Omega_I \quad \Omega_1=\Omega-\Omega_I$

Ω is the set of the first index of all sub scheme sets under all parent schemes. Ω_I is the set of the first index of the sub scheme sets under the parent scheme to which the sub scheme with subscript I belongs.

According to the calculation results, the optimal sub scheme $A_{ij_i^*}$ can be found under each parent scheme A_i, thus producing the equilibrium set $D^*=\{A_{1j_1^*},\cdots,A_{ij_i^*},\cdots A_{nj_n^*}\}$.

Next, the grey comprehensive perceived utility value of each $A_{ij_i^*}$ is calculated on basis of (12). Here

$$\Delta_1(\otimes)=max\{X_{A_{1j_1^*}}(\otimes),\cdots,X_{A_{(i-1)j_{i-1}^*}}(\otimes), X_{A_{(i+1)j_{i+1}^*}}(\otimes),\cdots,X_{A_{1j_1^*}}(\otimes)\}-X_{A_{ij_i^*}}(\otimes) \tag{16}$$

$$\Delta_2(\otimes)=0 \tag{17}$$

4 CASE STUDY

A multinational enterprise wants to invest and develop new products abroad. After the preliminary investigation of the team, the sales volume C_1, market share C_2 and development investment cost C_3 are determined as the investigation attributes, and the two cities A_{11}、A_{12}in country A_1 and the three cities A_{21}、A_{22}、A_{23} in country A_2 are identified as candidates. Although different countries may face different future market conditions, they can be divided into "good", "medium" and "poor", which is shown in Table.2. Due to the complexity and uncertainty of decision-making environment information and the subjective cognition of decision-makers, the attribute performance values of these five cities are represented by general grey numbers, as shown in the Table.3. Now we want to select the best investment city according to the above conditions and considering the regret avoidance of decision-makers.

Table 2: The natural state of the two countries

	$S^{(1)}$	$S^{(2)}$	$S^{(3)}$
A_1	0.3	0.4	0.3
A_2	0.45	0.35	0.2

Table 3: Attribute value of five cities in two countries under the "good" natural state

	C_1	C_2	C_3
A_{11}	$[400,500]$	$[0.3,0.4]\cup[0.42,0.48]$	$[75,85]\cup[91,95]$
A_{12}	$[350,400]\cup[404,410]$	$[0.1,0.2]\cup\{0.3\}$	$[60,70]\cup[72,76]$
A_{21}	$[420,440]\cup[480,500]$	$[0.1,0.2]\cup\{0.3\}$	$[70,80]\cup\{85\}$
A_{22}	$[420,450]$	$[0.3,0.4]$	$[80,90]\cup[95,97]$
A_{23}	$[330,340]\cup[350,380]$	$[0.2,0.3]\cup\{0.35\}$	$[70,76]$

Table 4: Attribute value of five cities in two countries under the "medium" natural state.

	C_1	C_2	C_3
A_{11}	$[380,400]\cup\{420\}$	$\{0.3\}\cup[0.36,0.42]$	$[75,80]\cup[85,90]$
A_{12}	$[320,380]\cup[390,400]$	$[0.1,0.2]\cup\{0.3\}$	$[65,75]\cup\{80\}$
A_{21}	$[400,420]\cup\{430\}$	$[0.4,0.6]\cup\{0.65\}$	$[80,85]\cup\{90\}$
A_{22}	$[380,400]$	$[0.55,0.65]$	$[75,80]$
A_{23}	$[340,360]\cup\{375\}$	$\{0.2\}\cup[0.3,0.4]$	$[75,80]\cup\{85\}$

Table 5: Attribute value of five cities in two countries under the "poor" natural state.

	C_1	C_2	C_3
A_{11}	$[350,380]\cup[400,420]$	$\{0.2\}\cup[0.25,0.35]$	$[75,85]\cup\{90\}$
A_{12}	$[300,350]\cup\{370\}$	$[0.3,0.4]$	$\{75\}\cup[80,85]$
A_{21}	$[380,420]$	$[0.3,0.5]$	$[85,90]$
A_{22}	$[400,410]\cup\{420\}$	$[0.3,0.5]$	$[80,85]\cup\{87\}$
A_{23}	$[340,350]\cup[360,380]$	$\{0.5\}\cup[0.6,0.7]$	$[70,80]\cup\{82\}$

Step1 The fields of sales volume, market share and investment cost are determined as $[300,500]$, $[0,1]$, $[50,100]$ respectively. The simplified form of the general grey number (GGN) can be calculated and obtained by definition 1,2. Thus the decision matrixes of the two countries in three states "good", "medium" and "poor" can be obtained as follows:

$$A_1^{(1)}(\otimes)=\begin{pmatrix} 425_{0.25} & 0.4_{0.155} & 86.5_{0.271} \\ 391_{0.27} & 0.225_{0.067} & 69.5_{0.272} \end{pmatrix}$$

$$A_2^{(1)}(\otimes)=\begin{pmatrix} 460_{0.2} & 0.225_{0.067} & 80_{0.188} \\ 435_{0.15} & 0.35_{0.1} & 90.5_{0.23} \\ 350_{0.204} & 0.3_{0.083} & 73_{0.12} \end{pmatrix}$$

$$A_1^{(2)}(\otimes)=\begin{pmatrix} 405_{0.096} & 0.345_{0.068} & 82.5_{0.2} \\ 372.5_{0.335} & 0.225_{0.067} & 75_{0.187} \end{pmatrix}$$

$$A_2^{(2)}(\otimes)=\begin{pmatrix} 420_{0.098} & 0.575_{0.174} & 86.25_{0.096} \\ 390_{0.1} & 0.6_{0.1} & 77.5_{0.1} \\ 362.5_{0.097} & 0.275_{0.127} & 81.25_{0.095} \end{pmatrix}$$

$$A_1^{(3)}(\otimes)=\begin{pmatrix} 387.5_{0.247} & 0.25_{0.12} & 85_{0.188} \\ 347.5_{0.234} & 0.35_{0.1} & 78.75_{0.105} \end{pmatrix}$$

$$A_2^{(3)}(\otimes)=\begin{pmatrix} 400_{0.2} & 0.575_{0.113} & 87.5_{0.1} \\ 412.5_{0.049} & 0.4_{0.2} & 84.75_{0.097} \\ 357.5_{0.152} & 0.4_{0.2} & 78.5_{0.191} \end{pmatrix}$$

Step2 It's clear there are 9 combined states. The set of the combined states is $S^* = \{S_1, \cdots, S_h, \cdots, S_9\}$. And the corresponding probability vector can be calculated on basis of (4).

$$\vec{P} = (P_1, \cdots, P_h, \cdots, P_9)$$
$$=(0.135, 0.105, 0.06, 0.18, 0.14, 0.08, 0.135, 0.105, 0.06)$$

Step3 The sub schemes in the second stage are matrix combined under the combination state according to (5).

The decision matrix under the combined state S_1 ("good"+"good") is

$$Y^{(1)}(\otimes) = \begin{pmatrix} A_1^{(1)}(\otimes) \\ A_2^{(1)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(1)}(\otimes)\right)_{5\times3}$$

The decision matrix under the combined state S_2 ("good"+"medium") is

$$Y^{(2)}(\otimes) = \begin{pmatrix} A_1^{(1)}(\otimes) \\ A_2^{(2)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(2)}(\otimes)\right)_{5\times3}$$

The decision matrix under the combined state S_3 ("good"+"poor") is

$$Y^{(3)}(\otimes) = \begin{pmatrix} A_1^{(1)}(\otimes) \\ A_2^{(3)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(3)}(\otimes)\right)_{5\times3}$$

S_4（"medium"+"good"）:

$$Y^{(4)}(\otimes) = \begin{pmatrix} A_1^{(2)}(\otimes) \\ A_2^{(1)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(4)}(\otimes)\right)_{5\times3}$$

S_5（"medium"+"medium"）:

$$Y^{(5)}(\otimes) = \begin{pmatrix} A_1^{(2)}(\otimes) \\ A_2^{(2)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(5)}(\otimes)\right)_{5\times3}$$

S_6（"medium"+"poor"）:

$$Y^{(6)}(\otimes) = \begin{pmatrix} A_1^{(2)}(\otimes) \\ A_2^{(3)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(6)}(\otimes)\right)_{5\times3}$$

S_7（"poor"+"good"）:

$$Y^{(7)}(\otimes) = \begin{pmatrix} A_1^{(3)}(\otimes) \\ A_2^{(1)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(7)}(\otimes)\right)_{5\times3}$$

S_8（"poor"+"medium"）:

$$Y^{(8)}(\otimes) = \begin{pmatrix} A_1^{(3)}(\otimes) \\ A_2^{(2)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(8)}(\otimes)\right)_{5\times3}$$

S_9（"poor"+"poor"）:

$$Y^{(9)}(\otimes) = \begin{pmatrix} A_1^{(3)}(\otimes) \\ A_2^{(3)}(\otimes) \end{pmatrix} \triangleq \left(y_{Ij}^{(9)}(\otimes)\right)_{5\times3}$$

Step4 As the beneficial attribute, the sales volume $C_1$and the market share$C_2$are normalized by (6). As the cost attribute, development investment cost $C_3$is normalized by (7). The normalized decision matrix under each state is obtained by (8) as follows:

$$Z^{(h)}(\otimes) = \left(z_{Ij}^{(h)}(\otimes)\right)_{5\times3}, \text{ here } h = 1, \cdots, 9$$

Step5 The attributes are weighted to obtain a new decision matrix under each state by (9) as follows:

$$X^{(h)}(\otimes) = \left(x_I^{(h)}(\otimes)\right)_{5\times1}, \quad \text{here} \quad h = 1, \cdots, 9,$$

$$x_I^{(h)}(\otimes) = \sum_{j=1}^{m} \omega_j z_{Ij}^{(h)}(\otimes).$$

Thus the grey comprehensive decision matrix on basis of (10) is

$$X(\otimes) = \left(X^{(1)}(\otimes), \cdots, X^{(h)}(\otimes), \cdots, X^{(9)}(\otimes)\right) = \left(x_{IJ}(\otimes)\right)_{5\times9}$$

as shown in Table.6.

Table 6: Grey Decision Matrix.

	S_1 0.135	S_2 0.105	S_3 0.06	S_4 0.18	S_5 0.14	S_6 0.08	S_7 0.135	S_8 0.105	S_9 0.06
$X_1(A_{11})$	$0.911_{0.272}$	$0.85_{0.272}$	$0.85_{0.272}$	$0.913_{0.2}$	$0.831_{0.2}$	$0.845_{0.2}$	$0.809_{0.247}$	$0.768_{0.247}$	$0.783_{0.247}$
$X_2(A_{12})$	$0.809_{0.272}$	$0.785_{0.272}$	$0.785_{0.272}$	$0.809_{0.335}$	$0.767_{0.335}$	$0.779_{0.335}$	$0.88_{0.234}$	$0.801_{0.234}$	$0.819_{0.234}$
$X_3(A_{21})$	$0.829_{0.272}$	$0.937_{0.272}$	$0.915_{0.272}$	$0.867_{0.2}$	$0.948_{0.187}$	$0.945_{0.2}$	$0.867_{0.2}$	$0.957_{0.174}$	$0.957_{0.2}$
$X_4(A_{22})$	$0.871_{0.272}$	$0.949_{0.272}$	$0.843_{0.272}$	$0.92_{0.23}$	$0.962_{0.187}$	$0.874_{0.2}$	$0.92_{0.23}$	$0.971_{0.1}$	$0.887_{0.2}$
$X_5(A_{23})$	$0.815_{0.272}$	$0.741_{0.272}$	$0.811_{0.272}$	$0.861_{0.204}$	$0.76_{0.187}$	$0.842_{0.2}$	$0.861_{0.204}$	$0.769_{0.127}$	$0.855_{0.2}$

Step6 According to Tversky and Kahneman(1992) and Chorus(2010), the coefficient of risk aversion of (13) is set as α=0.88 , and λ=1.25 , and θ_1=0.7, θ_2=0.3.

According to (12)(13)(14)(15), if I=1or 2, the grey comprehensive perceived utility value of the scheme can be obtained by the formula

$$U_I = \sum_{J=1}^{9} P_J\,[x_{IJ}(\otimes)^{\alpha} - \theta_1 R(\Delta_1(\overset{I=3,\,4,\,5}{max}\;x_{IJ}(\otimes)$$

$$-x_{IJ}(\otimes)) - \theta_2 R(\overset{I=1,\,2}{max}\,x_{IJ}(\otimes) - x_{IJ}(\otimes))]$$

Thus $U_1 = 0.85002_{0.272}$, $U_2 = 0.80334_{0.272}$.

According to (12)(13)(14)(15), if I=3 or 4 or 5, the grey comprehensive perceived utility value of the scheme can be obtained by the formula

$$U_I = \sum_{J=1}^{9} P_J\,[x_{IJ}(\otimes)^{\alpha} - \theta_1 R(\Delta_1(\overset{I=1,\,2}{max} x_{IJ}(\otimes$$

$$-x_{IJ}(\otimes)) - \theta_2 R(\overset{I=3,\,4,\,5}{max}\;x_{IJ}(\otimes) - x_{IJ}(\otimes))]$$

Thus $U_3 = 0.91022_{0.272}$, $U_4 = 0.924997_{0.272}$, $U_5 = 0.81751_{0.272}$.

Step7 The equilibrium set is $D^* = \{X_1, X_4\}$ according to the above results. Then calculate the grey comprehensive perceived utility values of the scheme X_1, X_4 on basis of (12)(13)(16)(17) .

$$U_{X_1} = \sum_{J=1}^{9} P_J\,[x_{1J}(\otimes)^{\alpha} - \theta_1 R(x_{4J}(\otimes) - x_{1J}(\otimes)) - \theta_2 R(x_{1J}(\otimes) - x_{1J}(\otimes))]$$
$$= 0.853_{0.272}$$

$$U_{X_4} = \sum_{J=1}^{9} P_J\,[x_{4J}(\otimes)^{\alpha} - \theta_1 R(x_{1J}(\otimes) - x_{4J}(\otimes)) - \theta_2 R(x_{4J}(\otimes) - x_{4J}(\otimes))]$$
$$= 0.926_{0.272}$$

So the best investment city is the city A_{22} of the country A_2 .

5 CONCLUSIONS

Aiming at the multi-stage risk multi-attribute decision-making problem, this paper proposes a decision-making method based on general grey number information and dynamic regret theory. Considering the impact of regret avoidance psychology on decision-making results, the method calculates the probability of the combined market state and constructs the grey regret function as well as grey comprehensive perceived utility function on basis of the dynamic regret theory. The set of regret equilibrium solutions is established according to the value of grey perceived utility function, and then the optimal scheme is selected by further comparing the comprehensive utility value.

The method model proposed in this paper not only considers the psychological and behavioral state of regret avoidance of decision makers in the dynamic decision-making process, but also takes into account the uncertainty and gray of the dynamic environment. Compared with the traditional risky multi-attribute decision-making method, it is closer to reality and has stronger operability and practicability, which provides method support and theoretical guidance for more uncertain decision-making problems in reality. At the same time, this paper also broadens the research scope and application space of regret theory and dynamic regret theory.

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