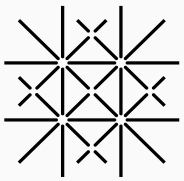
A Comparison of Abstraction Heuristics for Rubik's Cube

Clemens Büchner, Patrick Ferber, Jendrik Seipp, Malte Helmert

June 16, 2022



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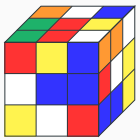
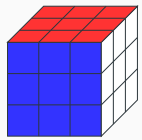
Introduction





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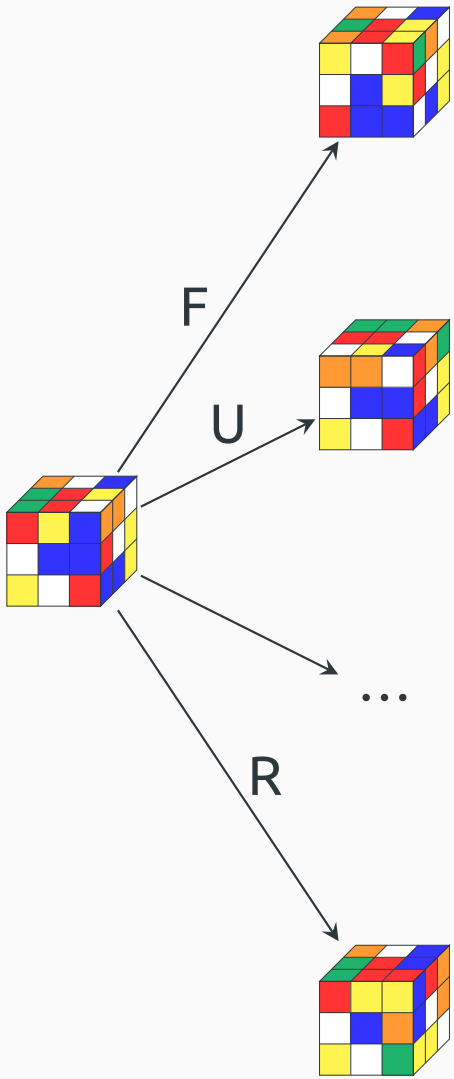

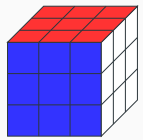

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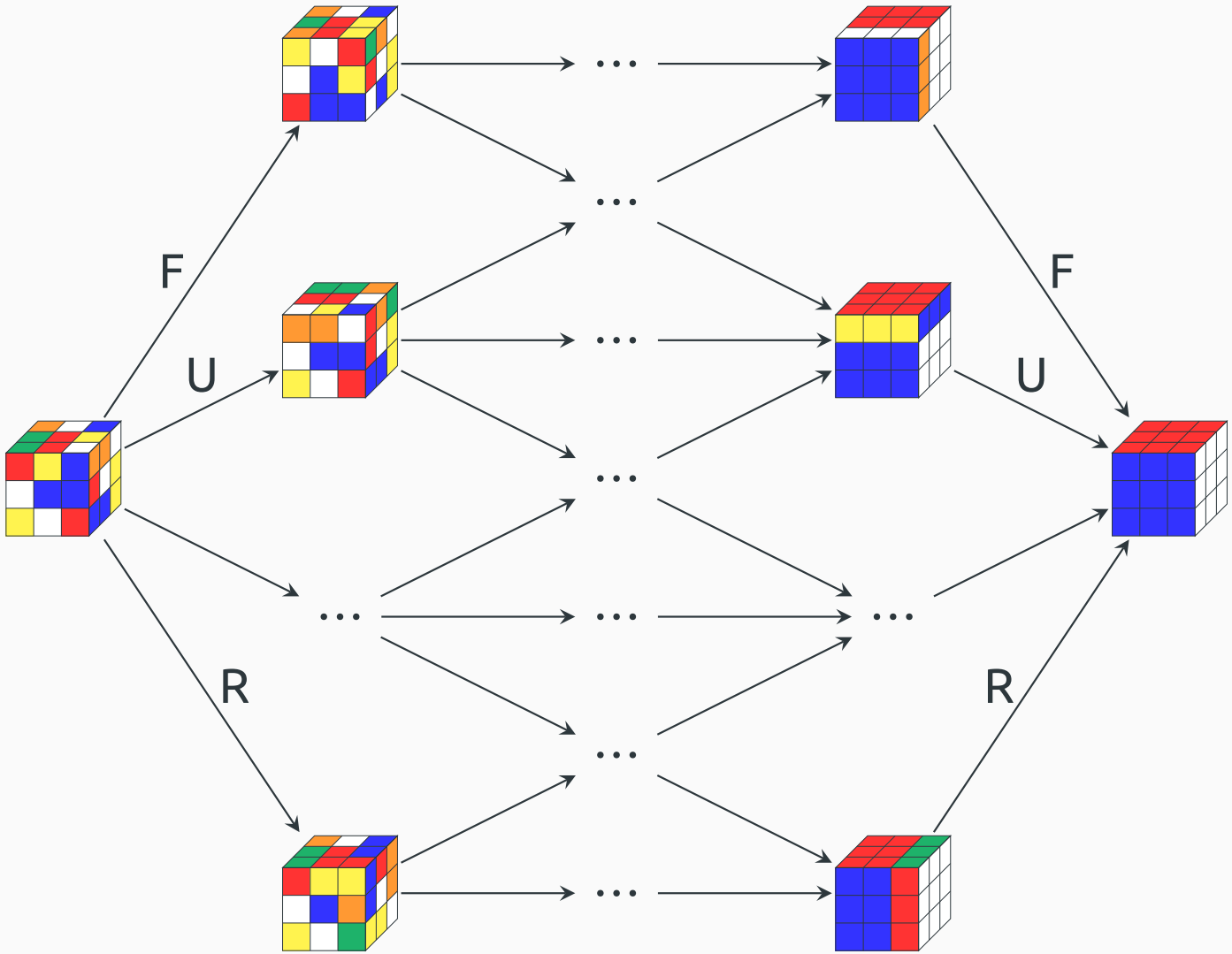

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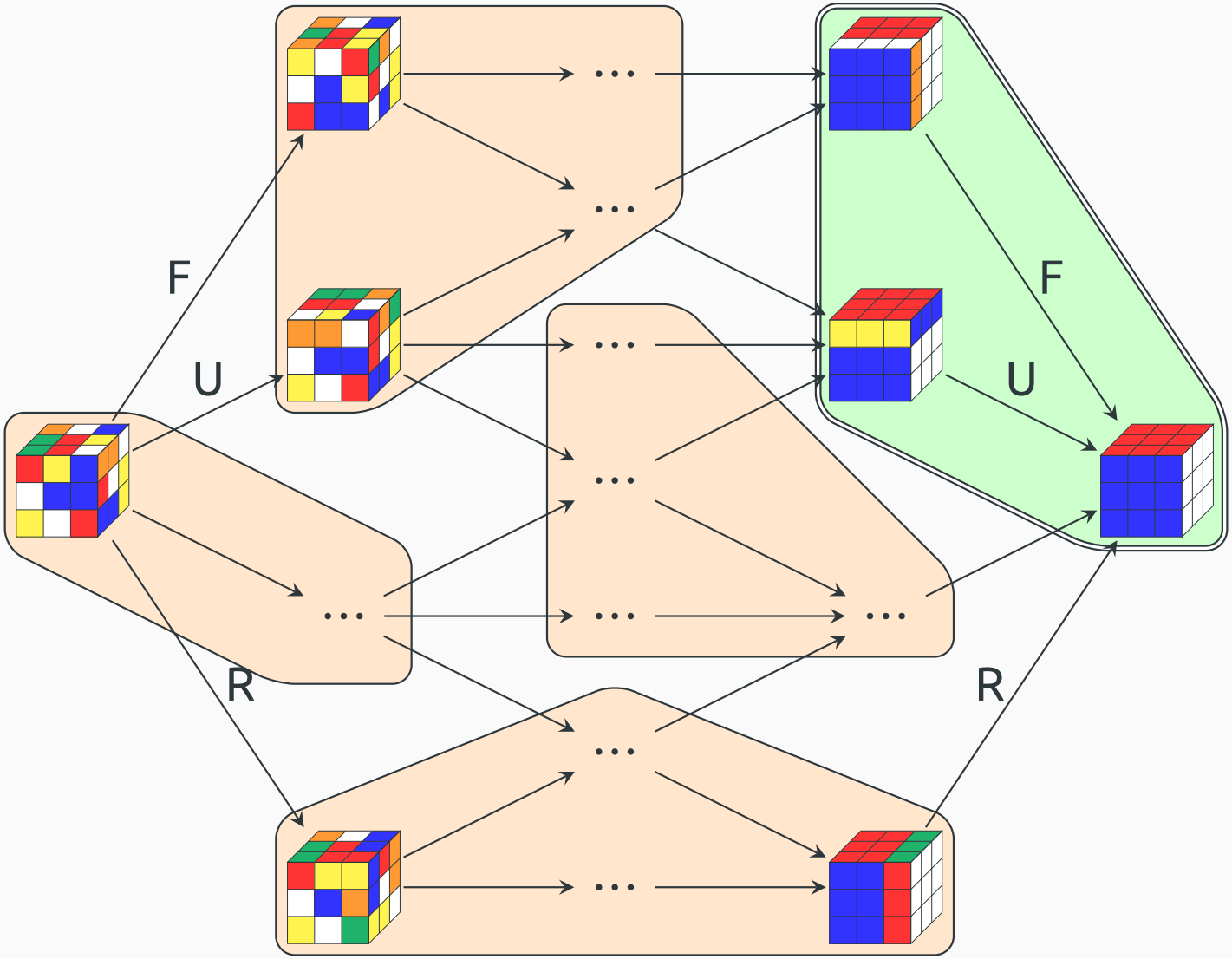


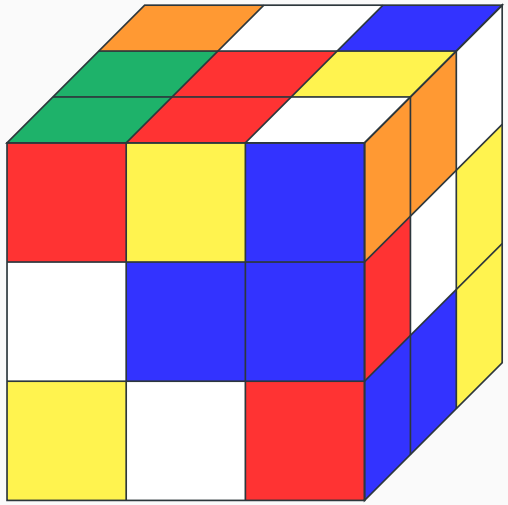
Contributions

- model of Rubik's Cube for general problem solvers
- generate Cartesian abstractions with conditional effects
- evaluate modern abstraction heuristics on Rubik's Cube

Rubik's Cube Model

20 variables, one per corner and edge

- domains $=$ positions $\times$ orientations



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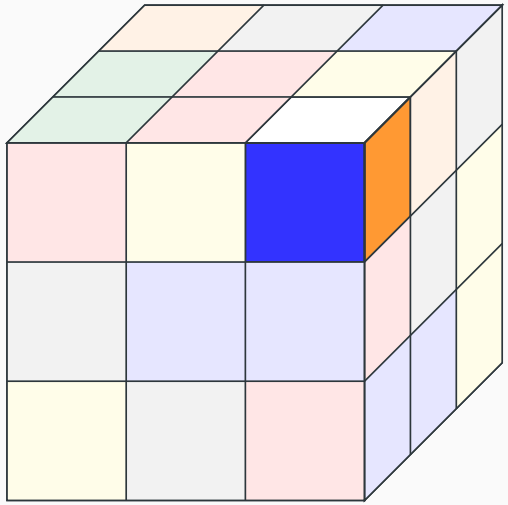

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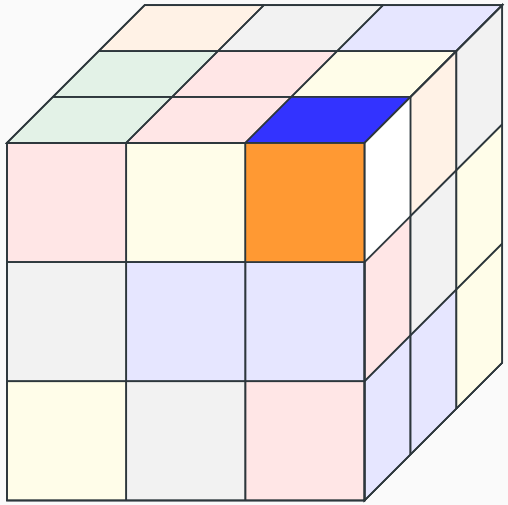

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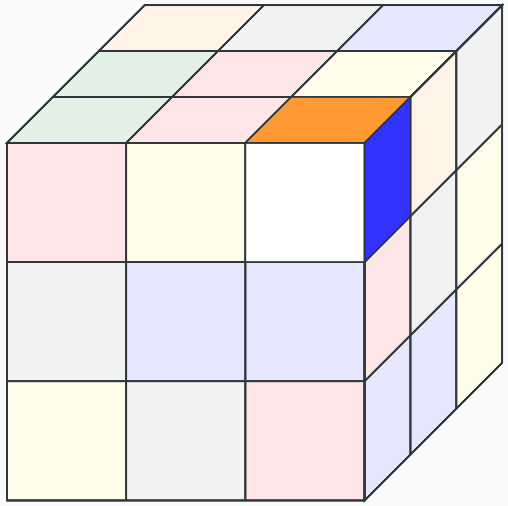


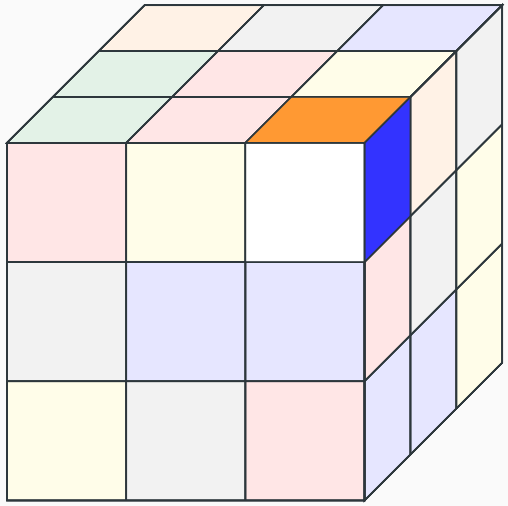
Rubik's Cube Model

20 variables, one per corner and edge

- domains = positions × orientations

18 operators, three per face

- no preconditions
- conditional effect for each cubie at each position in each orientation



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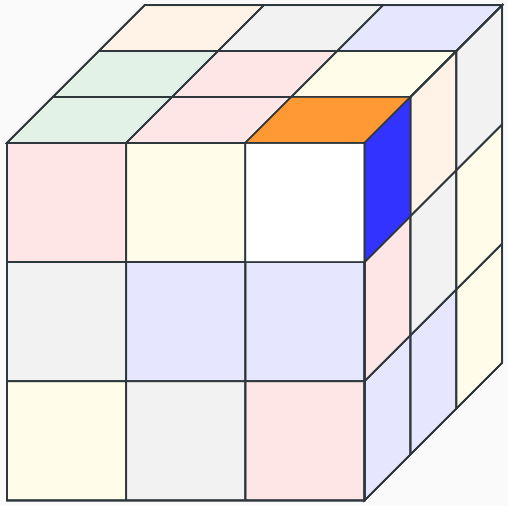
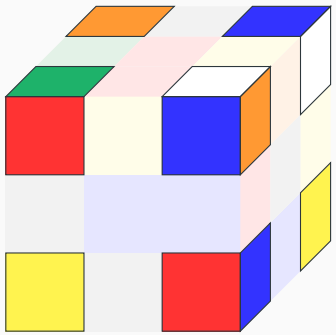
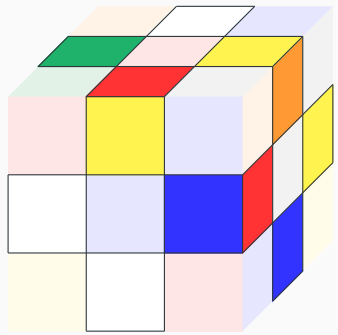
$\approx 4.3 \cdot 10^{19}$ reachable states



Solving Rubik's Cube Optimally

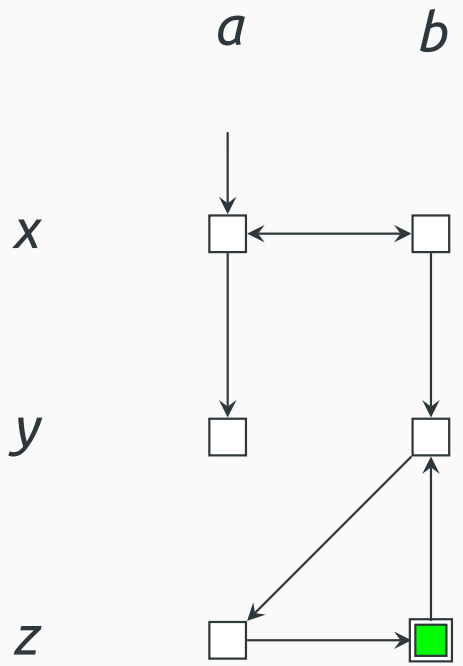
state of the art: pattern database heuristics (Korf 1997)





- split variables to obtain two simpler problems
- compute all goal distances in simplified problems
- use maximum as admissible heuristic in IDA* search

Overview of Abstraction Heuristics



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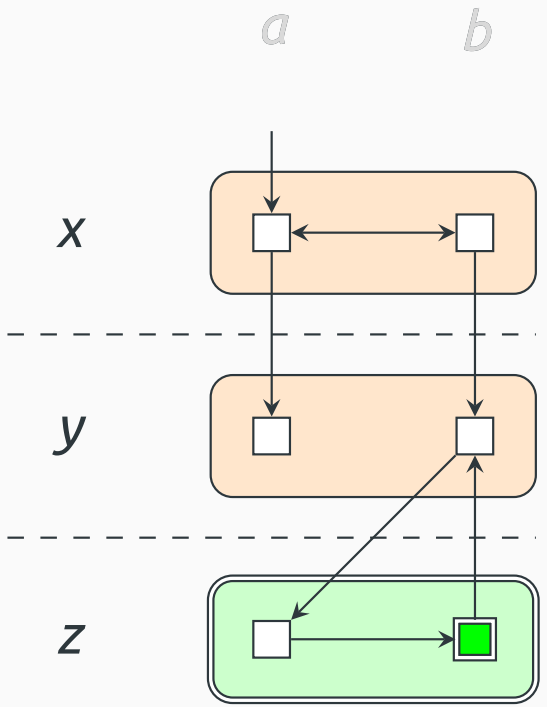
projections $\rightarrow$ PDBs

(Culberson and Schaeffer 1998)



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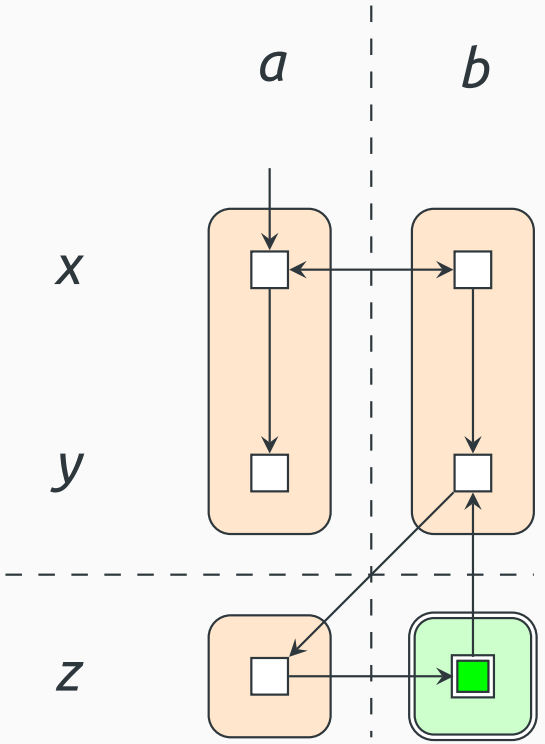

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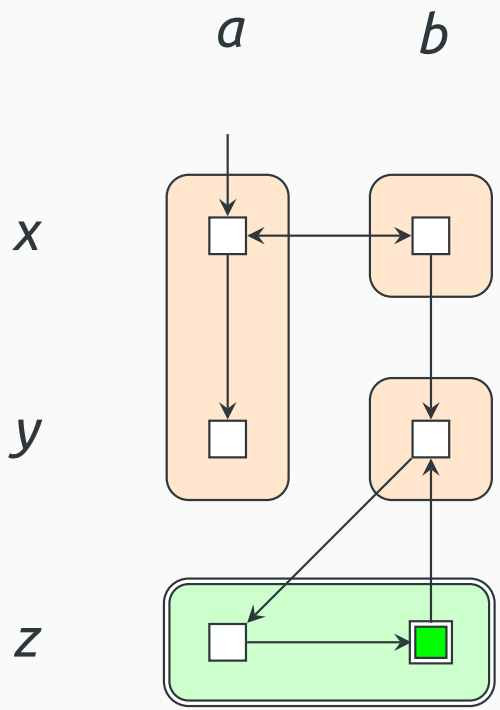

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merge-and-shrink abstractions
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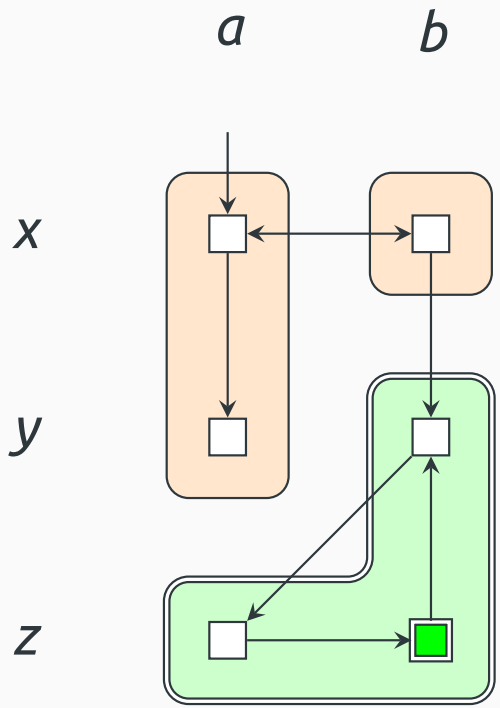

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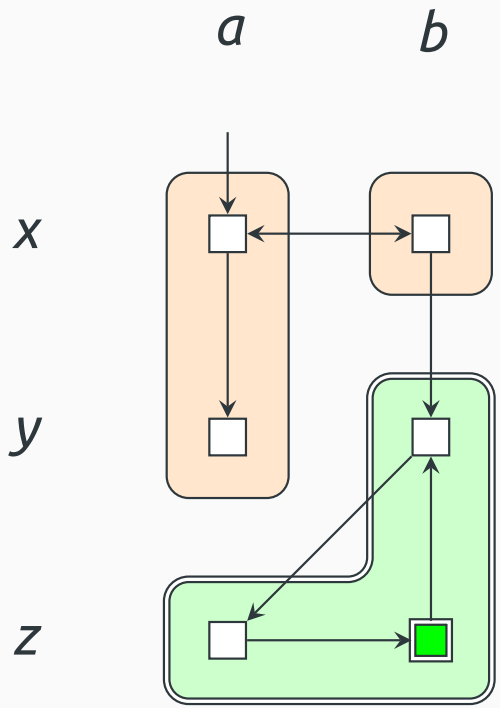
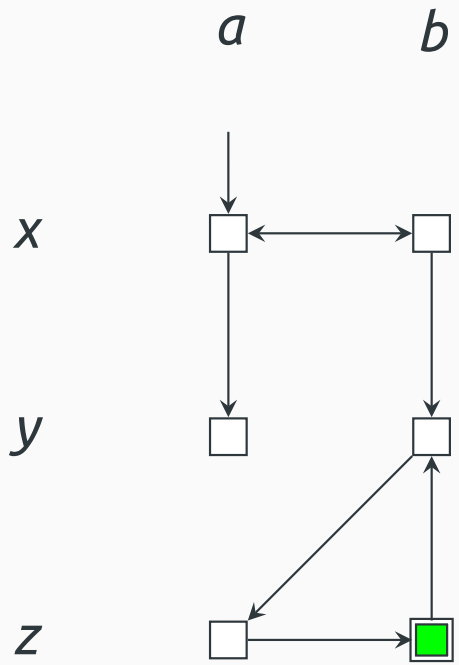


Cartesian CEGAR

Counterexample-Guided Cartesian Abstraction Refinement

Start with coarsest abstraction and iterate:

- find abstract plan
- execute in original
- if fails: fix flaw and repeat
- else: return solution



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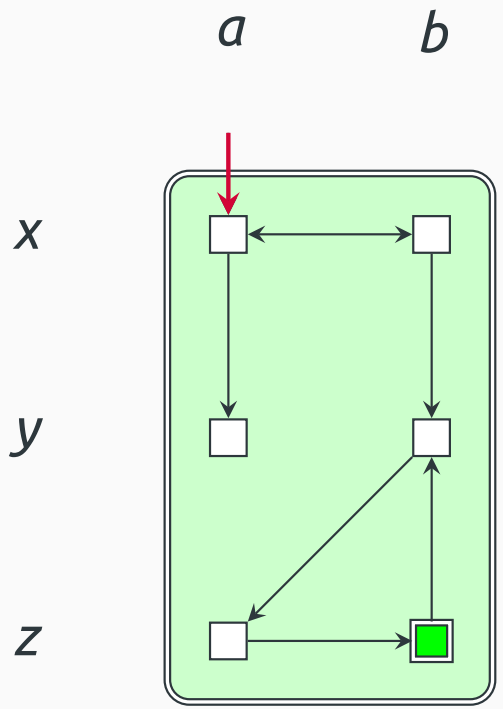

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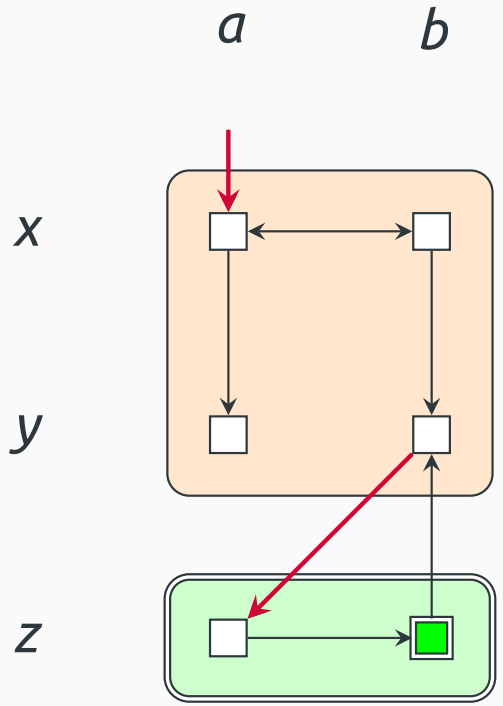

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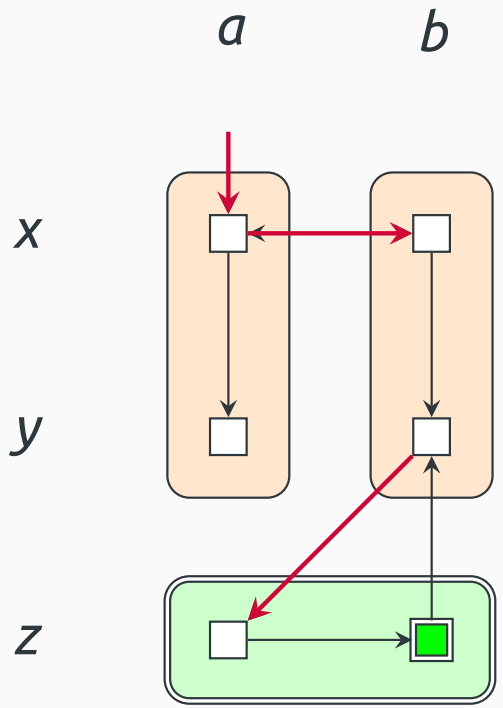

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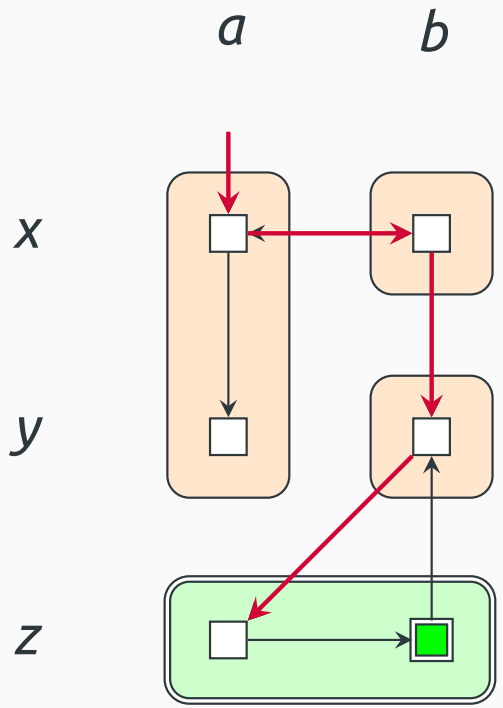

Cartesian CEGAR and Factored Effect Tasks

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Regression through operators with general conditional effects is not Cartesian.

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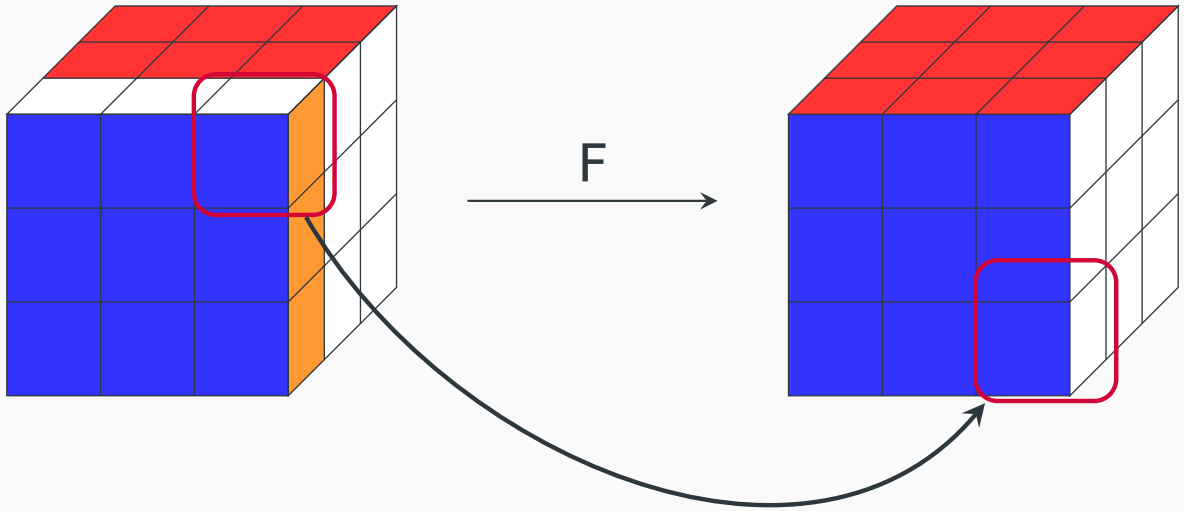
Special case Rubik's Cube: new position and orientation of cubie depends only on old position and orientation of itself.



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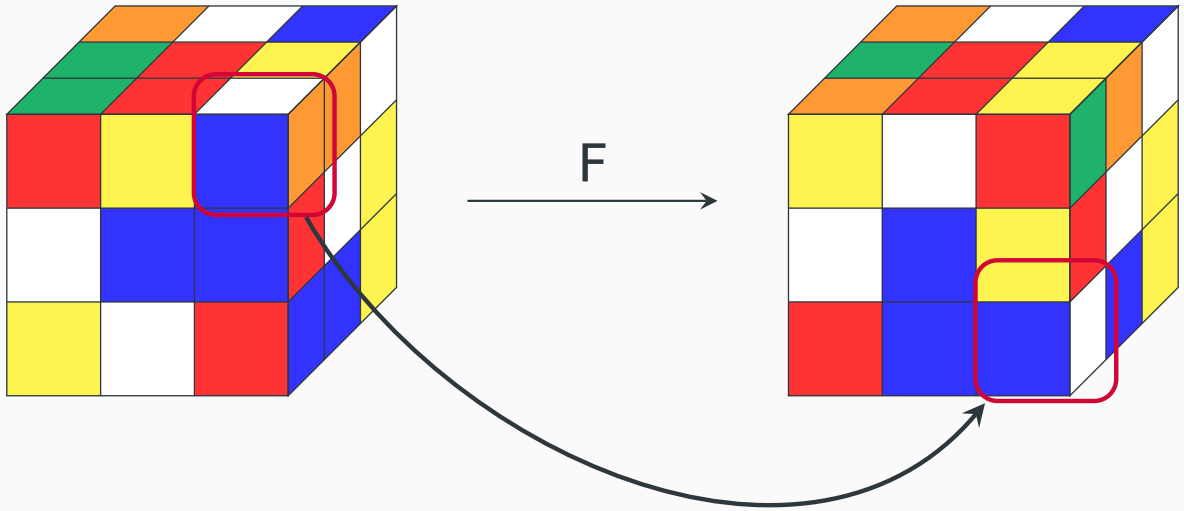

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Definition (Factored Effect Tasks)

A factored effect operator specifies in each effect condition exactly the variable changed by the effect.
A factored effect task has exclusively factored effect operators.

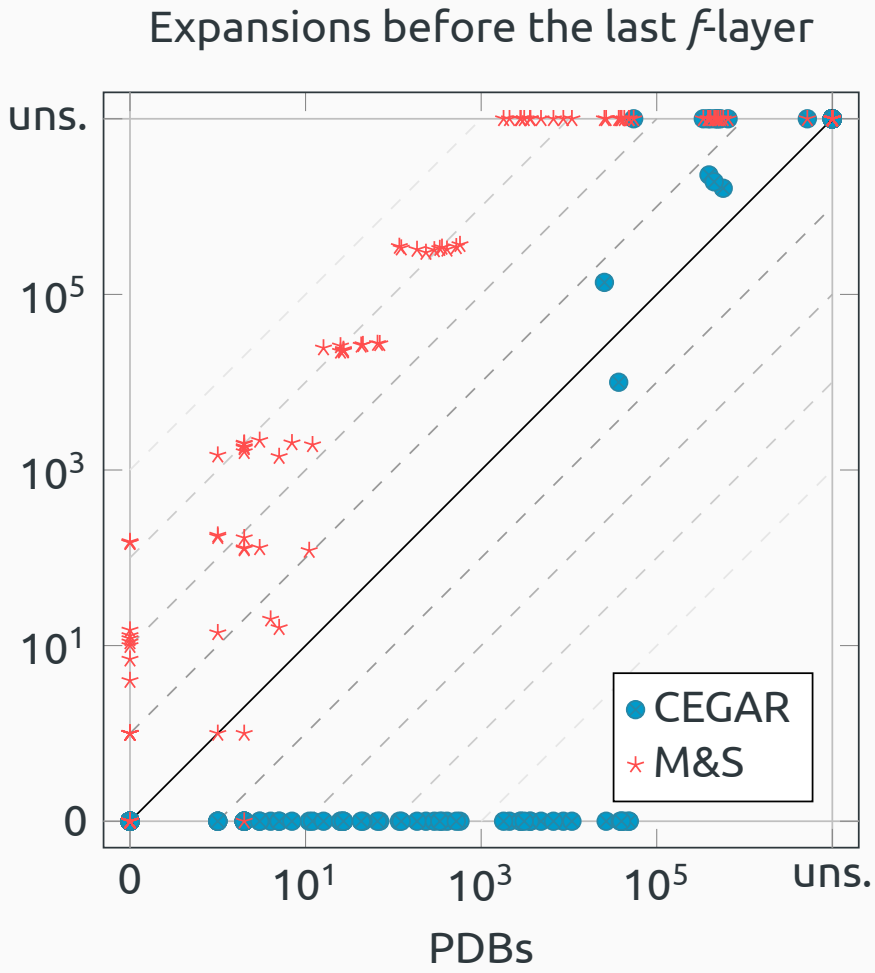
Theorem

Regression through factored effect operators is Cartesian.

Experiments

200 Rubik's Cube problems of increasing difficulty

	coverage
PDBs	123
CEGAR	113
M&S	90



Summary

- new Rubik's Cube benchmarks for planning in SAS$^+$
- Cartesian CEGAR adapted for factored effect tasks
- CEGAR yields perfect heuristic up to certain difficulty
- yet, fewer tasks solved by more general abstractions

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Future work:

- consider more permutation domains for analysis
- extend with domain abstractions