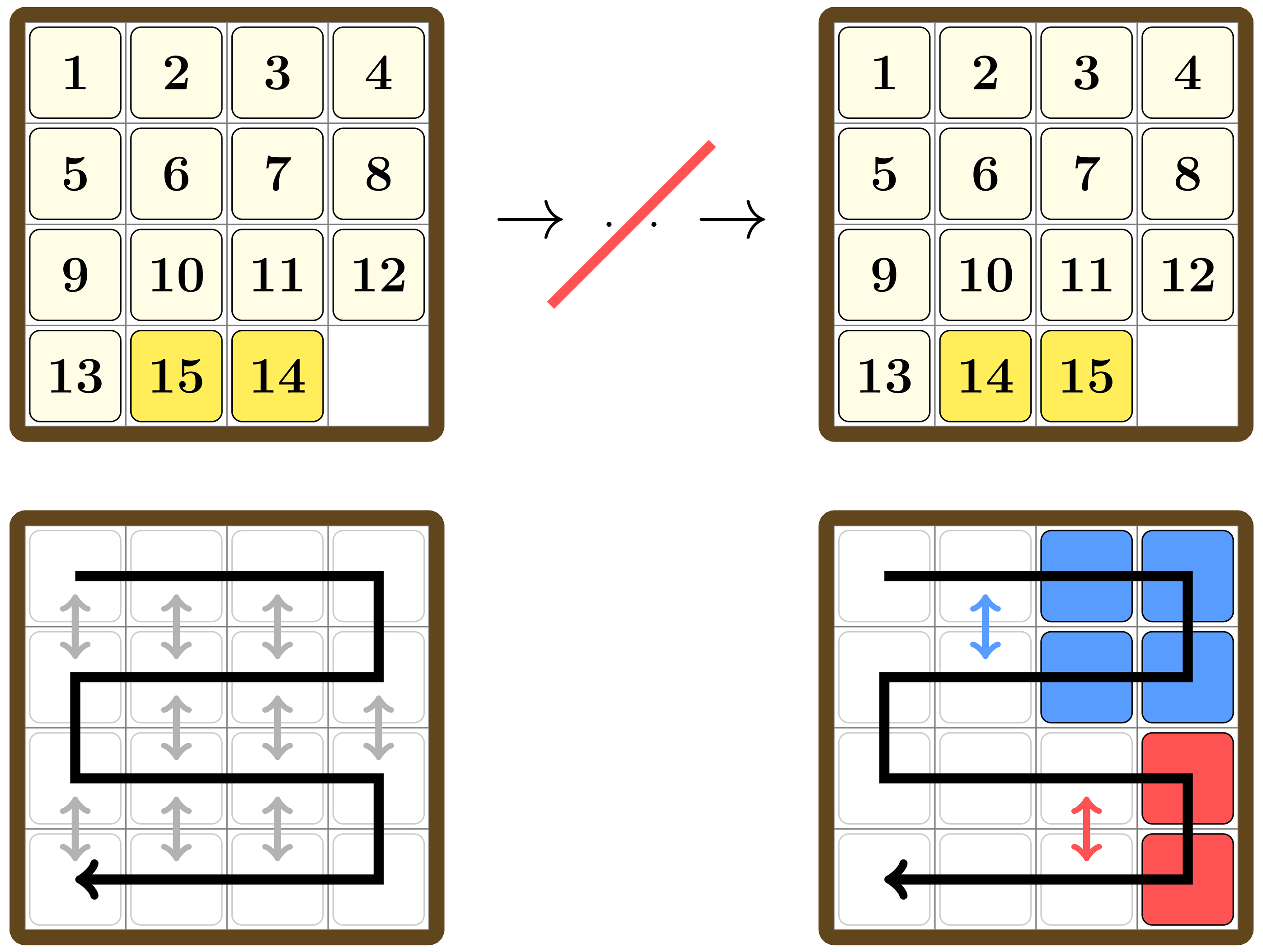
Separating functions can capture various unsolvability arguments.

General Idea



- every move **preserves** the **parity** of the number of incorrectly ordered pairs
- **initial** state has **1** incorrectly ordered pair
- **goal** state has **0** incorrectly ordered pairs

Separating Function Example

A function $f : S \to \{\mathbf{E}, \mathbf{O}\}$ such that

- $f(s_{\text{init}}) \neq f(s_{\text{goal}})$
- $f(s) = f(s')$ for all transitions $s \to s'$

codomain: $\mathbb{F}_2, \mathbb{F}_3, \mathbb{R}, \ldots$

relation: $>$ and $\leq, \ldots$

condition: for all transitions $s \to s'$ reachable from $s_{\text{init}}, \ldots$

Potential Functions

Constraints

$$\sum_{f \in \mathcal{F}} w(f)[s_{\text{init}} \models f] \neq \sum_{f \in \mathcal{F}} w(f)[s_{\text{goal}} \models f]$$

$$\sum_{f \in \mathcal{F}} w(f)[s \models f] = \sum_{f \in \mathcal{F}} w(f)[s' \models f]$$

for all transitions $s \to s'$

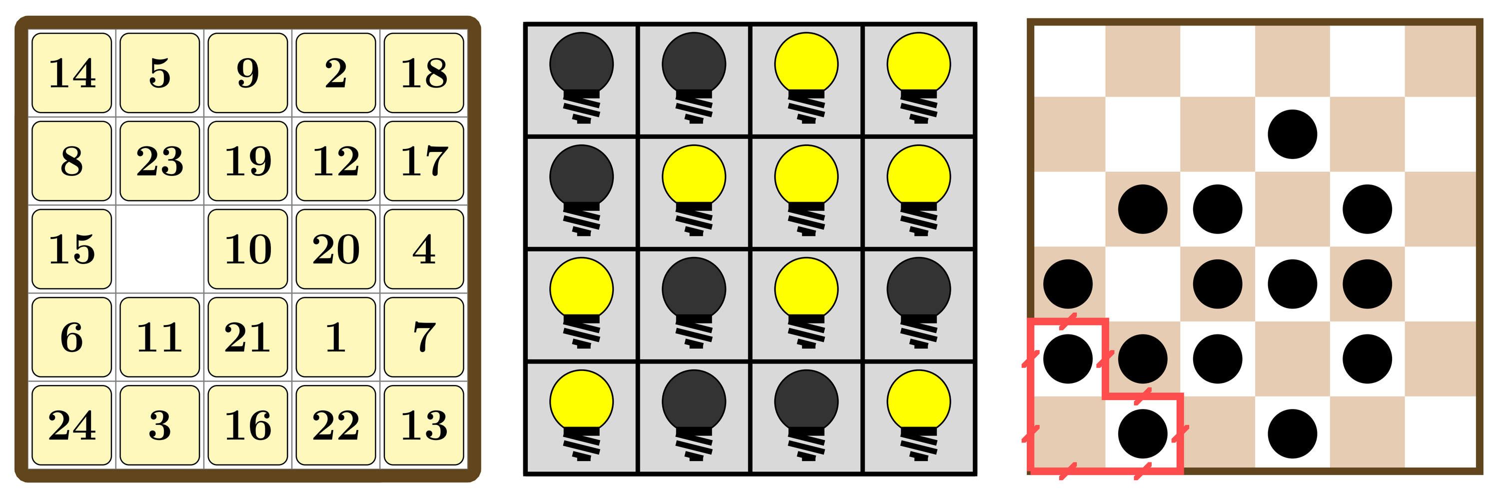
- constraints **satisfiable** $\Rightarrow$ task unsolvable
- **compact** representation for one- and two-dimensional features

Efficient Satisfiability Checks

$\langle \mathbb{F}_2, = \rangle \Rightarrow$ XOR-constraints $\Rightarrow$ Gaussian elimination

$\langle \mathbb{R}, \leq \rangle \Rightarrow$ linear inequalities $\Rightarrow$ LP-solver

Results



- detects unsolvability in **all instances** of `sliding-tiles`, `lights-out`, and `chessboard-pebbling`

Detecting Unsolvability Based on Separating Functions

Remo Christen, Salomé Eriksson, Florian Pommerening, and Malte Helmert

Link to paper

University of Basel