# Novelty vs. Potential Heuristics: A Comparison of Hardness Measures for Satisficing Planning

**Simon Dold** Malte Helmert

University of Basel

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# Planning

SAS$^+$ Planning Task $\Pi = \langle V, I, O, \gamma \rangle$

- state variables $V$ with finite domain
- initial state $I$
- operators $O$
- goal $\gamma$

Satisficing planning ignores plan cost.

# Planning

- PSPACE hard in general
- Some tasks are easier than others
- Use measure to quantify hardness

# Planning

Task induces a directed graph called state space

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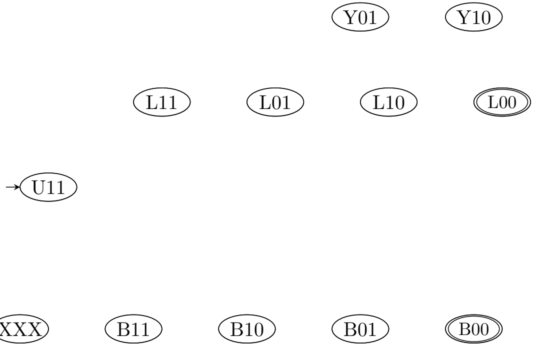
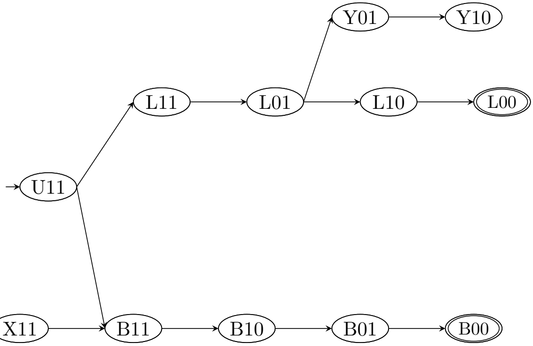
- Nodes correspond to states



# Planning

Task induces a directed graph called state space

- Nodes correspond to states
- Arcs correspond to operators



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A heuristic $h$ assigns a value to each state.

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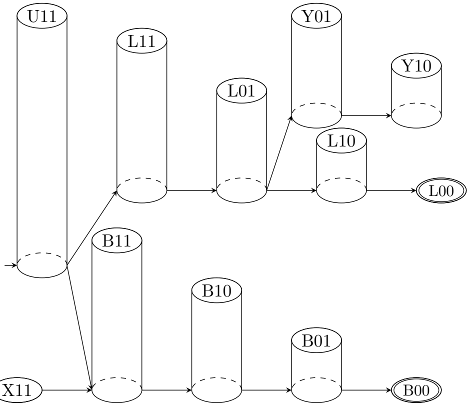
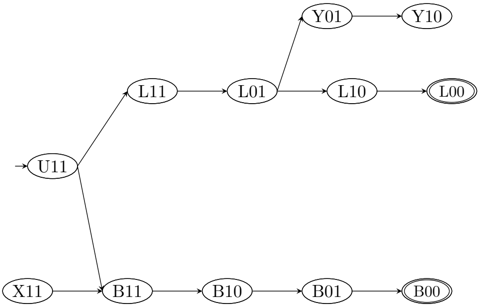


# Heuristic Properties

- alive state: reachable and solvable



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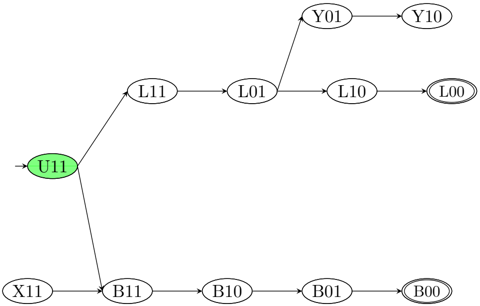

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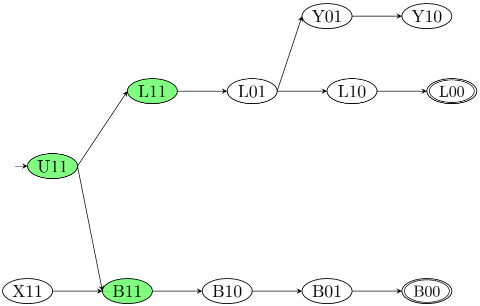

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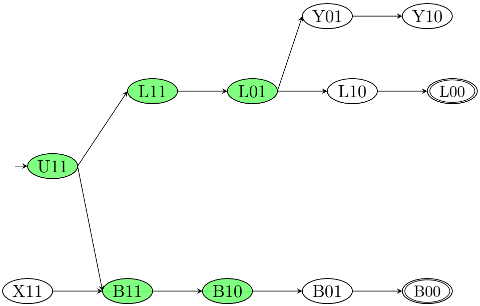

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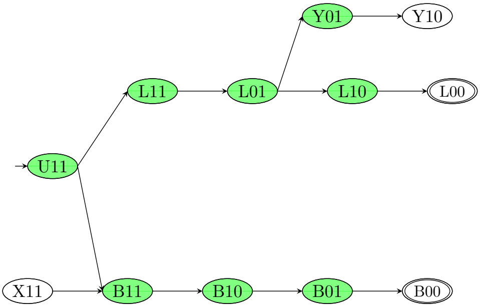

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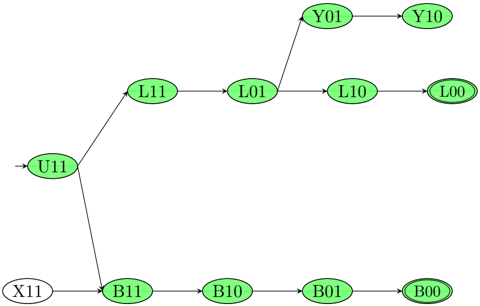

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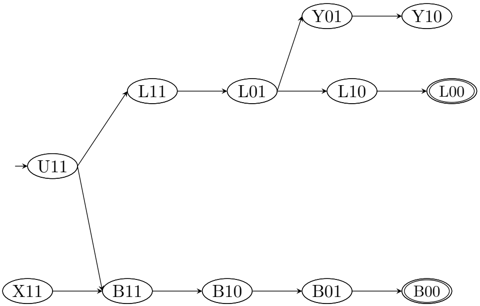

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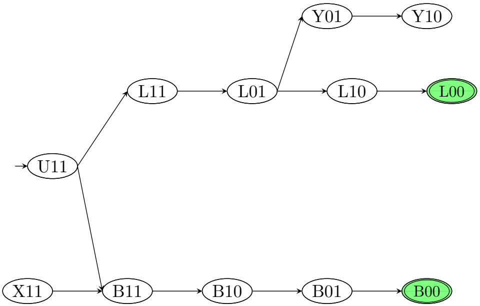

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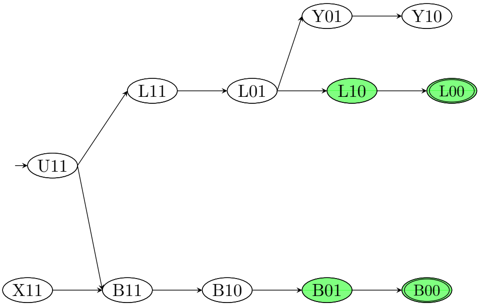

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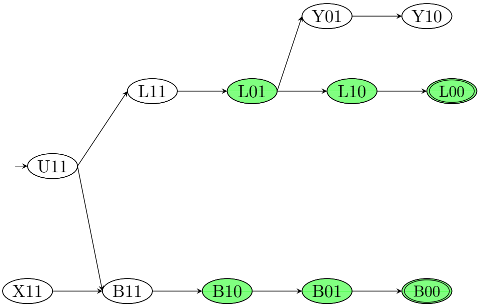

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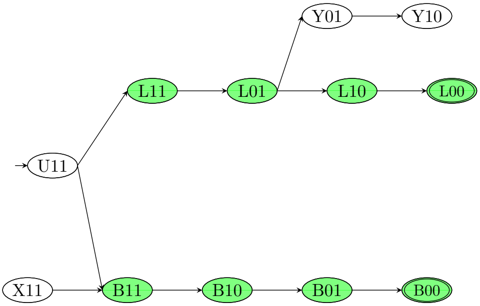

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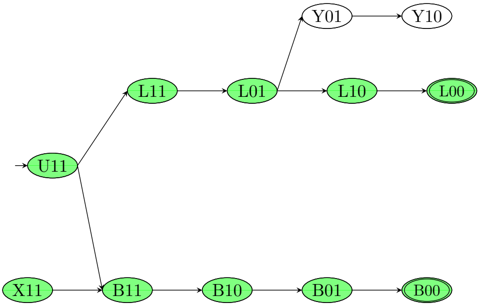
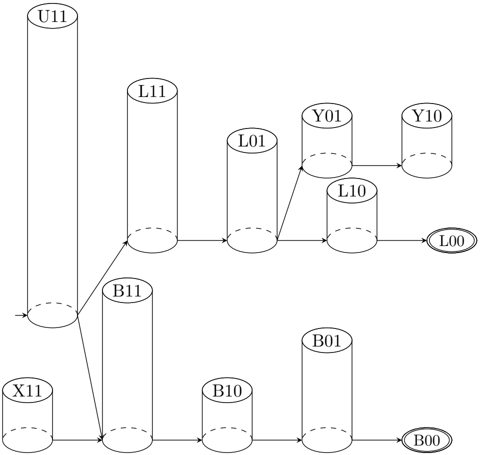


# Heuristic Properties

- descending heuristic: all (non-goal) alive states have an improving successor



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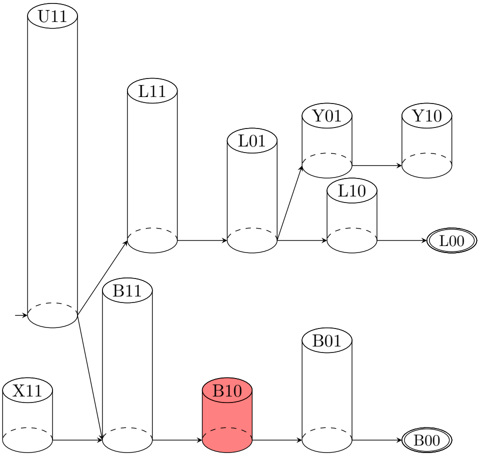

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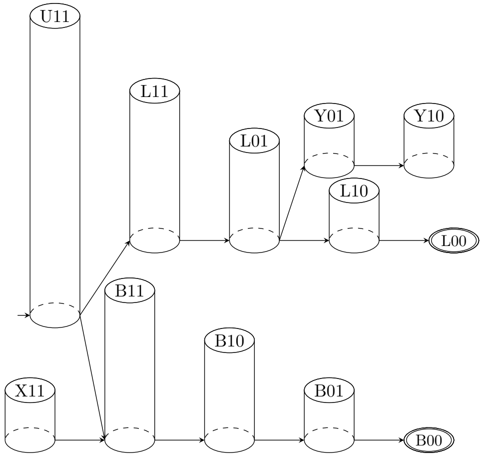
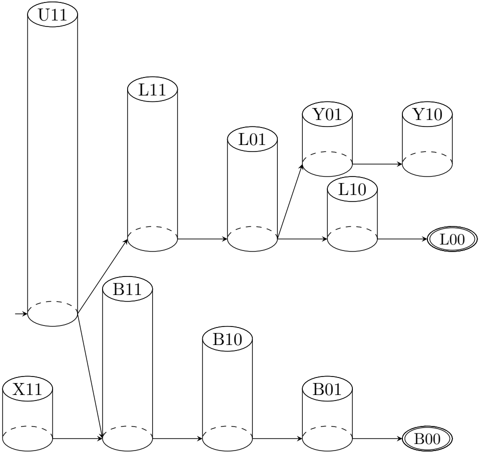


# Heuristic Properties

- dead-end avoiding heuristic: all improving successors of (non-goal) alive states are solvable



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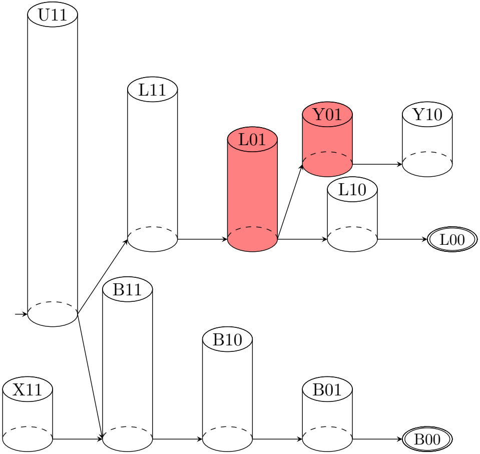

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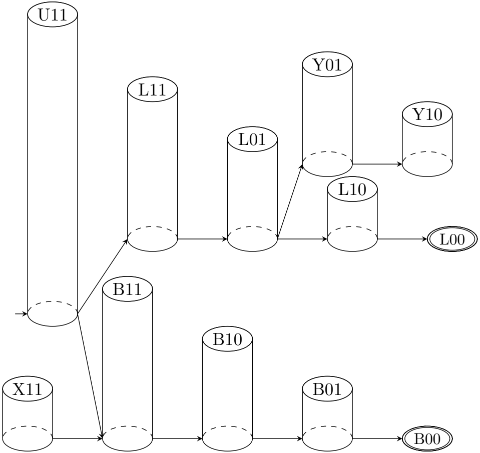

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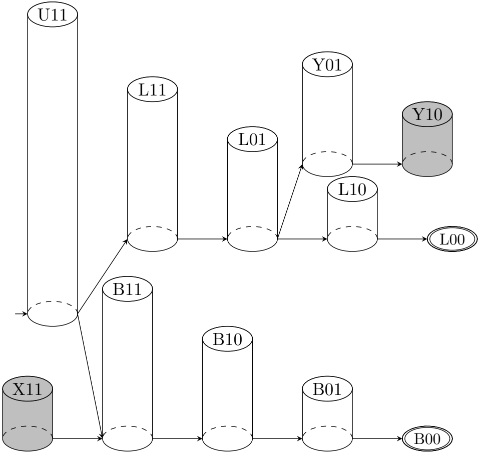


# Potential Heuristics

## Potential Heuristic

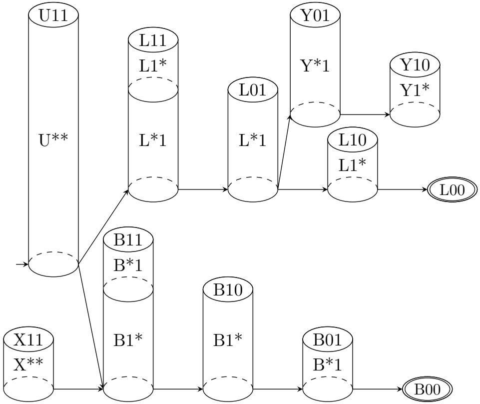
A **potential heuristic** is a heuristic that is computed with a weighted count of the partial assignments that agree with the given state.

$$h^{pot}(s) = \sum_{p \in \mathcal{P}} (w(p) \cdot [p \subseteq s])$$

The **dimension** of $h^{pot}$ is $max_{p \in \mathcal{P}, w(p) \neq 0} |p|$.

# Correlation Complexity

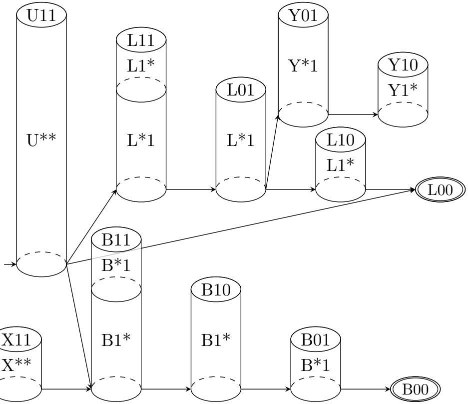
**Correlation Complexity** (Seipp et. al. 2016) asks: "What dimension is required to construct a descending and dead-end avoiding potential heurisitc?"




| $p$ | $w(p)$ |
|---|---|
| $U^{**}$ | 5 |
| $L1^{*}$ | 1 |
| $L^{*}1$ | 2 |
| $B1^{*}$ | 2 |
| $B^{*}1$ | 1 |
| $Y^{*}1$ | 2 |
| $Y1^{*}$ | 1 |
| $X^{**}$ | 1 |

# Correlation Complexity

Some easier tasks are not detected as easier.

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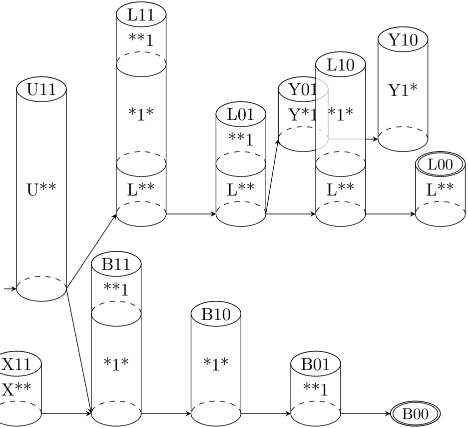
# River Measure

Correlation complexity:
All descending paths from all alive states reach a goal.

River measure (this work):
All descending paths from the initial state reach a goal.

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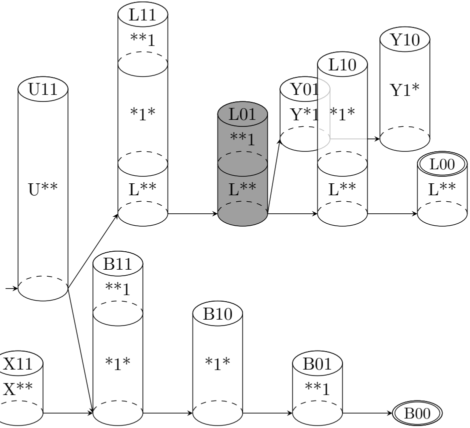


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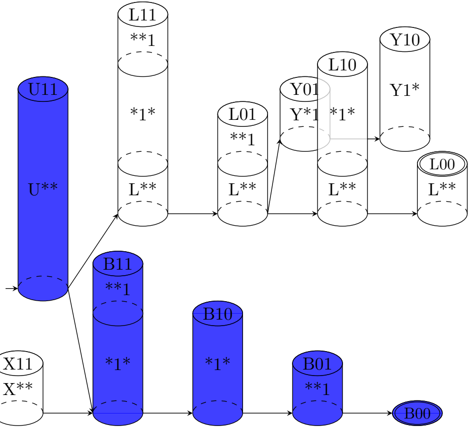


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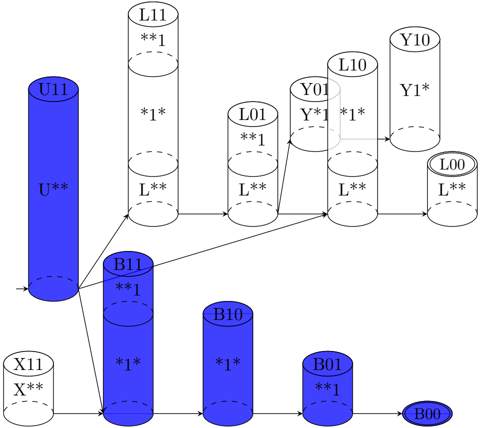


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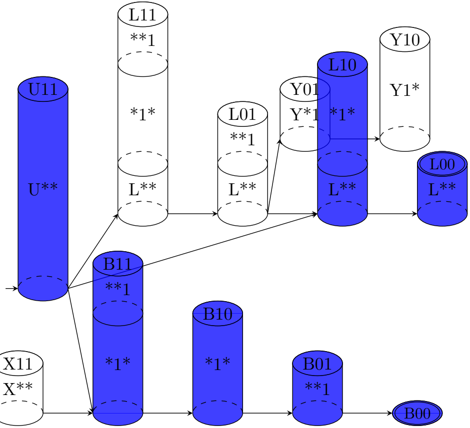


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# Measures

Conditions for the river measure are less restrictive than for the correlation complexity.

$$RM(\Pi) \leq CC(\Pi)$$

# Novelty Width

Novelty width (Lipovetzky and Geffner, 2012; 2014)

- Based on a modification of Breadth First Search
- Not states in the closed set but partial assignments of size k
- If p is not in the closed set, then p is novel
- Novelty width is the smallest k that guarantees to finds a plan
- Measures how 'hard' a planning task is

# Novelty Width Search

## Novelty Width Search

**if** $\gamma \in I$:
    **return** $I$
*open* $:= [I]$
*closed* $:= \{p \mid p \subseteq I, |p| = k\}$
**while** *open* $\neq \emptyset$:
    $s :=$ pop fist element of *open*
    **foreach** $s' \in$ *succ*$(s)$:
        **if** $\gamma \subseteq s'$:
            **return** $s'$
        **if** $\exists q \subseteq s'$ with $|q| = k, q \notin$ *closed*:
            insert each $p \subseteq s'$ with $|p| = k$ in *closed*
            append $s'$ to *open*

# Comparison

Novelty width is not comparable to correlation complexity.

- $\text{NW}(\Pi') < \text{CC}(\Pi')$ for some $\Pi'$
- $\text{NW}(\Pi'') > \text{CC}(\Pi'')$ for some $\Pi''$

River measure is comparable to both.

- $\text{RM}(\Pi) \leq \text{NW}(\Pi) + 1$
- $\text{RM}(\Pi) \leq \text{CC}(\Pi)$

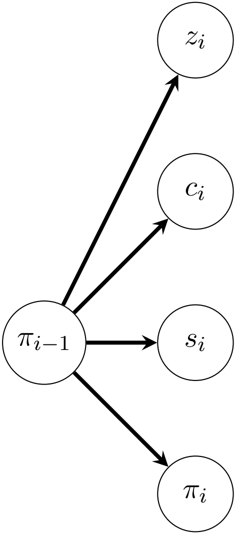
# Proof Sketch

Proof sketch:

- States of plan found with novelty width algorithm: $\pi_0, \pi_1, \ldots, \pi_L$
- Chose weights such that $\pi_i$ is the only improving successor of $\pi_{i-1}$

# Proof Sketch

Part of the search tree:



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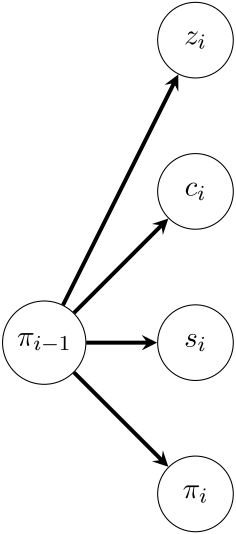


$\pi_i \supseteq q_i$

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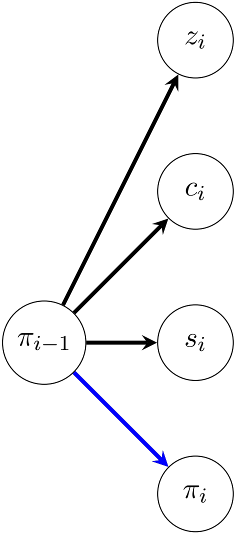


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$w(q_i) = -\Omega^{2i}$

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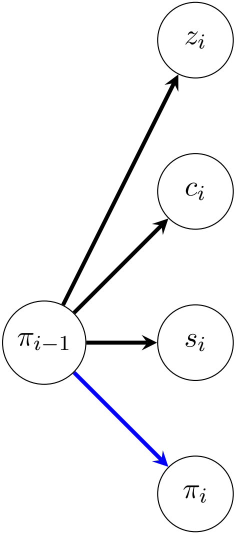


$s_i \supseteq q_i$

$\pi_i \supseteq q_i$

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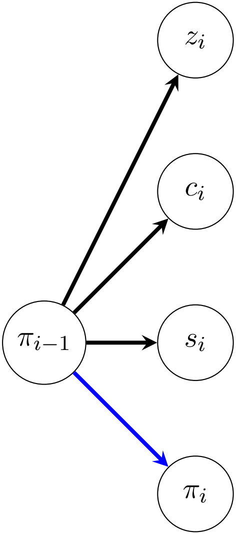


$s_i \supseteq q_i$

$s_i \supseteq \{\delta^s\}$

$\pi_i \supseteq q_i$

$\pi_i \not\supseteq \{\delta^s\}$

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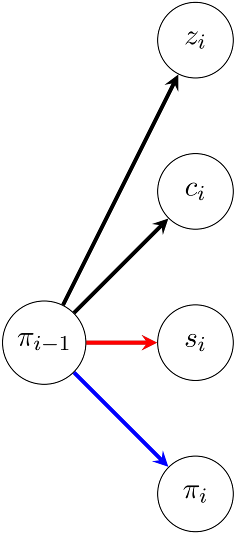


$s_i \supseteq q_i$
$s_i \supseteq \{\delta^s\}$

$w(q_i \cup \{\delta^s\}) = +\Omega^{2i+1}$

$\pi_i \supseteq q_i$
$\pi_i \not\supseteq \{\delta^s\}$

$w(q_i) = -\Omega^{2i}$

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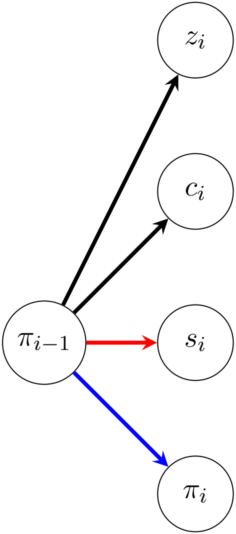


$c_i \not\supseteq q_i$
$c_i \supseteq q_{i-1}$

$s_i \supseteq q_i$
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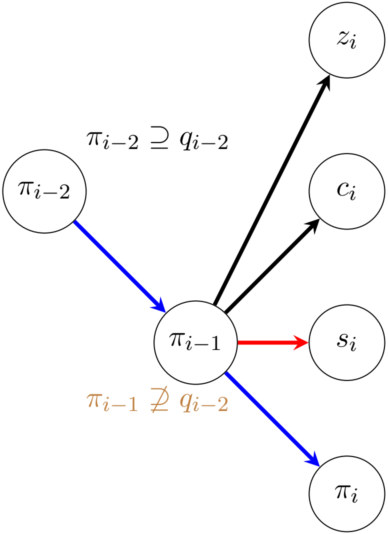


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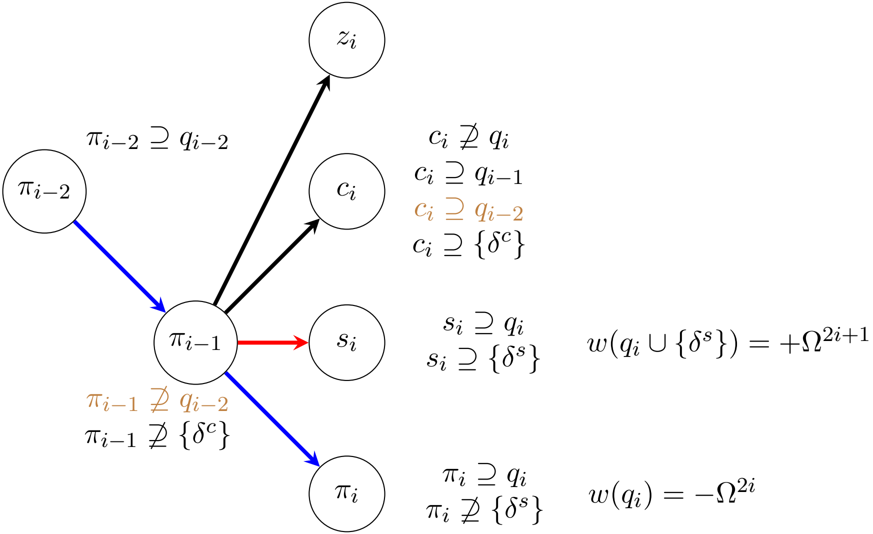

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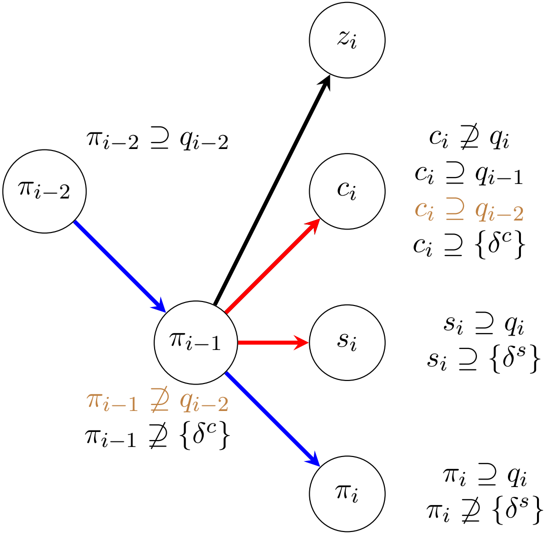


$w(q_{i-1} \cup \{\delta^c\}) = +\Omega^{2i-1}$

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$w(q_i) = -\Omega^{2i}$

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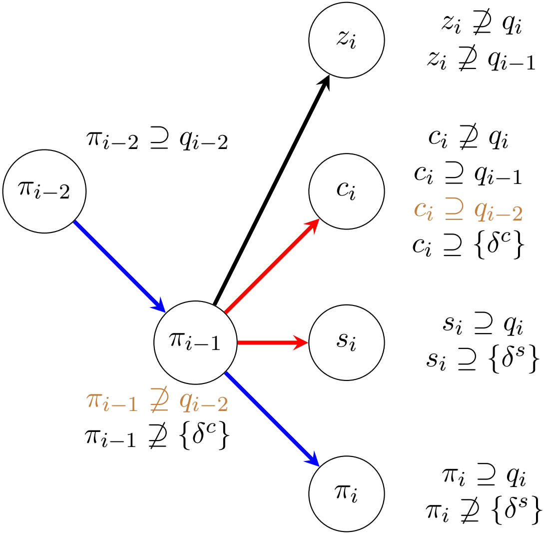


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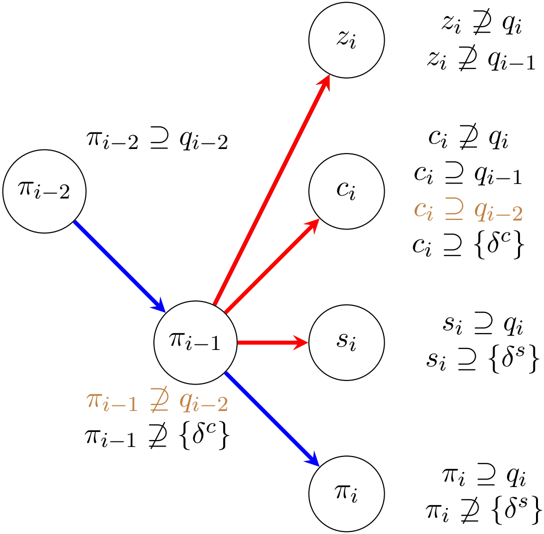


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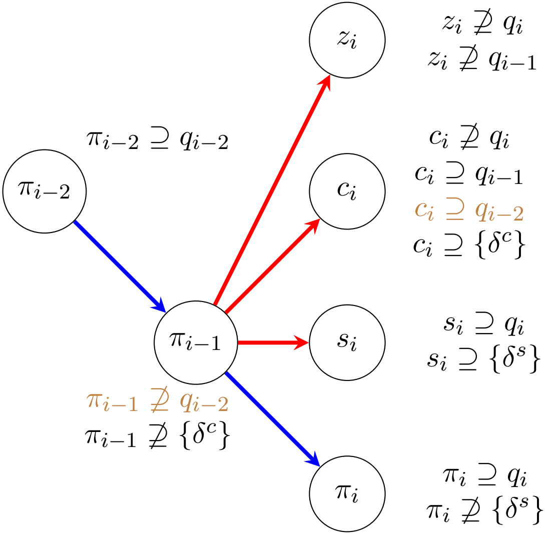


$$|q_i| = \mathsf{NW}(\Pi)$$
$$\mathsf{RM}(\Pi) \leq \mathsf{NW}(\Pi) + 1$$

$$w(q_{i-1} \cup \{\delta^c\}) = +\Omega^{2i-1}$$

$$w(q_i \cup \{\delta^s\}) = +\Omega^{2i+1}$$

$$w(q_i) = -\Omega^{2i}$$

# Conclusion

- River measure is a bridge to compare novelty width and correlation complexity
- $RM(\Pi) \leq NW(\Pi) + 1$
- $RM(\Pi) \leq CC(\Pi)$

# The end

Thank you for your attention!