On Variable Dependencies and Compressed Pattern Databases

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Introduction

Quotation

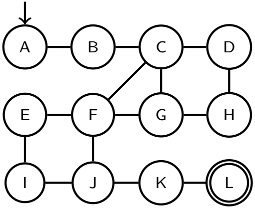
previous work on compressed pattern databases:

Sturtevant, Felner and Helmert (SoCS 2014)

"This approach worked very well for the 4-peg Towers of Hanoi, for instance, but its success for the sliding tile puzzles was limited and no significant advantage was reported for the Top-Spin domain (Felner et al., 2007)."

this paper: try to understand why

Compressed PDBs



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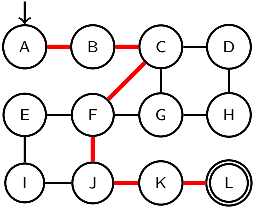
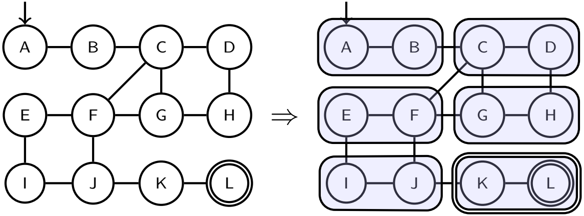


$$h^*(\mathsf{A}) = 6$$

Compressed PDBs



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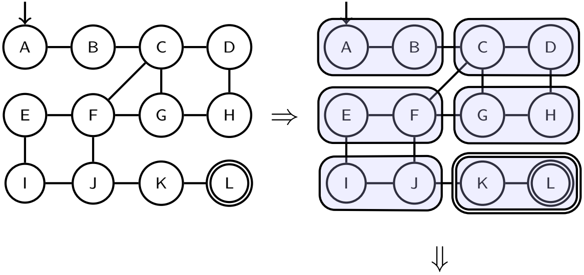
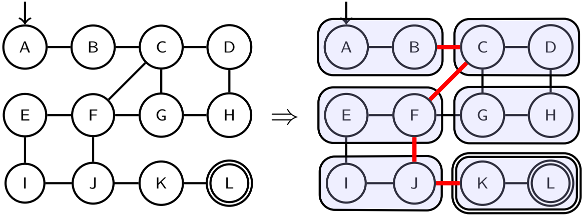


$h^*(A) = 6$

AB	4
CD	3
EF	2
GH	3
IJ	1
KL	0

Compressed PDBs



⇓

$$h^*(A) = 6$$

$$h_{\text{PDB}}(A) = 4$$

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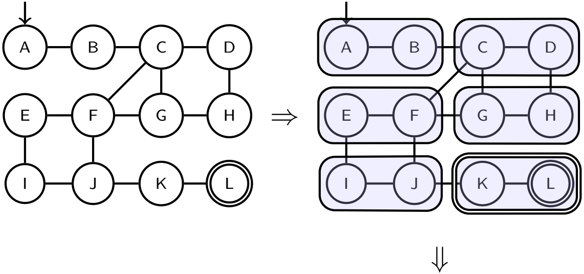

$$h^*(\mathrm{A}) = 6$$

$$h_{\mathrm{PDB}}(\mathrm{A}) = 4$$

$$h_{\mathrm{PDB}}^{comp}(\mathrm{A}) = 3$$

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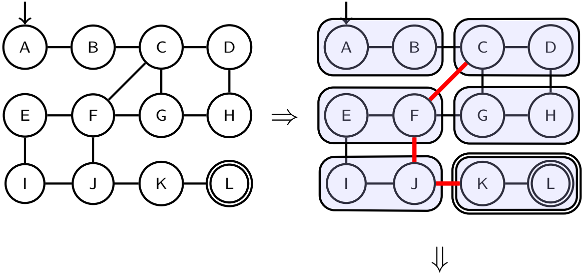

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Comparing PDBs to Compressed PDBs

Assume we have N units of memory.

Consider three heuristics:

- h_F: fine-grained PDB ($M \gg N$ entries)
- h_F^{comp}: compressed fine-grained PDB (N entries)
- h_C: coarse-grained PDB (N entries)

Which one should we use, h_F^{comp} or h_C?

Experimental Results

State Space	M/N	h_F	h_F^{comp} MOD	h_F^{comp} DIV	h_F^{comp} random	h_C
Hanoi	4	104.32	87.04	103.76	90.08	87.04
Sliding Tiles A	10	34.99	29.89	32.08	26.38	32.08
Sliding Tiles B	10	34.99	30.50	32.84	26.38	15.29
TopSpin	12	10.78	9.29	9.59	8.73	9.59

- Hanoi: 4 pegs and 16 disks; pattern with 15 disks
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all use lexicographic ranking

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Good News

Dominance of Compressed PDBs

Theorem (dominance of compressed PDBs)

Let h_F and h_C be heuristics such that h_F is a refinement of h_C.

Consider compressed heuristics with a compression regime that is compatible with h_F and h_C.

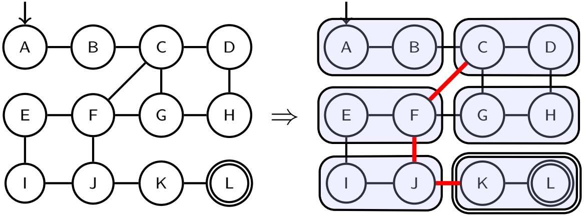
Then

$$h_F^{comp}(s) \geq h_C(s)$$

for all states s.

informally: compression step applies further abstraction on top of the abstraction h_F

Dominance of Compressed PDBs: Proof Idea



⇓

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$$h_F^{comp}(\mathrm{A}) = 3$$

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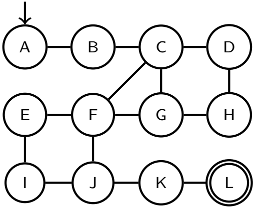

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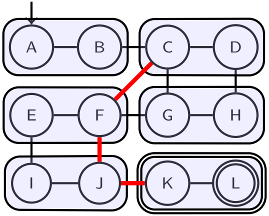

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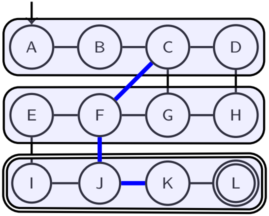

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⇓

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⇓

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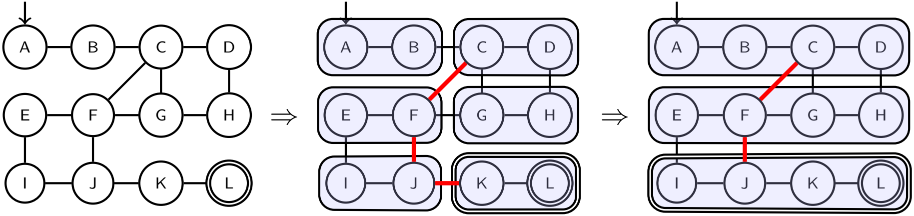

$$h^*(A) = 6$$
$$h_F(A) = 4$$
$$h_F^{comp}(A) = 3$$
$$h_C(A) = 2$$

⇓

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⇓

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$h_F^{comp}(s) \geq h_C(s)$ for all states according to the theorem

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Bad News

State Variables

States are described in terms of state variables.

Examples:

- Towers of Hanoi: position of one disk
- sliding tiles: position of a tile (or blank)
- TopSpin: position of a token

PDBs project to a subset of variables (the "pattern").

Variable Dependencies

Variable u depends on variable v if changing u is conditioned in any way on v.

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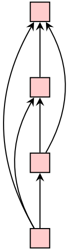
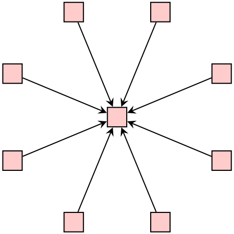
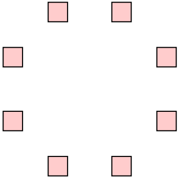


Towers of Hanoi



sliding tiles



TopSpin

Improvements vs. Dependencies

Theorem (no improvements without dependencies)

Consider the patterns $F \supseteq C$ in an undirected state space.

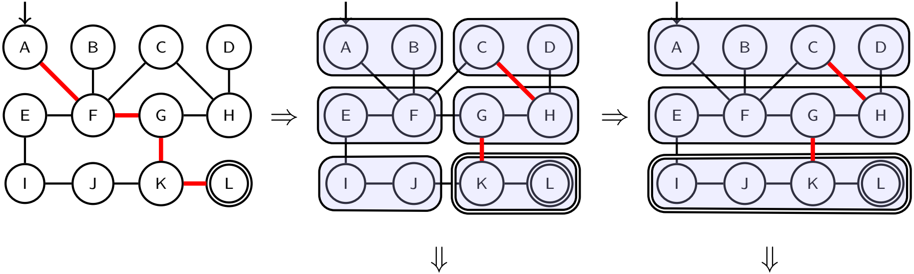
Let h_F^{comp} be a compressed PDB heuristic with a compression regime compatible with the refinement relation between F and C.

If no variable in C depends on any variable in $F \setminus C$, then

$$h_F^{comp}(s) = h_C(s)$$

for all states s.

Improvements vs. Dependencies: Proof Idea



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Related Work in Classical Planning

our result:

- $h_F^{comp} = h_C$
- for undirected state spaces
- under certain dependency conditions

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literature (Haslum et al. 2007; Pommerening et al. 2013):

- $h_F = h_C$
- for arbitrary state spaces
- under certain (different) dependency conditions

neither result entails the other

$\rightsquigarrow$ many more details in paper

Conclusion

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When is entry compression a good idea?

- never bad when compatible with refinement
- never good when refinement does not capture a dependency

What does this mean for the benchmarks?

- Towers of Hanoi: must compress smaller disks away
- sliding tile: compressing blank the only useful refinement
- TopSpin: no dependencies, hence no gain
  (ditto: Pancakes, Rubik's Cube)

Thank You

Thank you for your attention!