A Theory of Merge-and-Shrink for Stochastic Shortest Path Problems

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Overview of this Paper

Based on compositional theory of M&S in classical planning (Sievers and Helmert 2021)

Contributions

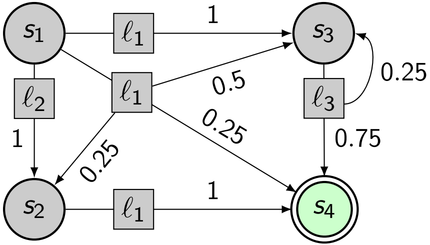
- Formalization of transformations of probabilistic TSs with desirable properties
- Contribution of a suitable factored representation for M&S
- Formalization of Merge-and-Shrink transformations on this representation

Background: Probabilistic Planning Tasks

Compact representation of TS as prob. planning task: $\Pi = \langle V, O, I, G \rangle$

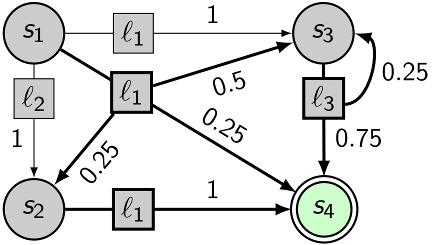
- Finitely many variables V with finite domains $\mathcal{D}(v)$, $v \in V$
- Finitely many operators O where each $o \in O$:
  - has a non-negative cost $\mathrm{cost}(o)$
  - has a precondition $\mathrm{pre}(o)$ (partial variable assignment)
  - has finitely many effects $\mathrm{eff}_\mathrm{i}(o)$ (partial variable assignments) with associated probability p_i
- Initial state I is a variable assignment
- Goal state G is a partial variable assignment

Background: Probabilistic Transition Systems

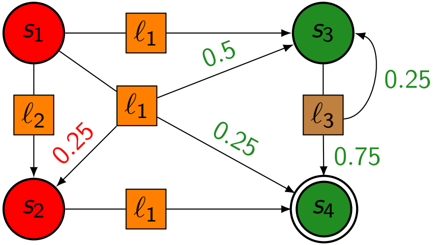


Background: Probabilistic Transition Systems

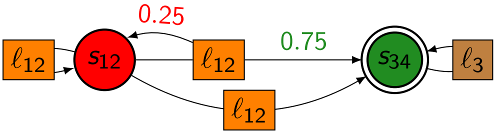
Cost: $\frac{23}{12}$



Transformations



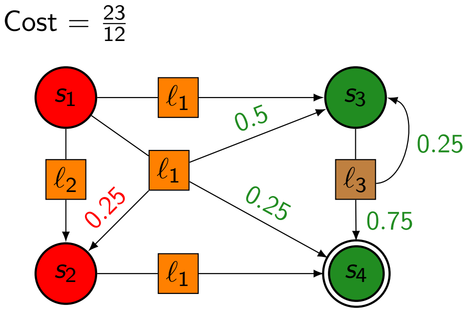
Original TS Θ



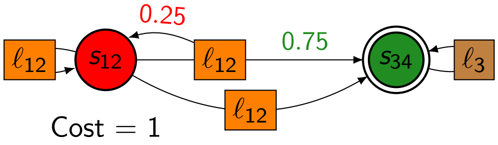
Transformed TS Θ'

$+$ State Mapping σ $+$ Label Mapping λ

Transformations



Original TS Θ



Transformed TS Θ'

$+$ State Mapping σ $+$ Label Mapping λ

Transformation Classes

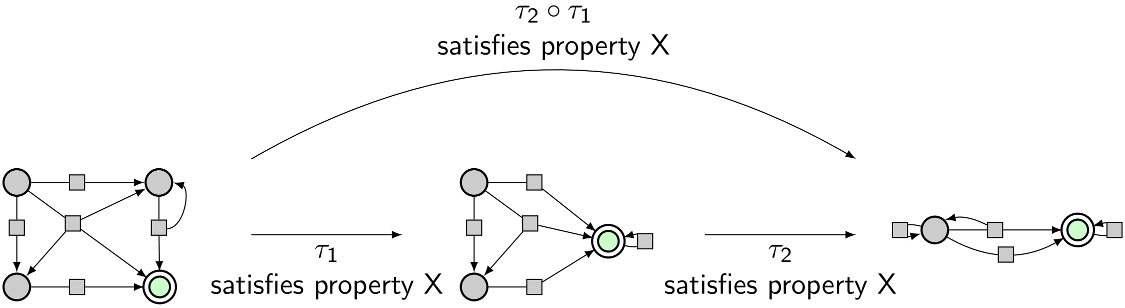
CONS_S	σ is total on S	Conservative Transformations (aka Abstractions)
CONS_L	λ is total on L	
CONS_C	$\forall \ell \in \text{dom}(\lambda).\, c'(\lambda(\ell)) \leq c(\ell)$	
CONS_T	$\text{ind}_\tau(T) \subseteq T'$	
CONS_G	$\sigma(S_\star) \subseteq S'_\star$	
IND_S	σ is surjective on S'	Induced Transformations
IND_L	λ is surjective on L'	
IND_C	$\forall \ell' \in L'.\, \exists \ell \in \lambda^{-1}(\ell').\, c(\ell) = c'(\ell')$	
IND_T	$\text{ind}_\tau(T) \supseteq T'$	
IND_G	$\sigma(S_\star) \supseteq S'_\star$	
REF_C	$\forall \ell' \in L'.\, \forall \ell \in \lambda^{-1}(\ell').\, c(\ell) = c'(\ell')$	Refinable Transformations
REF_T	$\forall s' \in S'.\, T'(s') \subseteq \bigcap_{s \in \sigma^{-1}(s')} \text{ind}_\tau(T(s))$	
REF_G	$\sigma^{-1}(S'_\star) \subseteq S_\star$	

From Syntactic Properties to Heuristic Guarantees

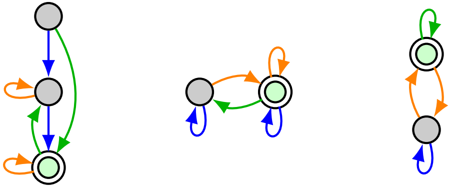
Heuristic Guarantees

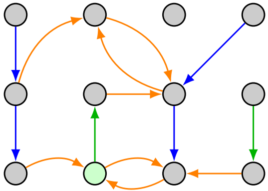
- Conservative transformations (aka abstractions) result in *admissible* heuristics
- Refinable transformations result in *pessimistic* heuristics
- Exact (conservative + refinable) transformations result in *perfect* heuristics

Composability



Merge-and-Shrink — Factored Representation

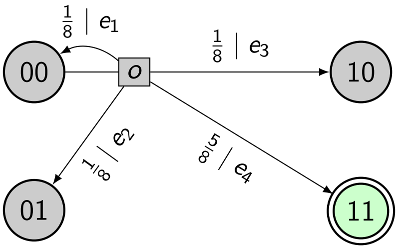


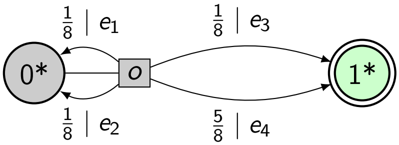


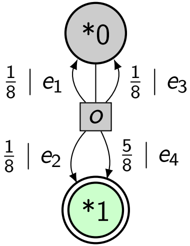
Merge-and-Shrink — Factored Representation

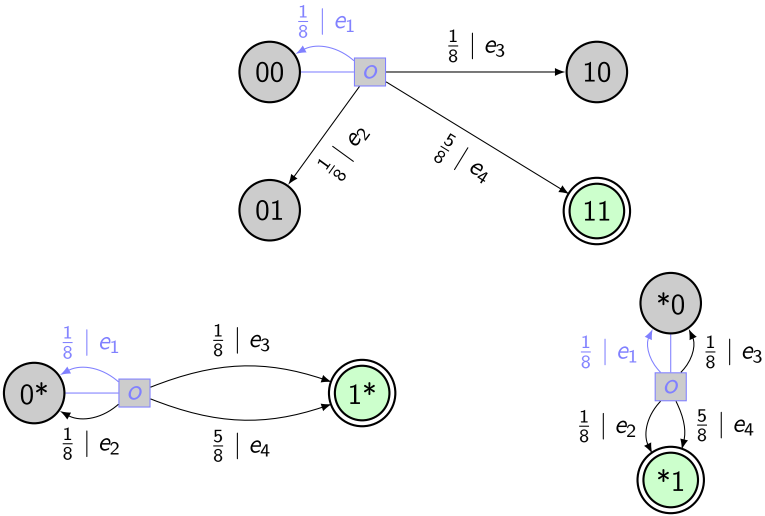
- Classical Planning: Synchronize on operators
- Probabilistic Planning: Synchronize on operators **and probabilistic outcomes**

Merge-and-Shrink — Factored Representation





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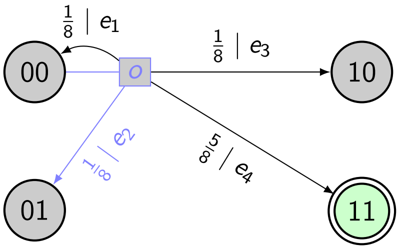
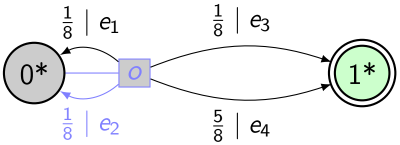
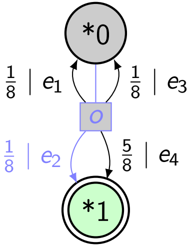
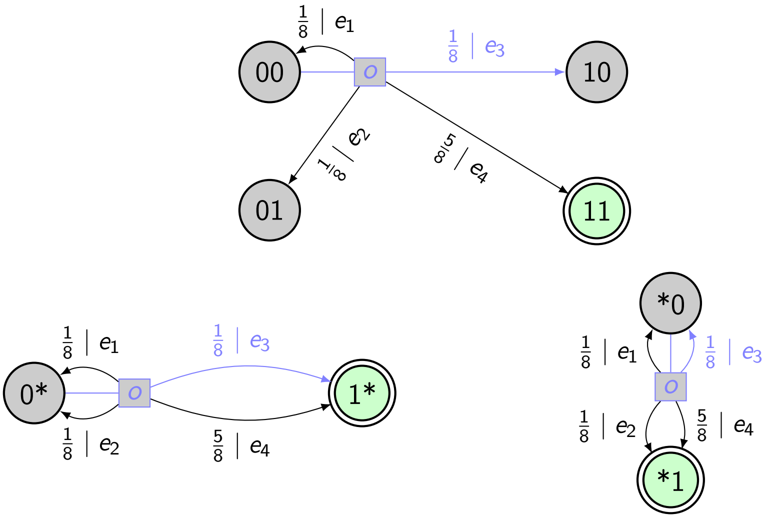
Merge-and-Shrink — Factored Representation

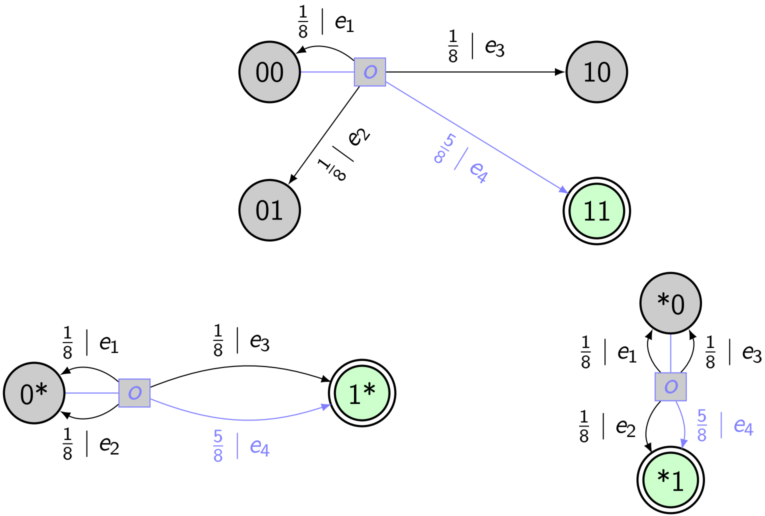






Merge-and-Shrink — Factored Representation





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1/8 | e2
5/8 | e4
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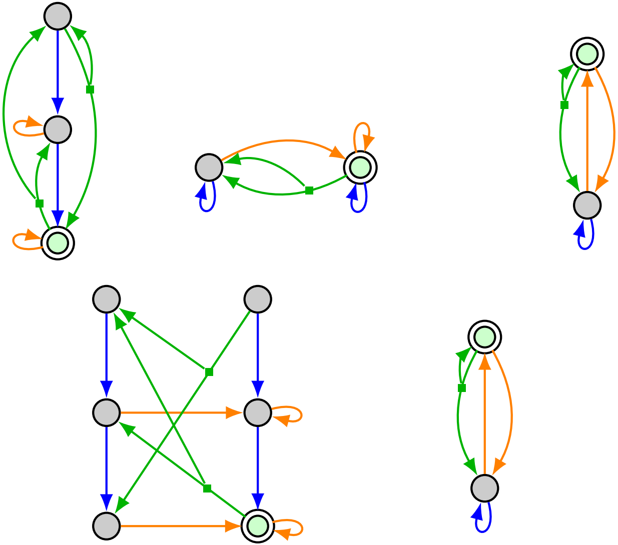
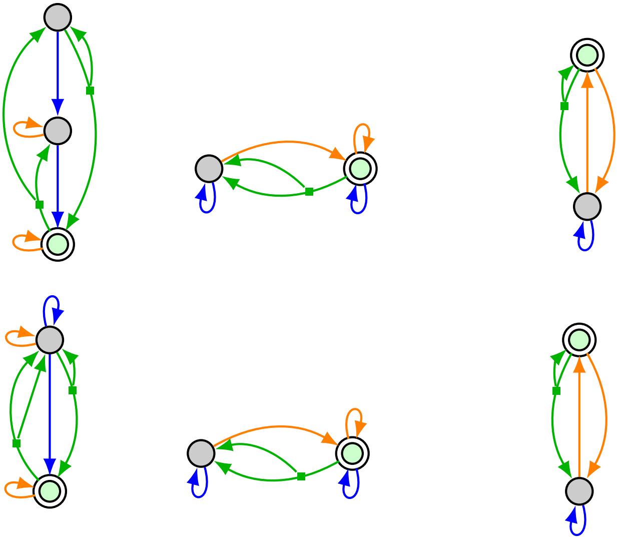
Merging



Shrinking

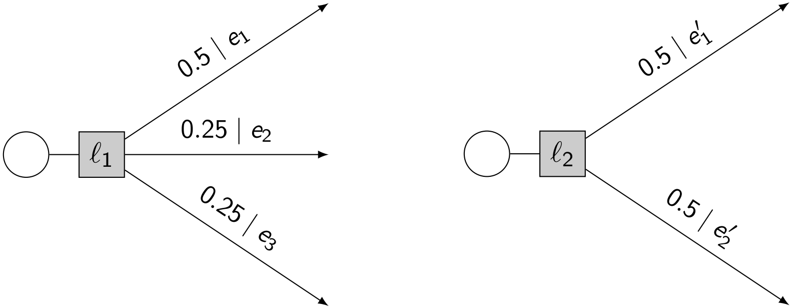


Shrinking: Properties

Properties of Shrink Transformations

- conservative and induced transformation
- **exact** if based on extension of bisimulation

Label Reduction



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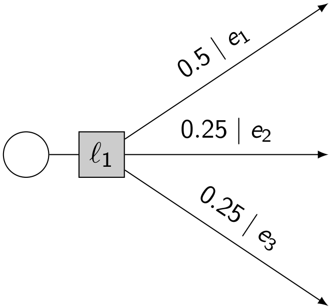

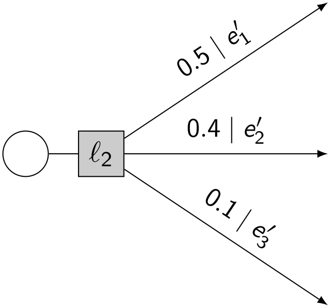

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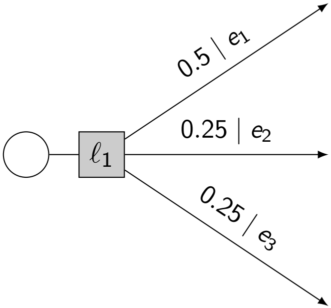

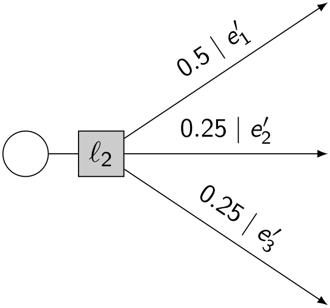

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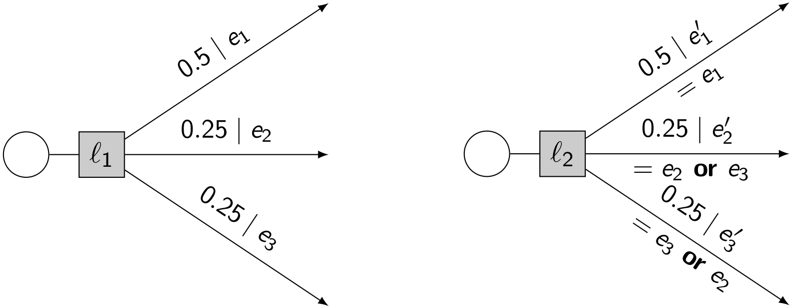

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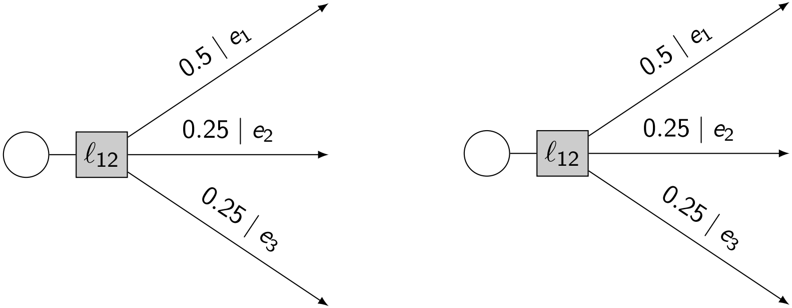

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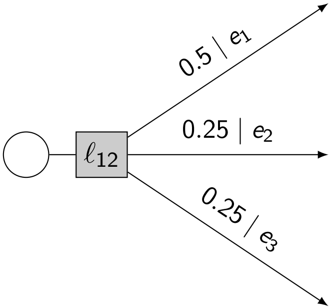

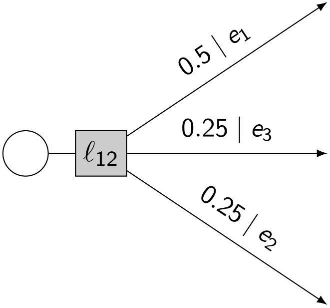

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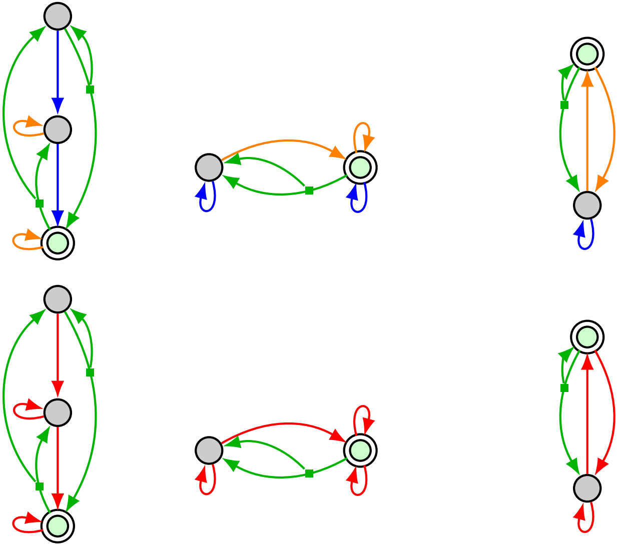

Label Reduction: Properties

Properties of Label Reduction

- conservative transformation
- exact iff induced/refinable and only labels with same cost are merged
- atomic label reduction exact **if** based on extension of Θ-combinability

Conclusion

- Purely theoretic paper on the foundations of merge-and-shrink for SSPs
- Introduces a new factored representation suitable for merge-and-shrink
- Generalizes many results from the classical theory
- Not covered in this theory yet: Pruning transformations

References I

Sievers, S.; and Helmert, M. 2021. Merge-and-Shrink: A Compositional Theory of Transformations of Factored Transition Systems. 71: 781–883.