

# Counterexample-guided Cartesian Abstraction Refinement and Saturated Cost Partitioning for Optimal Classical Planning

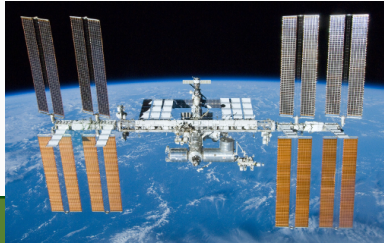
---

Jendrik Seipp

February 28, 2018

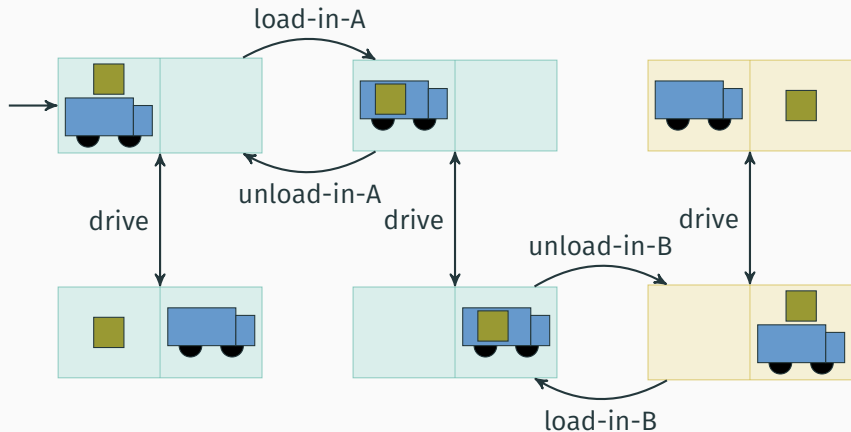
University of Basel

# Planning

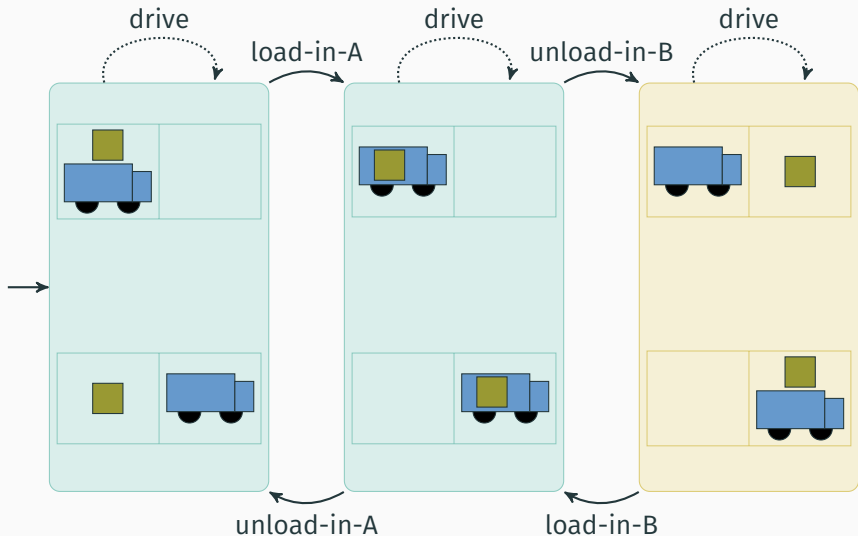


Find a sequence of actions that achieves a goal.

# Optimal Classical Planning



# Optimal Classical Planning: Example Abstraction



- abstraction heuristics never overestimate → **admissible**
- $A^*$  + admissible heuristic → **optimal** plan
- higher accuracy → better guidance

- abstraction heuristics never overestimate → **admissible**
- $A^*$  + admissible heuristic → **optimal** plan
- higher accuracy → better guidance
- **how to create abstractions?**

# Counterexample-guided Cartesian Abstraction Refinement

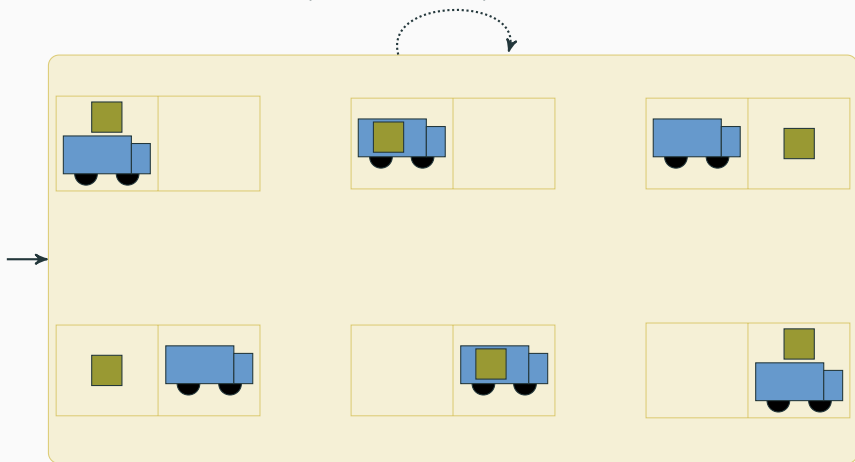
## CEGAR Algorithm

- start with coarse abstraction
- until finding concrete solution or running out of time:
  - find abstract solution
  - check if and why it fails in the real world
  - refine abstraction

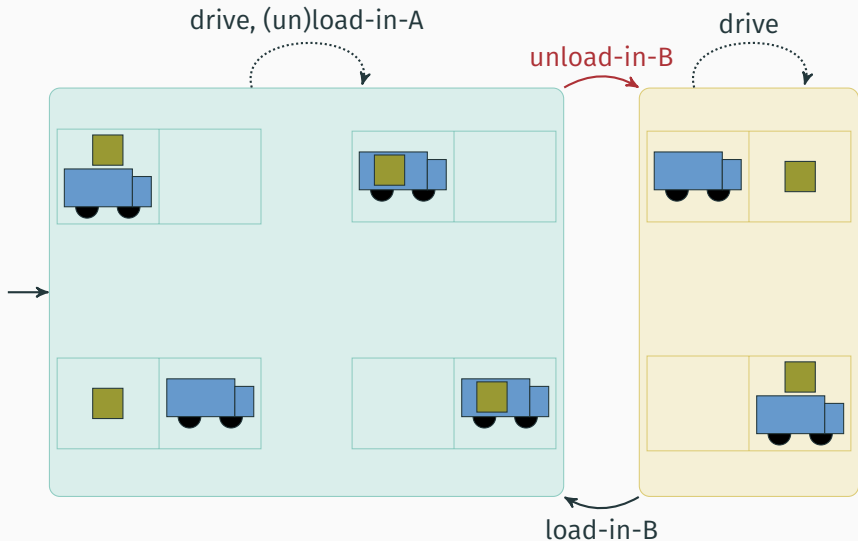


# Example Refinement

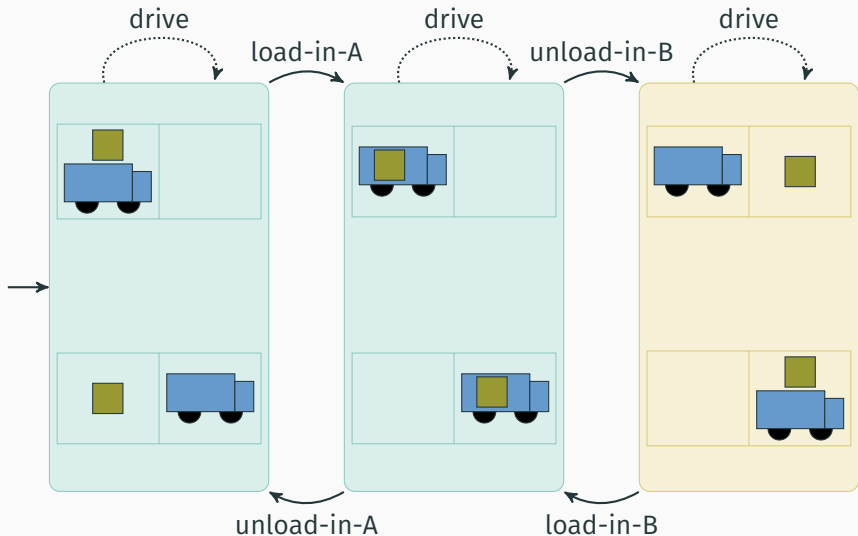
drive, (un)load-in-A, (un)load-in-B



# Example Refinement



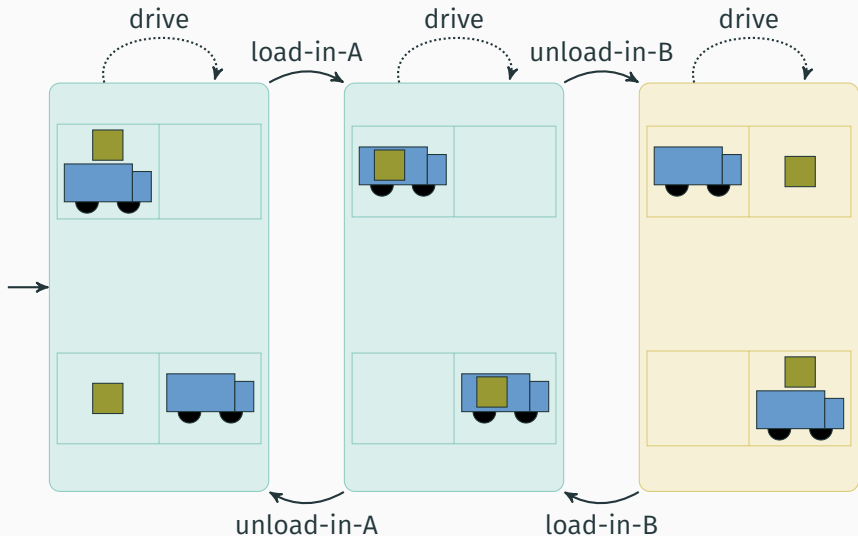
# Example Refinement



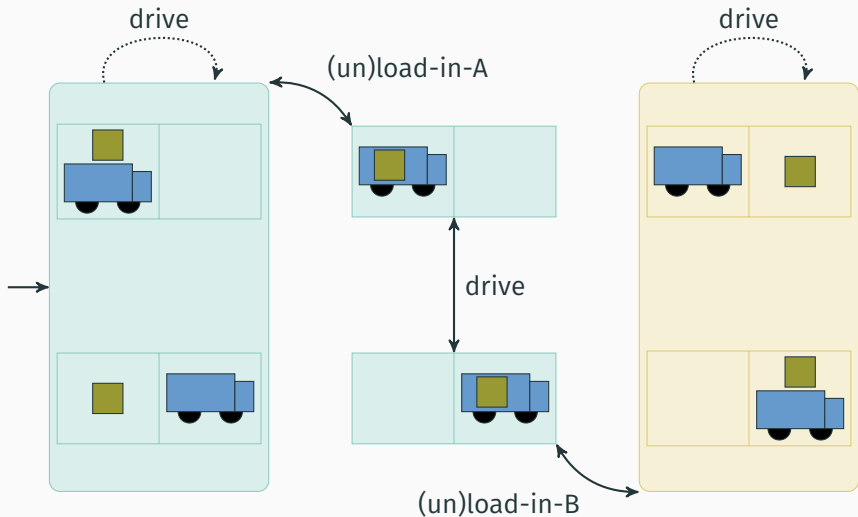
## Cartesian Abstractions

- relation to other classes of abstractions?

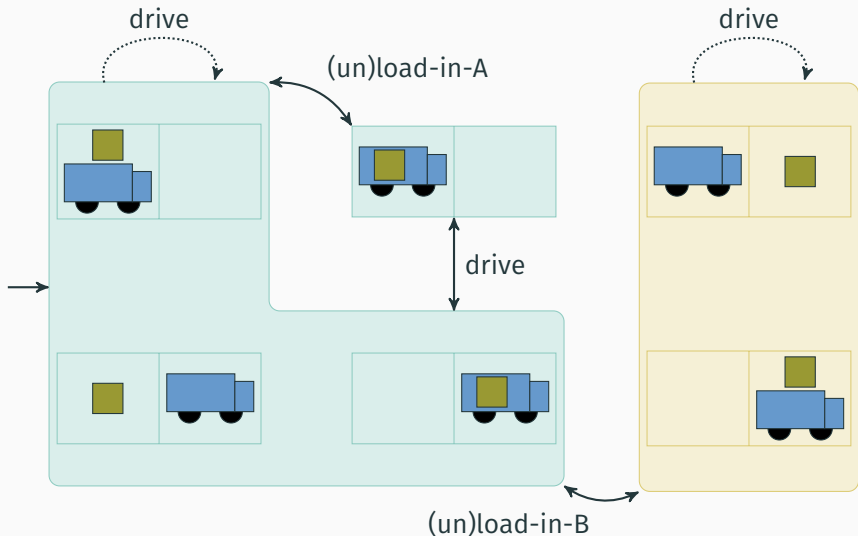
# Projection (PDB)



# Cartesian Abstraction



# Merge-and-shrink Abstraction

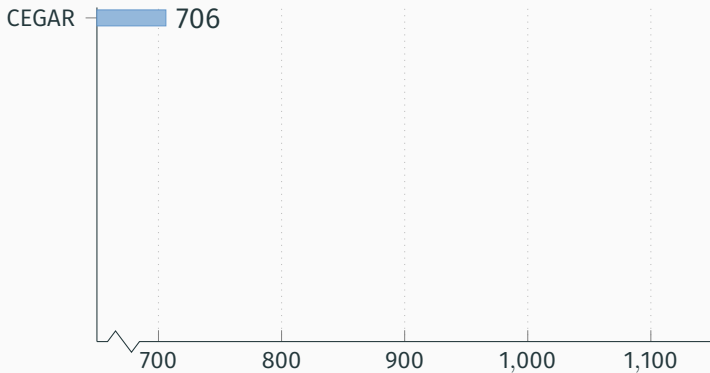
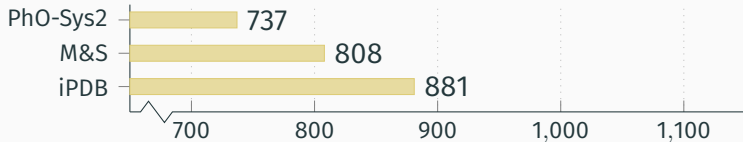


# Classes of Abstractions

- **Projections (PDBs)**  
refinement at least doubles number of states
- **Cartesian Abstractions**  
allow efficient and fine-grained refinement
- **Merge-and-shrink Abstractions**  
refinement complicated and expensive



# Solved Tasks



## Diminishing Returns

- finding solutions takes longer
- heuristic values only increase logarithmically

## Diminishing Returns

- finding solutions takes longer
- heuristic values only increase logarithmically

→ multiple smaller abstractions

- build abstraction for each **goal fact**

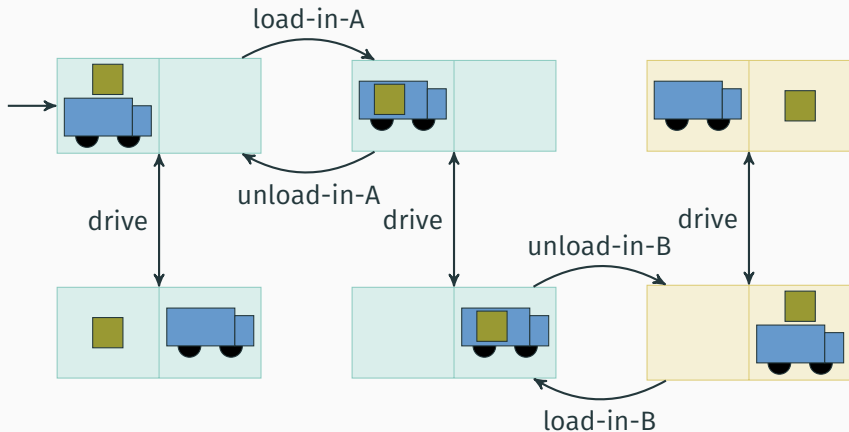
- build abstraction for each **goal fact**
- **problem**: tasks with single goal fact

## Task Decomposition by Landmarks

- build abstraction for each **fact landmark**

# Task Decomposition by Landmarks

- build abstraction for each **fact landmark**



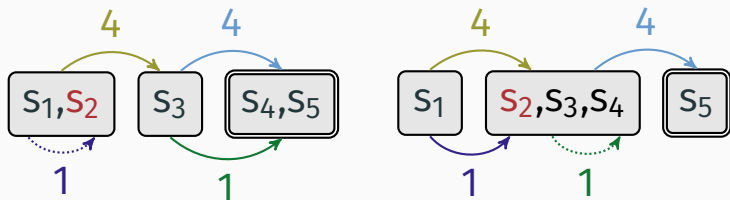
# Multiple Heuristics

how to combine multiple heuristics?



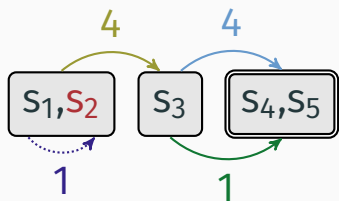
# Multiple Heuristics

how to combine multiple heuristics?

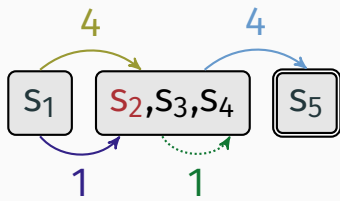


# Multiple Heuristics

how to combine multiple heuristics?



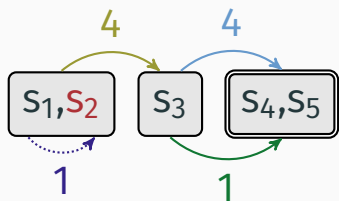
$$h_1(s_2) = 5$$



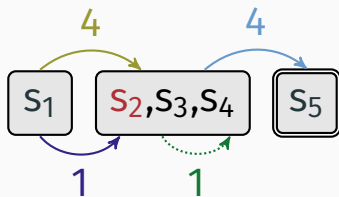
$$h_2(s_2) = 4$$

# Multiple Heuristics

how to combine multiple heuristics?



$$h_1(s_2) = 5$$



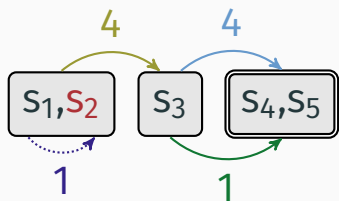
$$h_2(s_2) = 4$$

maximize over estimates:

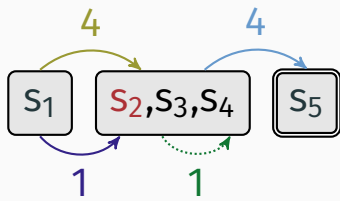
- $h(s_2) = 5$

# Multiple Heuristics

how to combine multiple heuristics?



$$h_1(s_2) = 5$$



$$h_2(s_2) = 4$$

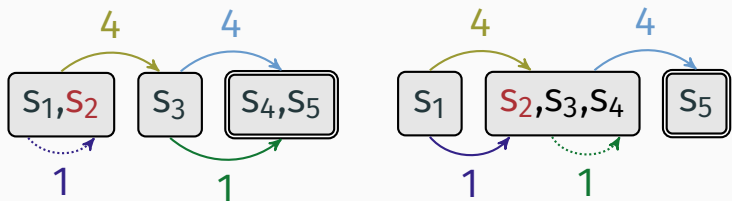
**maximize** over estimates:

- $h(s_2) = 5$
- only **selects** best heuristic
- does not **combine** heuristics

# Multiple Heuristics: Cost Partitioning

## Cost Partitioning

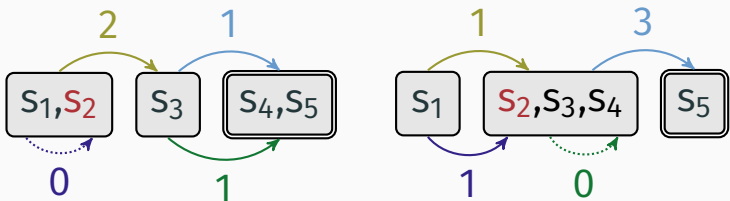
- split operator costs among heuristics
- sum of costs must not exceed original cost



# Multiple Heuristics: Cost Partitioning

## Cost Partitioning

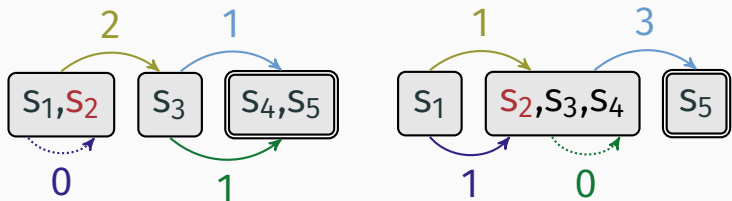
- split operator costs among heuristics
- sum of costs must not exceed original cost



# Multiple Heuristics: Cost Partitioning

## Cost Partitioning

- split operator costs among heuristics
- sum of costs must not exceed original cost



$$h(s_2) = 3 + 3 = 6$$

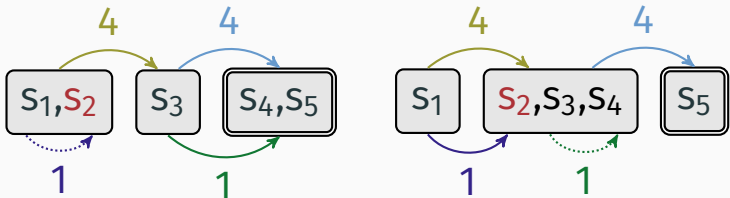
# Saturated Cost Partitioning



# Saturated Cost Partitioning

## Saturated Cost Partitioning Algorithm

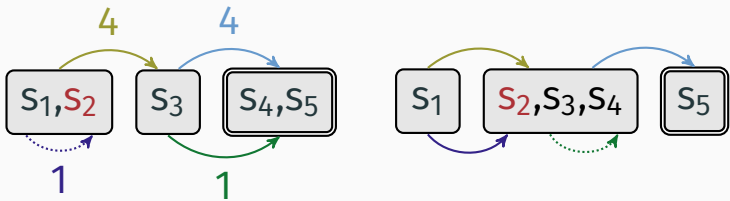
- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics



# Saturated Cost Partitioning

## Saturated Cost Partitioning Algorithm

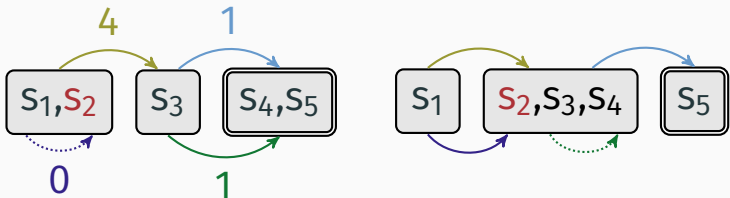
- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics



# Saturated Cost Partitioning

## Saturated Cost Partitioning Algorithm

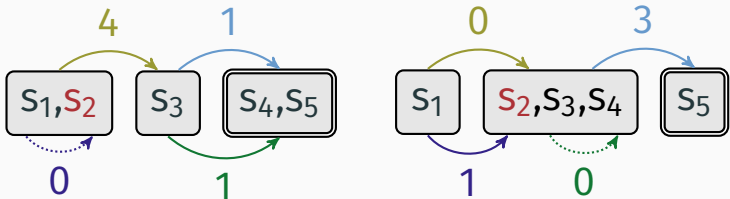
- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics



# Saturated Cost Partitioning

## Saturated Cost Partitioning Algorithm

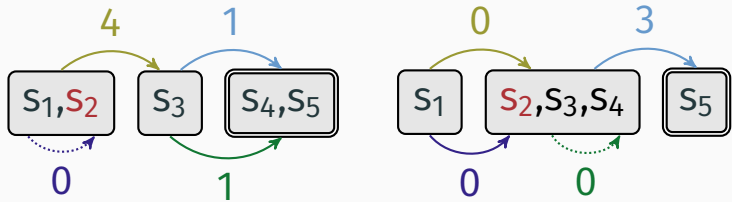
- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics



# Saturated Cost Partitioning

## Saturated Cost Partitioning Algorithm

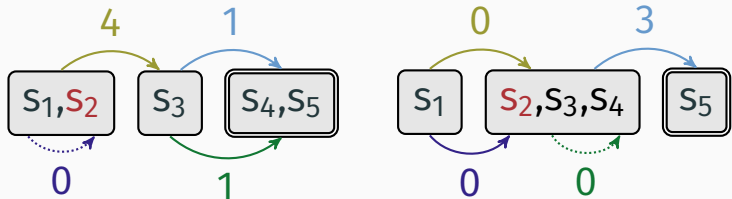
- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics



# Saturated Cost Partitioning

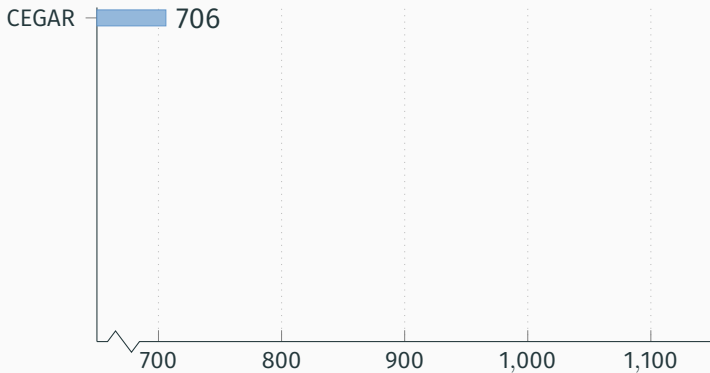
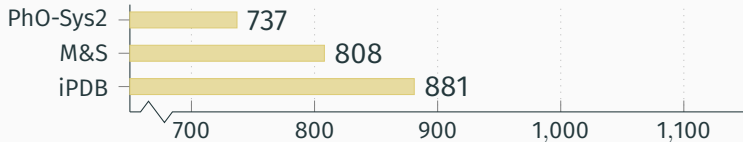
## Saturated Cost Partitioning Algorithm

- order heuristics, then for each heuristic  $h$ :
  - use minimum costs preserving all estimates of  $h$
  - use remaining costs for subsequent heuristics

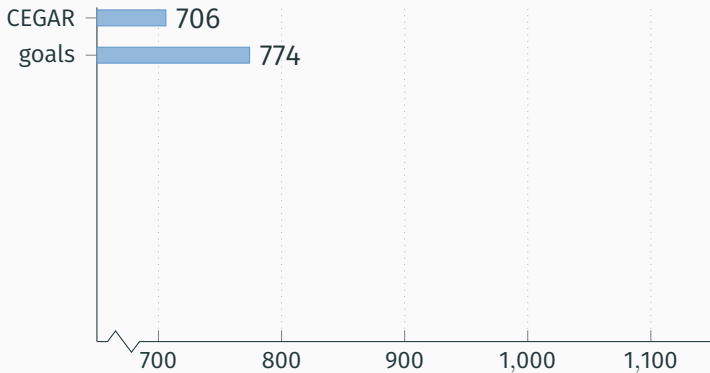
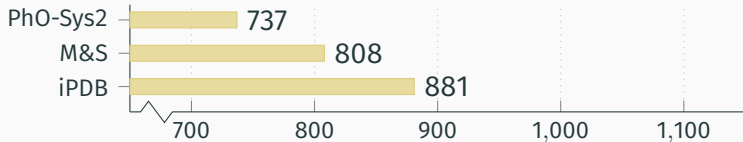


$$h(s_2) = 5 + 3 = 8$$

# Solved Tasks

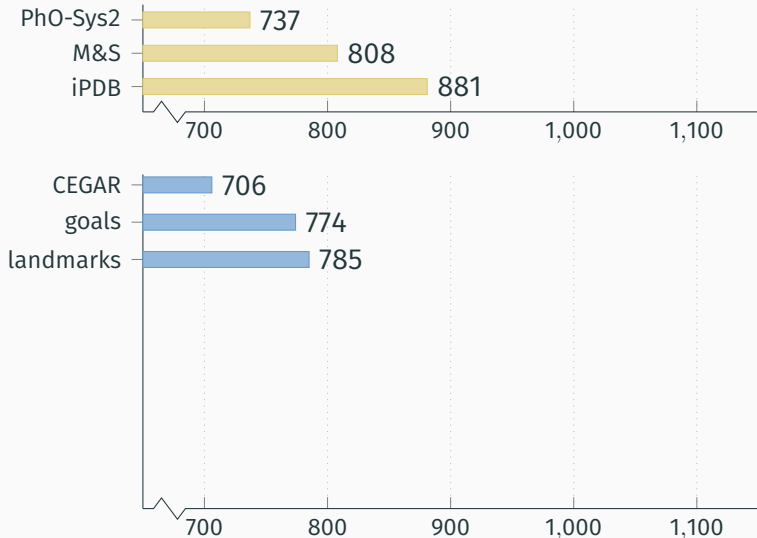


# Solved Tasks

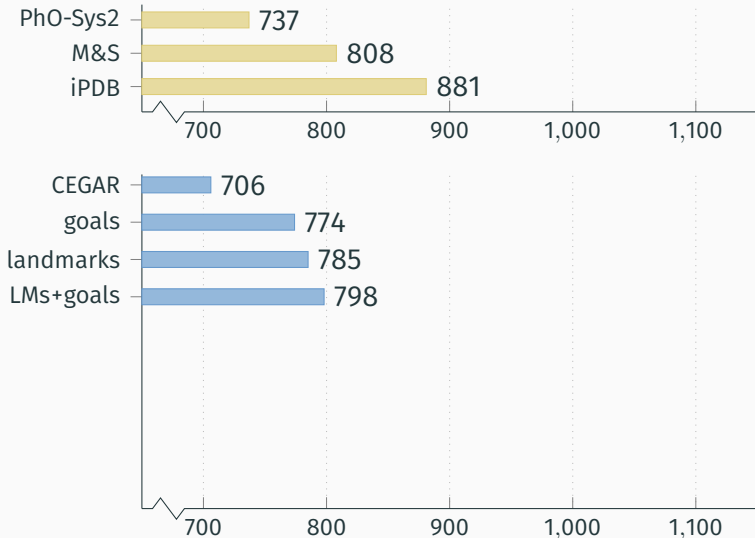




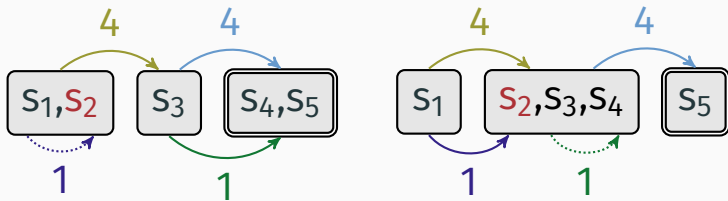
# Solved Tasks



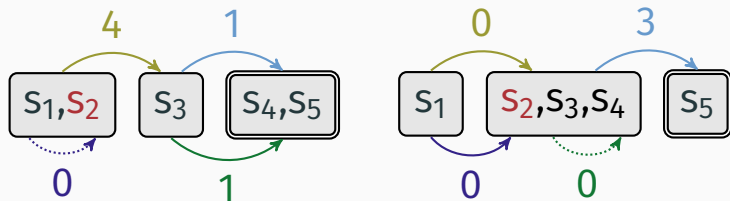
# Solved Tasks



## Order of Heuristics Is Important

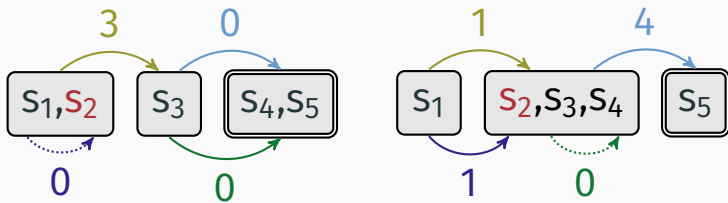


## Order of Heuristics Is Important



$$h_{\rightarrow}^{\text{SCP}}(s_2) = 5 + 3 = 8$$

## Order of Heuristics Is Important



$$h_{\rightarrow}^{\text{SCP}}(s_2) = 5 + 3 = 8$$

$$h_{\leftarrow}^{\text{SCP}}(s_2) = 3 + 4 = 7$$

## Finding a Good Order

- $n$  heuristics  $\rightarrow n!$  orders

## Finding a Good Order

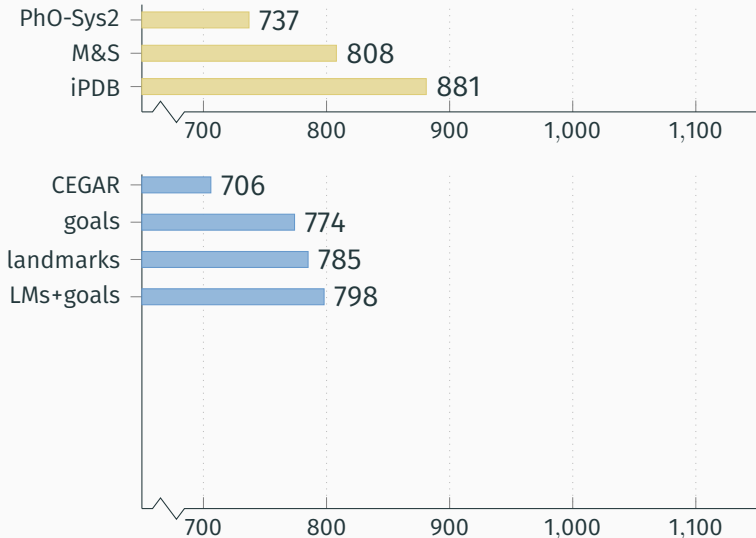
- $n$  heuristics  $\rightarrow n!$  orders
- $\rightarrow$  **search** for good order: greedy initial order + optimization

**Goal:** high estimates and low costs

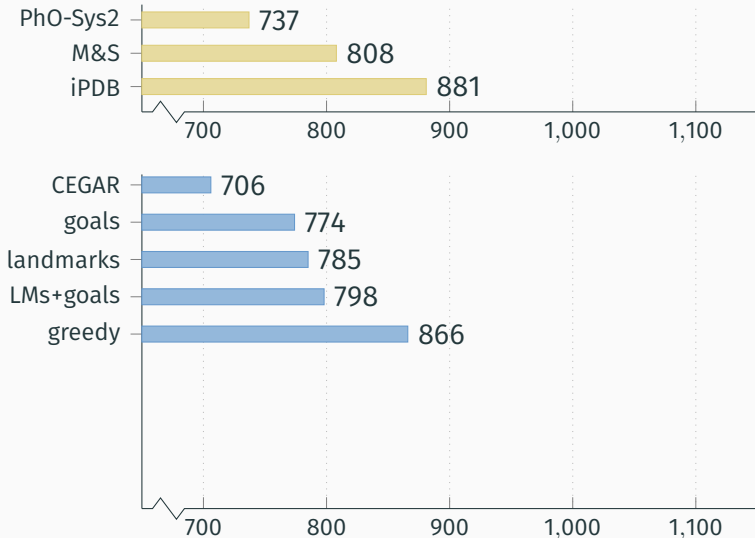


**Goal:** high estimates and low costs  
→ order by **heuristic/costs** ratio

# Solved Tasks



# Solved Tasks

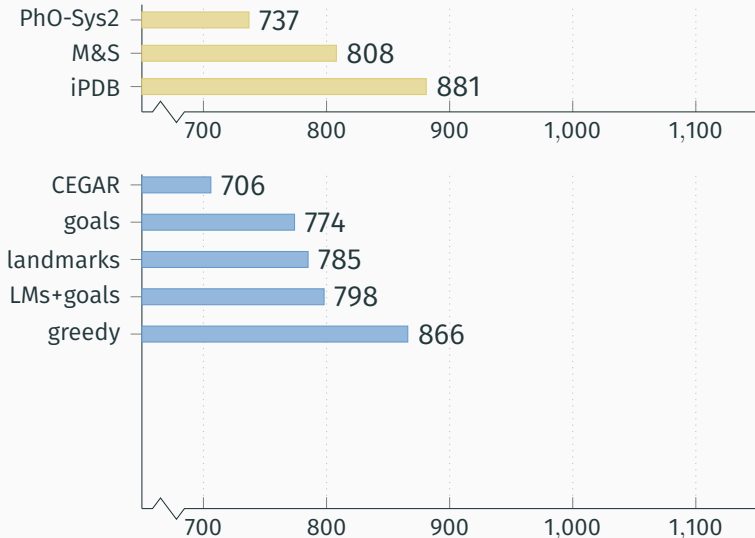


**Optimization:** finding initial order usually only first step

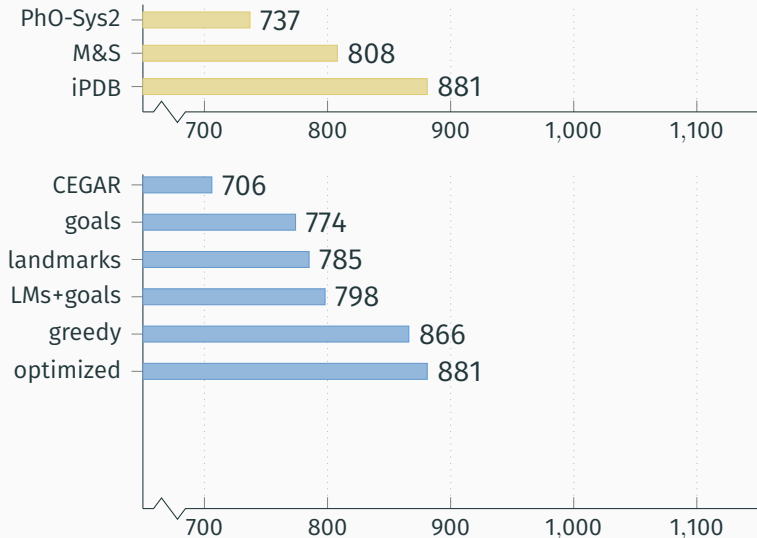
## Hill-climbing Search

- start with initial order
- until no better successor found:
  - switch positions of two heuristics
  - commit to first improving successor

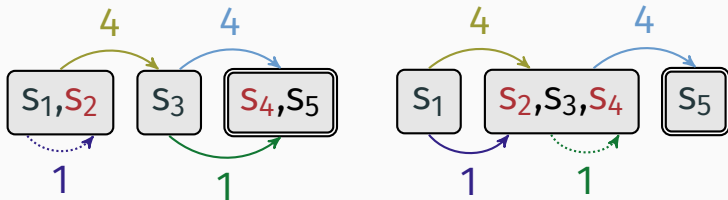
# Solved Tasks



# Solved Tasks



# One Order Is Not Enough



$$h_{\rightarrow}^{\text{SCP}}(s_2) = 8$$

$$h_{\leftarrow}^{\text{SCP}}(s_2) = 7$$

$$h_{\rightarrow}^{\text{SCP}}(s_4) = 3$$

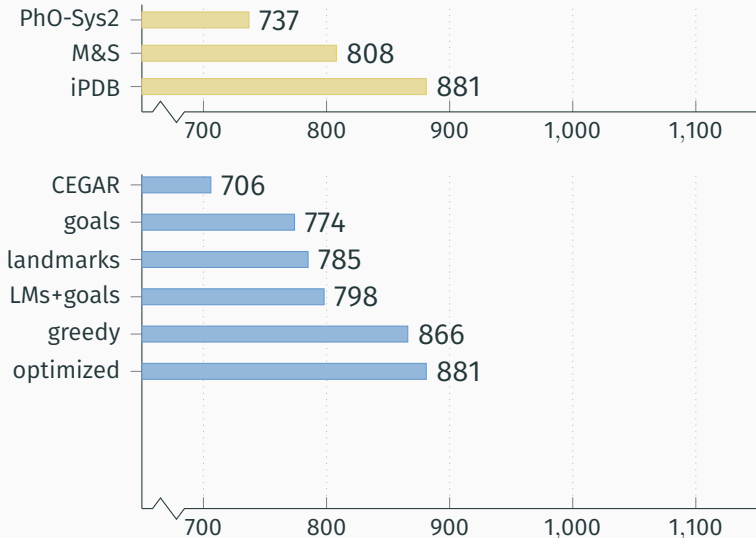
$$h_{\leftarrow}^{\text{SCP}}(s_4) = 4$$

Approach:

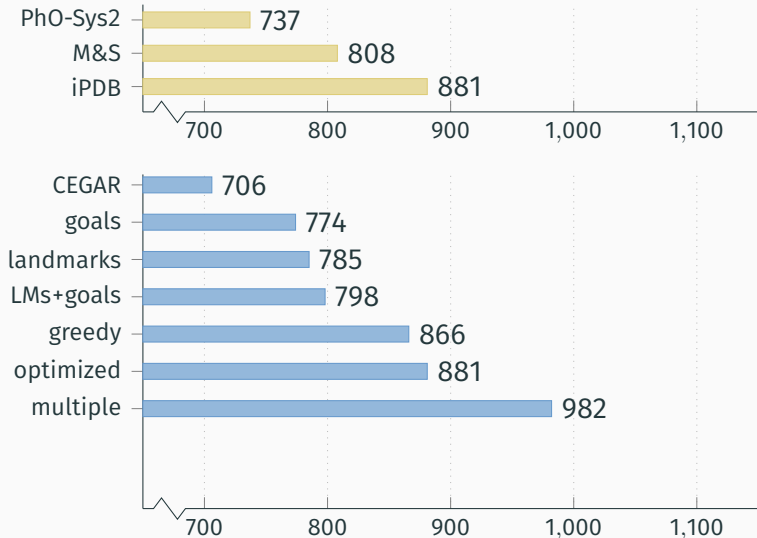
- compute saturated cost partitioning for **multiple orders**
- **maximize** over heuristic estimates



# Solved Tasks



## Solved Tasks



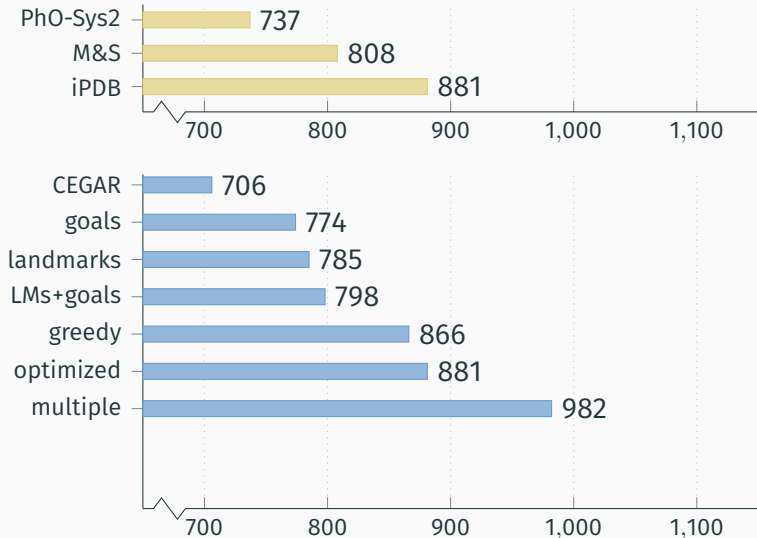
## Problems:

- many **useless orders**
- **slow** evaluation

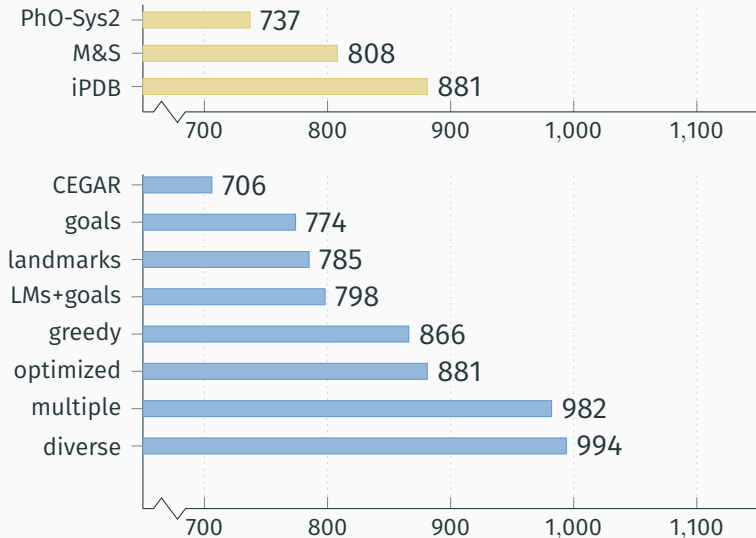
## Diversification Algorithm

- sample 1000 states
- start with empty set of orders
- until time limit is reached:
  - generate an optimized order
  - if a sample profits from it, keep it
  - otherwise, discard it

## Solved Tasks



# Solved Tasks



# Comparison of Cost Partitioning Algorithms

UCP

**Uniform Cost Partitioning**  
distribute costs evenly among relevant heuristics



GZOCP

UCP

Greedy Zero-one Cost Partitioning

order heuristics and give full cost to first relevant heuristic

GZOC

PhO

UCP

Post-hoc Optimization

compute weight for each heuristic and return weighted sum

GZOCP

PhO

CAN

UCP

Canonical Heuristic

maximum over sums of independent heuristic subsets

# Theoretical Comparison

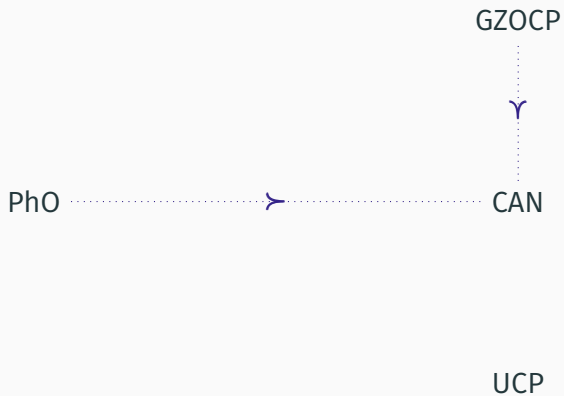
GZOCP

PhO .....  ..... CAN

UCP

Pommerening et al. 2013

# Theoretical Comparison



# Theoretical Comparison

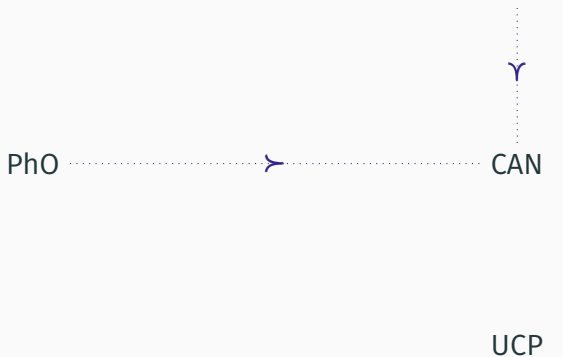
SCP

GZOCP

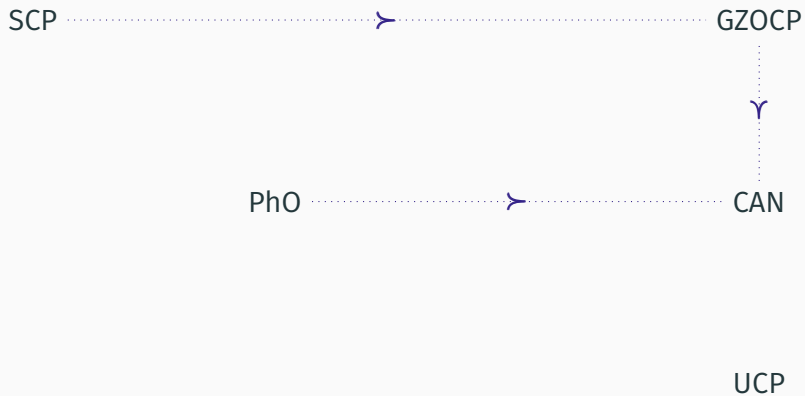
PhO

CAN

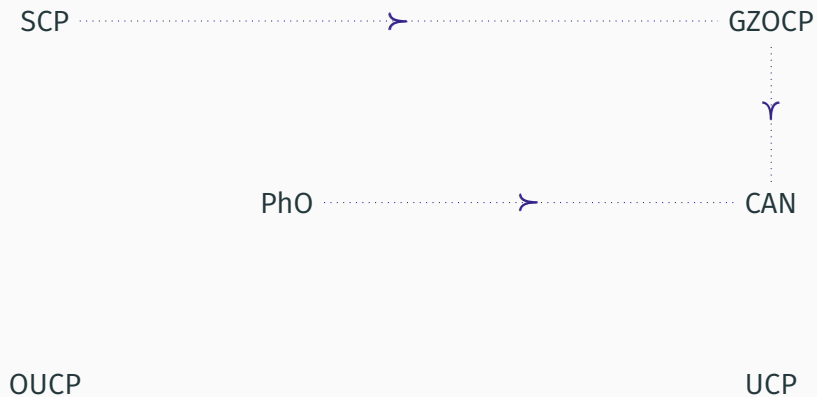
UCP



# Theoretical Comparison

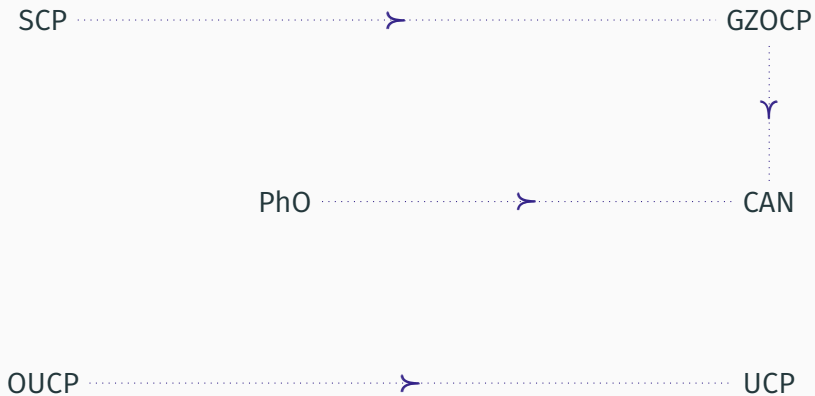


# Theoretical Comparison

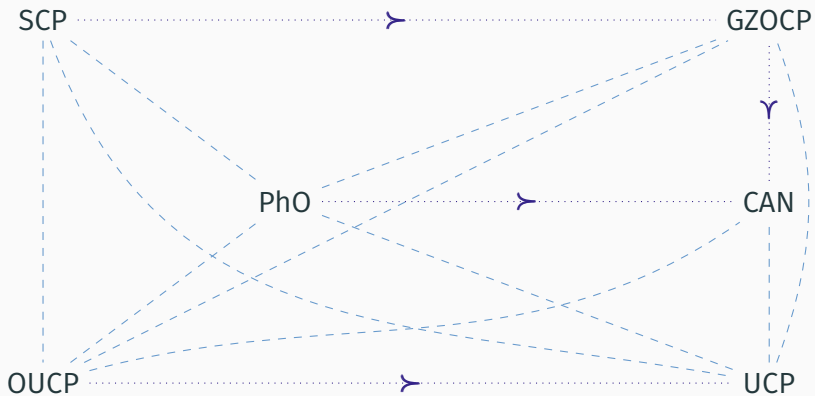




# Theoretical Comparison

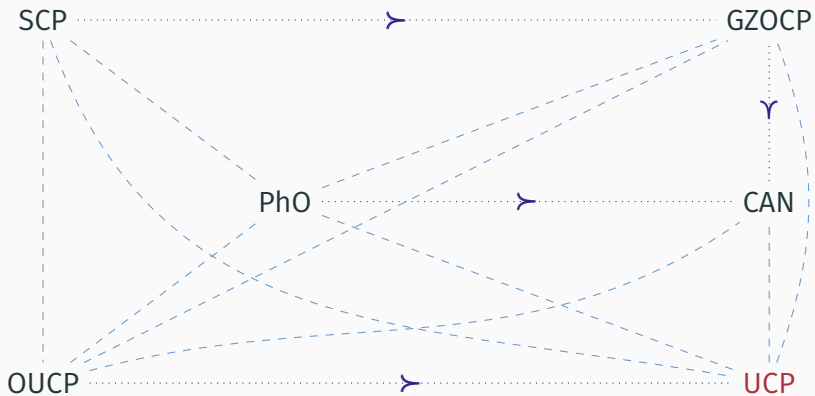


# Theoretical Comparison

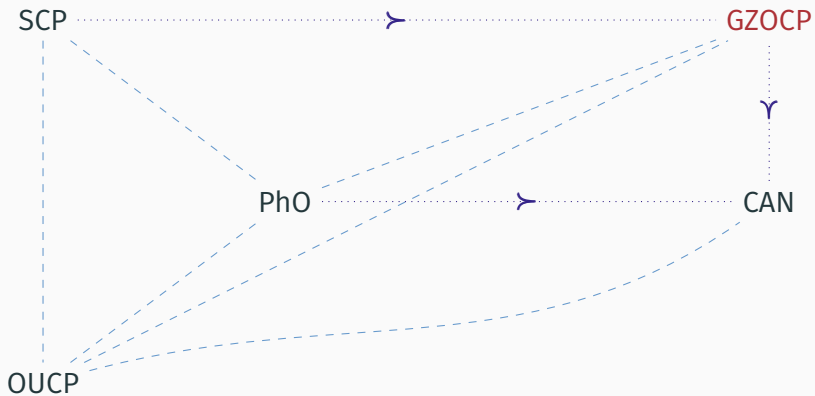


- Heuristics: Cartesian abstraction heuristics + PDBs

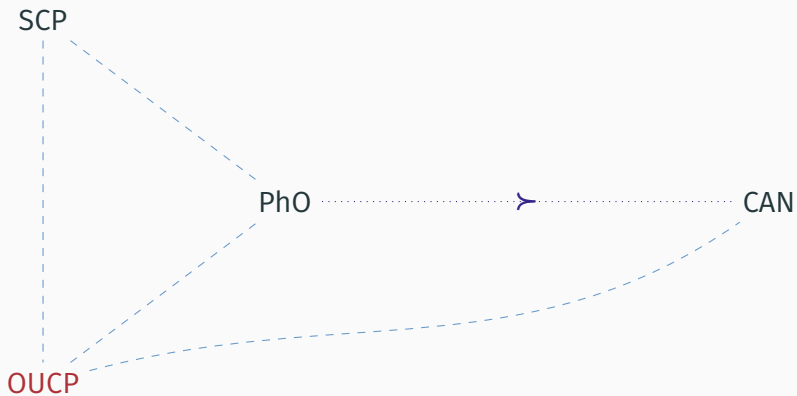
# Experimental Comparison



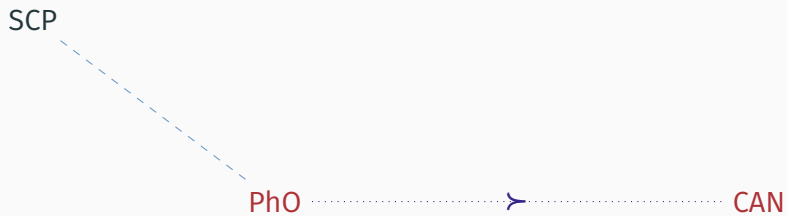
# Experimental Comparison



# Experimental Comparison



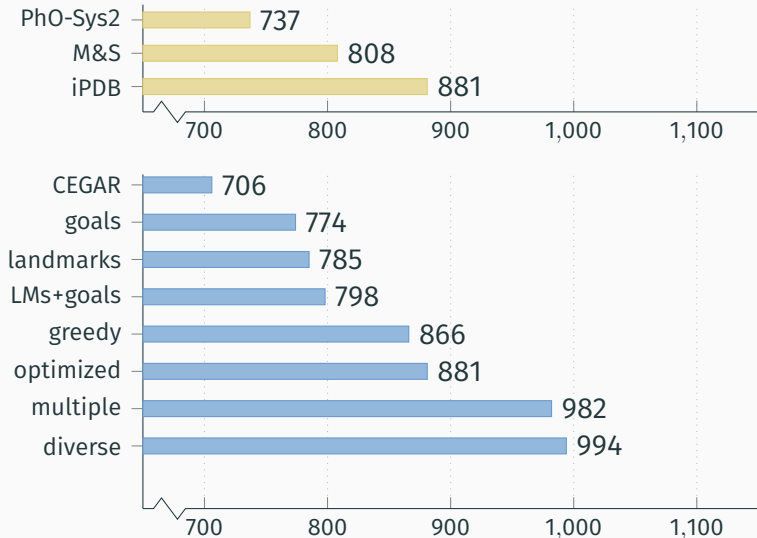
# Experimental Comparison



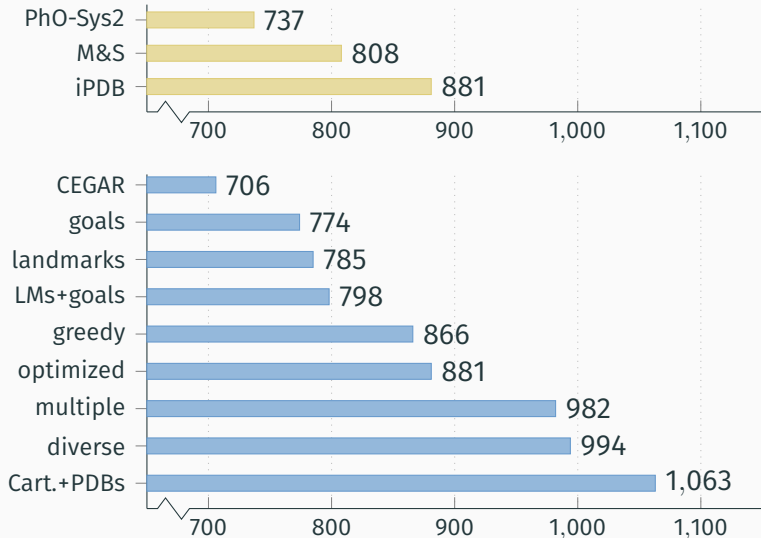
SCP



# Solved Tasks



# Solved Tasks



## Counterexample-guided Cartesian Abstraction Refinement

- refines abstraction only where needed
- decompositions yield complementary heuristics

## Counterexample-guided Cartesian Abstraction Refinement

- refines abstraction only where needed
- decompositions yield complementary heuristics

## Saturated Cost Partitioning

- assigns each heuristic only the costs it needs
- best results for diverse optimized orders

## Counterexample-guided Cartesian Abstraction Refinement

- refines abstraction only where needed
- decompositions yield complementary heuristics

## Saturated Cost Partitioning

- assigns each heuristic only the costs it needs
- best results for diverse optimized orders

## Comparison of Cost Partitioning Algorithms

- dominances and non-dominances
- saturated cost partitioning preferable in all settings