Theoretical Foundations for Structural Symmetries of Lifted PDDL Tasks

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July 13, 2019

Motivation

- Symmetries arise in many areas:
  - Model checking
  - SAT
  - Petri nets
  - Planning
- Planning symmetries (mostly) of ground representations
- Potential application of symmetries before grounding:
  - Invariant synthesis/Speed up grounding
  - Task transformations

Contributions

- Transfer structural symmetries to lifted planning tasks
- Investigate relationship between lifted and ground symmetries
- Provide graph representation of planning tasks for computing symmetries
- Quantitative analysis of IPC benchmarks

Outline

Abstract Structures

- S: set of symbols s, each with type $t(s)$
- Abstract structures over S:
  - $s \in S$ abstract structure
  - If $A_1, \ldots, A_n$ abstract structures, then also $\langle A_1, \ldots, A_n \rangle$ and $\{A_1, \ldots, A_n\}$ abstract structures

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- Example:
  - $S = \{a, b, c, d\}$, $t(a) = t(b) = t_1$, $t(c) = t(d) = t_2$
  - $A = \{\langle a, c \rangle, \langle b, d \rangle, \{c, d\}\}$

Structural Symmetries

- Symbol mapping σ: permutation of S with $t(\sigma(s)) = t(s)$
- Induced abstract structure mapping $\tilde{\sigma}$:

$$\tilde{\sigma}(A) := \begin{cases} \sigma(A) & \text{if } A \in S \\ \{\tilde{\sigma}(A_1), \dots, \tilde{\sigma}(A_n)\} & \text{if } A = \{A_1, \dots, A_n\} \\ \langle\tilde{\sigma}(A_1), \dots, \tilde{\sigma}(A_n)\rangle & \text{if } A = \langle A_1, \dots, A_n\rangle \end{cases}$$

- σ structural symmetry for abstract structure A if $\tilde{\sigma}(A) = A$

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- Symmetry:
  - Symbol mapping σ: swap *a* with *b*, swap *c* with *d*
  - σ is structural symmetry: $\tilde{\sigma}(A) = \{\langle b, d\rangle, \langle a, c\rangle, \{d, c\}\} = A$

Lifted Planning Tasks as Abstract Structures

- Lifted representation: normalized PDDL with action costs
- Lifted planning task Π as abstract structure:
  - Symbols with the following types: object, variable, predicate, function, negation, $n \in \mathbb{N}$
  - Abstract structures for modeling atoms, literals, function terms, operators, axioms

Example Operator (Spanner)

```
(:action pick-up
        :parameters (?s ?l)
        :precondition
            (and (LOCATION ?l)
                 (SPANNER ?s)
                 (bob-at ?l)
                 (spanner-at ?s ?l))
        :effect
            (and (not (spanner-at ?s ?l))
                 (carrying ?s)
                 (increase (total-cost) 1)))
```

$$\langle \{s, l\},$$
$$\{\langle location, l\rangle, \langle spanner, s\rangle, \langle bob\text{-}at, l\rangle, \langle spanner\text{-}at, s, l\rangle\},$$
$$\{\langle \emptyset, \emptyset, \langle \neg, \langle spanner\text{-}at, s, l\rangle\rangle, \langle \emptyset, \emptyset, \langle carrying, s\rangle\},$$
$$1\rangle$$

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Full Grounding

- *ground*(Π): fully grounded planning task Π

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Theorem

If σ is a structural symmetry for planning task Π, then σ is a structural symmetry for ground(Π).

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- Optimized grounding: remove some irrelevant part of the task representation (reachability analysis)
- Rational grounding ($ground_{rat}(\Pi)$): remove all or no symmetric irrelevant parts

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    - Example symmetry of STRIPS task Π: $\sigma(P(a)) = P(a)$ and $\sigma(P(b)) = Q(b)$
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- Other direction:
  - If σ symmetry of ground task Π (in our definition), then σ also symmetry of Π (in STRIPS)
  - If σ symmetry of lifted task Π, then σ also transition graph symmetry

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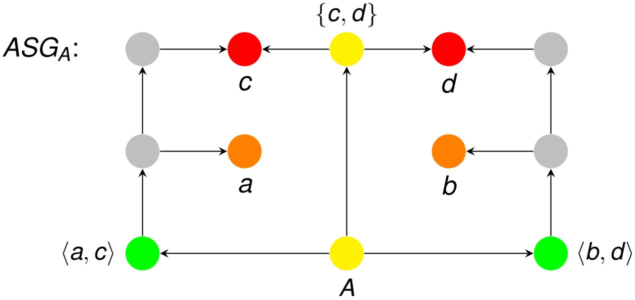

Properties

Let A be an abstract structure.

Theorem

Every colored graph automorphism of ASG_A induces a structural symmetry of A.

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Every structural symmetry of A induces a colored graph automorphism of ASG_A.

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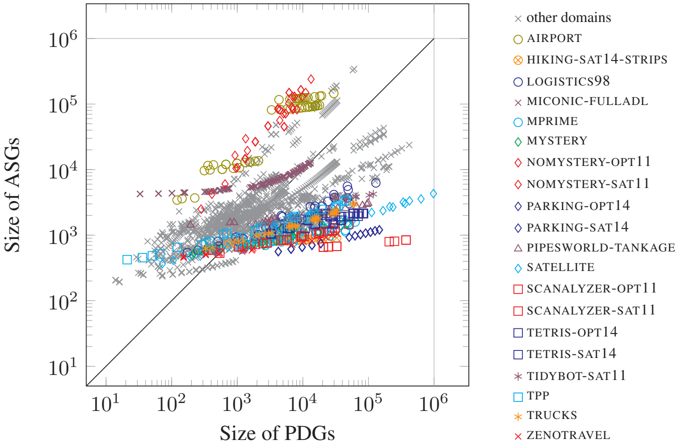
Summarized Results

- Roughly 53% of IPC tasks with lifted symmetries
- Ground symmetry groups often larger than lifted ones
- Quick computation using abstract structure graphs

Size of PDG vs. Abstract Structure Graph



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  - Structural symmetries of the lifted representation
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  - Lifted symmetries also ground symmetries
  - Graph representation of planning tasks
  - Many lifted symmetries in IPC benchmarks
- Future work:
  - Accelerated computation of invariants/grounding: consider only subset of (symmetric) objects (ICAPS 2018)
  - Task transformations