Additive Pattern Databases for Decoupled Search

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Motivation

- state of the art in optimal classical planning: explicit heuristic search with abstractions
- goal: use abstraction heuristics also with decoupled search
- contribution: pattern database (PDB) and their additive combination for decoupled search

Background

- planning tasks: finite-domain state variables for representing states

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- pattern database (PDB) heuristics:
  - project variables to a subset
  - store perfect heuristic values of abstraction

Decoupled Search in a Nutshell

- partition state variables to decompose the task: center factor and leaf factors
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  - pricing function: cost of reachable leaf states
  - $\rightarrow$ represents exponentially many (explicit) member states

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  - compile prices of current state into task
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- problems:
  - can be information-lossy (e.g., delete relaxation)
  - impractical for abstraction-based heuristics

Alternative Definition for Computing Decoupled Heuristics

explicit decoupled heuristic

$$h_{\mathcal{F},\mathrm{ex}}(s^{\mathcal{F}}) = \min_{s \in [s^{\mathcal{F}}]} \mathrm{price}(s^{\mathcal{F}}, s) + h(s)$$

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- problem: exponentially many member states

Single PDBs for Decoupled Search

reminder: explicit decoupled heuristic

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Decoupled PDB

$$\text{dPDB}(h^P, s^{\mathcal{F}}) = \min_{s^P \in S^P} \text{price}(s^{\mathcal{F}}, s^P) + h^P(s^P)$$

Combining Multiple PDBs

explicit search

- given: $\mathcal{H} = \{H_1, \dots, H_n\}$ with H_i additive set of (PDB) heuristics
- canonical combination:

$$h^{\mathcal{H}}(s) = \max_{H \in \mathcal{H}} \sum_{h \in H} h(s)$$

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- how to transfer to decoupled search?

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- compute decoupled PDBs individually:

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properties

- information-lossy: use different minimizing member state in each PDB
- inadmissible: may count prices of leaves multiple times in different heuristics

Explicit Decoupled Canonical Heuristic (1)

reminder: explicit decoupled heuristic

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- practical implementation via branch-and-bound
- incremental computation of member states allows pruning
- worst case: enumeration of all exponentially many member states

Polynomial-time Approximations (1)

reminder: explicit decoupled canonical heuristic

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- admissible, but lossy approximation

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Leaf-Disjoint PDBs

additive sets: pairwise leaf-disjoint PDBs

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Single-Leaf PDBs

each PDB affects at most one leaf

- minimize sum of prices and heuristic separately for each set of affected leaves
- heuristic value equals $h^{\mathcal{H}}_{\mathcal{F},\mathrm{ex}}$

Experiments

F					
MM					

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	explicit search				
		LD	SL		
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Experiments

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				expl. dec. heur.		
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F	284	206	293	206	212	
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		LD	SL	no pruning	pruning	LD	SL
F	284	206	293	206	212	210	**304**
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Conclusions

- alternative way of computing explicit heuristics for decoupled search
- efficient computation of PDBs for decoupled search
- admissible combination of sets of additive PDBs NP-complete
- practical implementation and polynomial-time approximations