

Additive Pattern Databases for Decoupled Search

Silvan Sievers¹ Daniel Gnad² Álvaro Torralba³

¹University of Basel, Switzerland

²Linköping University, Sweden

³Aalborg University, Denmark

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- ▶ state of the art in optimal classical planning: explicit heuristic search with **abstractions**
- ▶ goal: use abstraction heuristics also with **decoupled search**
- ▶ contribution: **pattern database (PDB)** and their **additive combination** for decoupled search

- ▶ planning tasks: finite-domain **state variables** for representing states

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- ▶ pattern database (PDB) heuristics:
 - ▶ **project** variables to a **subset**
 - ▶ store perfect heuristic values of abstraction

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- ▶ decoupled state $s^{\mathcal{F}}$:
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 - ▶ pricing function: cost of reachable leaf states
 - ▶ → represents exponentially many (explicit) member states

Heuristics for Decoupled Search So Far

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- ▶ **problems**:
 - ▶ can be **information-lossy** (e.g., delete relaxation)
 - ▶ **impractical** for abstraction-based heuristics

explicit decoupled heuristic

$$h_{\mathcal{F},\text{ex}}(s^{\mathcal{F}}) = \min_{s \in [s^{\mathcal{F}}]} \text{price}(s^{\mathcal{F}}, s) + h(s)$$

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- ▶ **problem:** exponentially many member states

reminder: explicit decoupled heuristic

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Single PDBs for Decoupled Search

reminder: explicit decoupled heuristic

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Decoupled PDB

$$\text{dPDB}(h^P, s^{\mathcal{F}}) = \min_{s^P \in \mathcal{S}^P} \text{price}(s^{\mathcal{F}}, s^P) + h^P(s^P)$$

Combining Multiple PDBs

explicit search

- ▶ given: $\mathcal{H} = \{H_1, \dots, H_n\}$ with H_i additive set of (PDB) heuristics
- ▶ **canonical** combination:

$$h^{\mathcal{H}}(s) = \max_{H \in \mathcal{H}} \sum_{h \in H} h(s)$$

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- ▶ how to **transfer** to decoupled search?

reminder: canonical heuristic

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Naïve Combination

reminder: canonical heuristic

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properties

- ▶ **information-lossy**: use different minimizing member state in each PDB
- ▶ **inadmissible**: may count prices of leaves multiple times in different heuristics

Explicit Decoupled Canonical Heuristic (1)

reminder: explicit decoupled heuristic

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Explicit Decoupled Canonical Heuristic (2)

complexity

computing $h_{\mathcal{F},\text{ex}}^{\mathcal{H}}$ is **NP**-complete

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- ▶ practical implementation via **branch-and-bound**
- ▶ incremental computation of member states allows **pruning**
- ▶ worst case: enumeration of all exponentially many member states

Polynomial-time Approximations (1)

reminder: explicit decoupled canonical heuristic

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- ▶ alternative: consider each $H \in \mathcal{H}$ **independently**, i.e., **swap min and max**:

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- ▶ **admissible, but lossy** approximation

Leaf-Disjoint PDBs

additive sets: pairwise leaf-disjoint PDBs

Polynomial-time Approximations (2)

Leaf-Disjoint PDBs

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Single-Leaf PDBs

each PDB affects at most one leaf

Polynomial-time Approximations (2)

Leaf-Disjoint PDBs

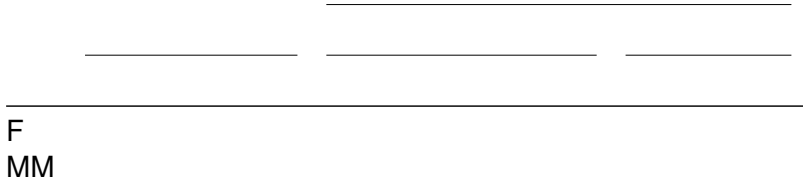
additive sets: pairwise leaf-disjoint PDBs

Single-Leaf PDBs

each PDB affects at most one leaf

- ▶ minimize sum of prices and heuristic separately for each set of affected leaves
- ▶ heuristic value equals $h_{\mathcal{F},\text{ex}}^{\mathcal{H}}$

Experiments



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	explicit search		
	LD	SL	
F	284	206	293
MM	749	662	743

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	explicit search		decoupled search	
	LD	SL	expl. dec. heur.	no pruning
F	284	206	293	206
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	explicit search		decoupled search		
	LD	SL	expl. dec. heur.		
			no pruning	pruning	
F	284	206	293	206	212
MM	749	662	743	596	628

Experiments

	explicit search			decoupled search		
	LD	SL		expl. dec. heur.		poly. approx.
				no pruning	pruning	LD
F	284	206	293	206	212	210
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Experiments

	decoupled search						
	explicit search		expl. dec. heur.		poly. approx.		
	LD	SL	no pruning	pruning	LD	SL	
F	284	206	293	206	212	210	304
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Conclusions

- ▶ **alternative way** of computing explicit heuristics for decoupled search
- ▶ **efficient computation of PDBs** for decoupled search
- ▶ admissible combination of sets of additive PDBs **NP-complete**
- ▶ practical implementation and **polynomial-time approximations**