Learning Heuristic Functions Through Supervised Learning

Master Thesis

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Outline

1. Background
   - MDP
2. Learning a Heuristic Function
   - Machine Learning
   - Gradient Descent Methods
3. Results
   - Experimental environment
   - Parameters
   - Final result
4. Conclusion and Future Work

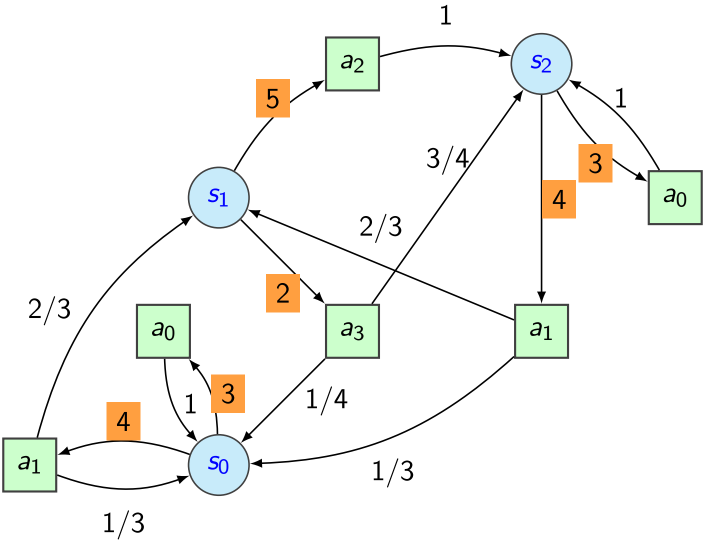
MDP

Definition (Markov Decision Process)

A MDP is a 6-tuple $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, H, s_0 \rangle$ where:

- $\mathcal{S}$ – the finite set of states
- $\mathcal{A}$ – the finite set of actions
- $\mathcal{T} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ – the **transition function**
- $\mathcal{R} : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ – the reward function
- $H \in \mathbb{N}$ – the **finite horizon**
- $s_0 \in \mathcal{S}$ – the **initial state**

Example of a MDP



MDP

A policy $\pi : \mathcal{S} \times \{1, ..., H\} \rightarrow \mathcal{A}$.
State-values: $V_\pi(s, d)$ and action value: $Q_\pi(s, \pi(s, d))$.

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A factored MDP:

- A finite-domain variable v associated with $\mathcal{D}_v$
- A finite set of finite-domain variables $\mathcal{V}$
- Define state using $\mathcal{V}$
- For state $s \in \mathcal{S}$ we have a set of facts $\mathcal{F}(s) = \{\langle v, d\rangle \mid v \in \mathcal{V} \wedge d \in \mathcal{D}_v\}$

MDP

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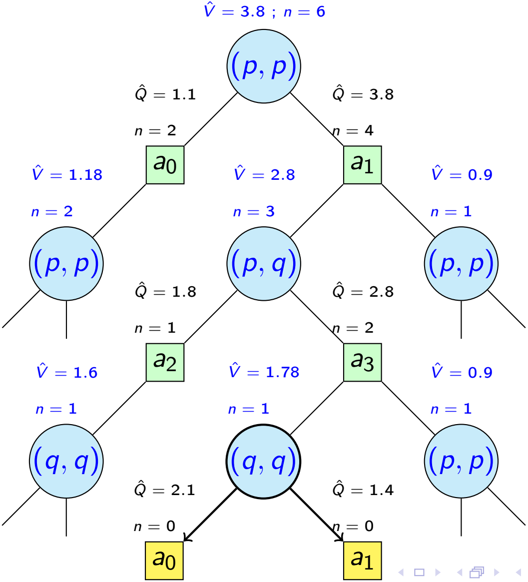
Search algorithm: UCT*

- Action Selection
- Expansion
- Backup

UCT* - Expansion



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Learning a Heuristic Function

Machine Learning

Automated learning from data.

Learning a Heuristic Function

Machine Learning

Automated learning from data.

For each action $a \in \mathcal{A}$:

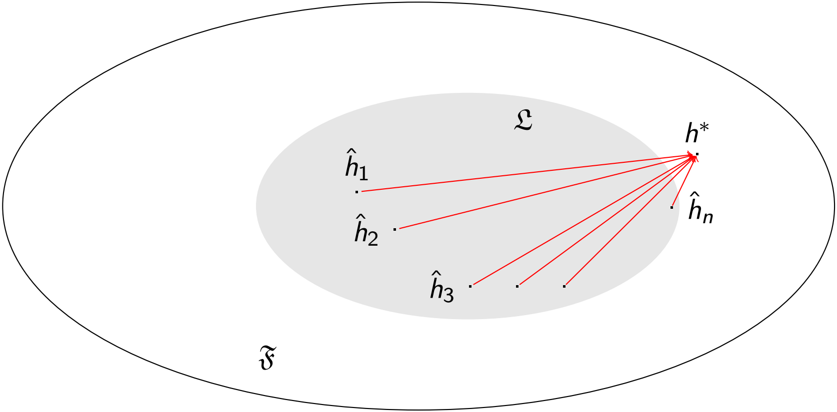
- We want to learn $h^* : \mathcal{S} \to \mathbb{R}$
- A given data set $\mathfrak{D} = \{(s, \hat{Q})\}_{i=1}^{m}$ with samples $h^*(s) = \hat{Q}$
- We want to find $\hat{h} : \mathcal{S} \to \mathbb{R}$ which approximates h^*
- Search in $\mathfrak{L}$ – the space of linear functions

$$\hat{h}(s) = \sum_{f \in \mathcal{F}(s)} w_f$$

- Evaluating $\hat{h}$ with the **Mean Square Error** function:

$$J(\mathcal{W}, s) = \sum_{(s,\hat{Q}) \in \mathcal{D}} (\hat{h}(s) - \hat{Q})^2$$

Learning a Heuristic Function



Gradient Descent Methods

Update of weights in the opposite direction of the MSE gradient.

$$w_f := w_f - \alpha \frac{\partial J(\mathcal{W}, s)}{\partial w_f}, \forall w_f \in \mathcal{W}$$

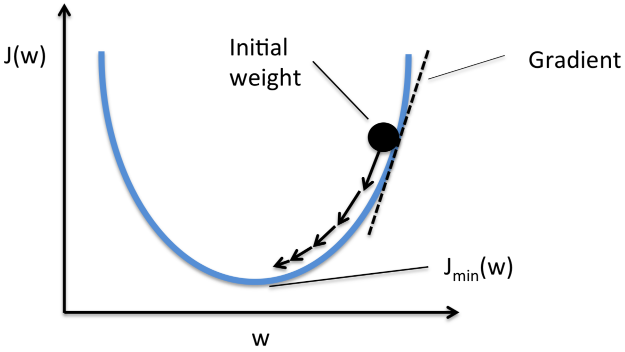
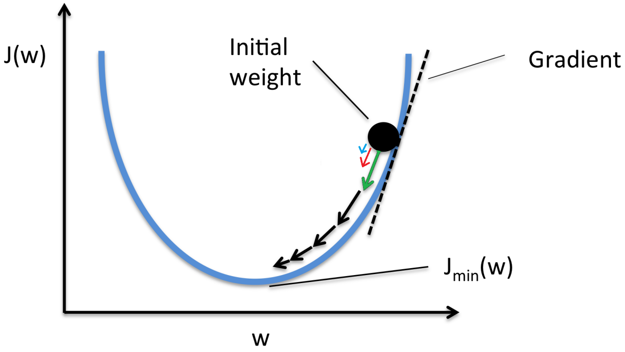


Image credits: Sebastian Raschka (https://sebastianraschka.com).

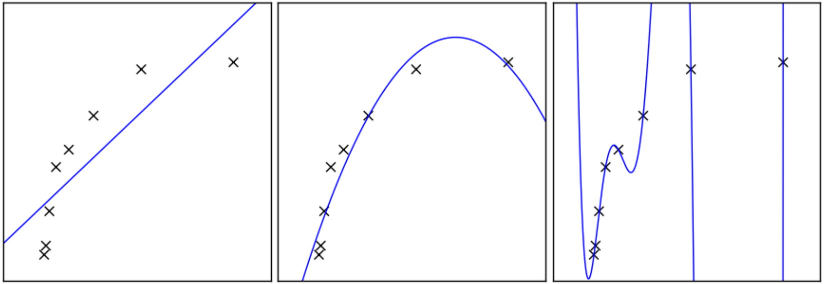
Gradient Descent Methods

Based on the portion of $\mathfrak{D}$ we have three gradient descent types:

- Stochastic Gradient Descent – each data set entry
- Batch Gradient Descent – whole data set
- Mini Batch Gradient Descent – subset of the data set



Underfitting and Overfitting



d = 1 (under-fit) · d = 2 · d = 6 (over-fit)

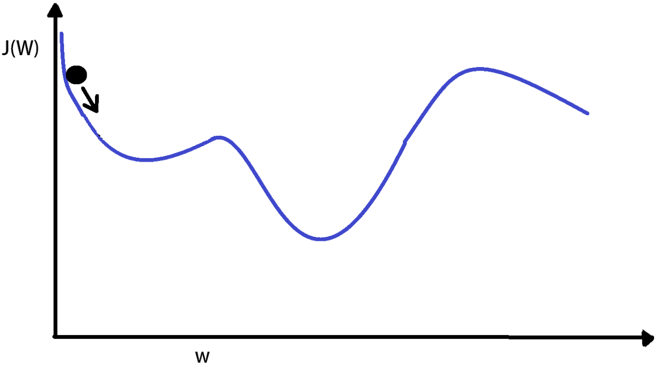
Image credits: Radim Rehurek (http://radimrehurek.com).

Gradient Descent Methods

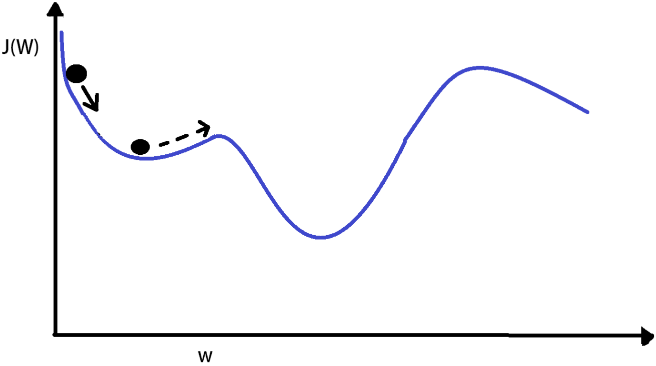
Improvements:

- Early stopping
- Learning rate decay
- Momentum

Momentum



Momentum



Gradient Descent Methods

To recap what we have:

- A data set $\mathfrak{D} = \{(s, \hat{Q})\}_{i=1}^{n}$
- Function approximation:

$$\hat{h}(s) = \sum_{f \in \mathcal{F}(s)} w_f$$

- Parameters – GD type, number of epochs, learning rate, momentum, learning rate decay, early stopping

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Results

Experimental environment:

- 12 domains and 10 instances per domain from IPPC 2014 and IPPC 2011
- Benchmarking against
  - DP-UCT
  - Winning algorithm of IPPC 2011
  - Winning algorithm of IPPC 2014

Goal

Find a parameter configuration for learning a heurstic function which outperforms baseline algorithms.

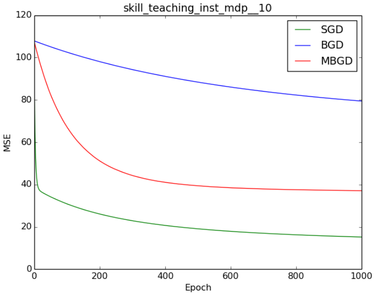
Results

Parameters:

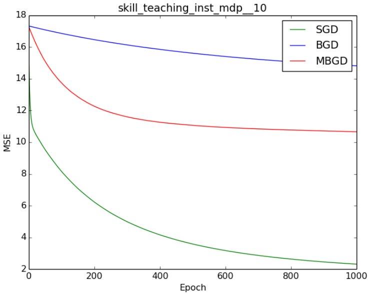
- Heuristic Function
- Data set size
- Gradient Descent Types
- Learning rate
- Number of epochs
- Momentum and learning rate decay
- Combination of features

Heuristic Functions

We wanted to learn two heuristic functions: IDS and UCT* using IDS.



a. IDS



b. UCT* using IDS

Figure: $\alpha = 0.00001$, $\sigma = 0.05$, $\gamma = 0$ and number of epochs 1000

Results

Parameters:

- Heuristic Function – **IDS** $\sim$ 75 and **UCT*** $\sim$ 35
- Data set size
- Gradient Descent Types
- Learning rate
- Number of epochs
- Momentum and learning rate decay
- Combination of features

Results

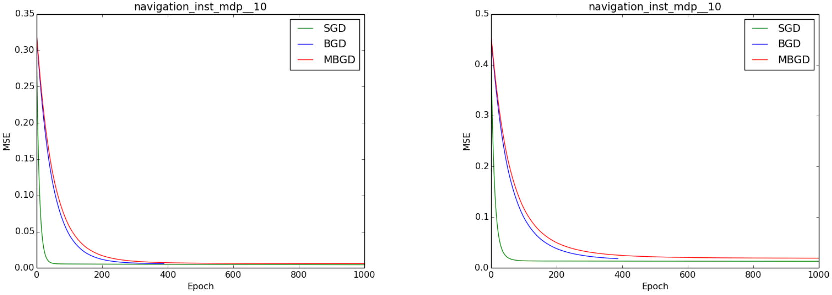
Parameters:

- Heuristic Function – **IDS** ~ 75 and **UCT*** ~ 35
- Data set size – **Bigger data set is better**
- Gradient Descent Types
- Learning rate
- Number of epochs
- Momentum and learning rate decay
- Combination of features

Gradient Descent Types

SGD performing best:

DP-UCT	IPPC2011	IPPC2014	SGD	BGD	MBGD
0.0	0.0	0.75	**1.0**	0.63	0.81



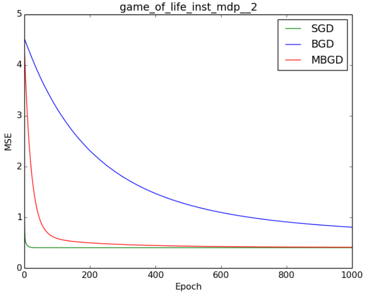
a. Training b. Testing

Figure: $\alpha = 0.0001$, $\sigma = 0$, $\gamma = 0$ and number of epochs 1000

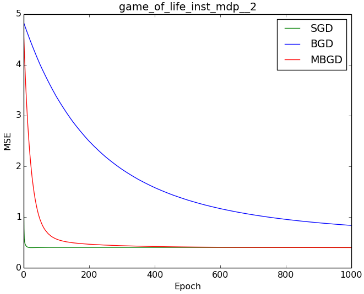
Gradient Descent Types

BGD performing best:

DP-UCT	IPPC2011	IPPC2014	SGD	BGD	MBGD
0.71	0.93	0.75	0.91	**1.0**	0.85



a. Training



b. Testing

Figure: $\alpha = 0.0005$, $\sigma = 0.05$, $\gamma = 0$ and number of epochs 1000

Results

Parameters:

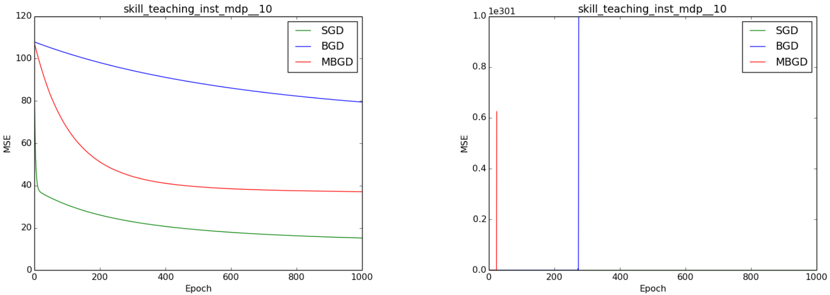
- Heuristic Function – **IDS** $\sim$ 75 and **UCT*** $\sim$ 35
- Data set size – **Bigger data set is better**
- Gradient Descent Types – **Best choice depends on the task**
- Learning rate
- Number of epochs
- Momentum and learning rate decay
- Combination of features

Results

Parameters:

- Heuristic Function – **IDS** $\sim$ 75 and **UCT*** $\sim$ 35
- Data set size – **Bigger data set is better**
- Gradient Descent Types – **Best choice depends on the task**
- Learning rate – **Best choice depends on the task**
- Number of epochs – **Best choice depends on the task**
- Momentum and learning rate decay
- Combination of features

Momentum



a. $\gamma = 0$ b. $\gamma = 0.3$

Figure: IDS, $\alpha = 0.00001$, $\sigma = 0.05$, $\gamma = 0$ and number of epochs 1000

Results

Parameters:

- Heuristic Function – **IDS** $\sim$ 75 and **UCT*** $\sim$ 35
- Data set size – **Bigger data set is better**
- Gradient Descent Types – **Best choice depends on the task**
- Learning rate – **Best choice depends on the task**
- Number of epochs – **Best choice depends on the task**
- Momentum and learning rate decay – **No momentum and small learning rate decay**
- Combination of features

Combination of features

Combining state features into pairs and generating new features.

$$\mathcal{F}^2(s) =$$
$$\{\langle v_1, d_1, v_2, d_2\rangle \mid v_1, v_2 \in \mathcal{V} \wedge d_1, d_2 \in \mathcal{D}_v \wedge s[v_1] = d_1 \wedge s[v_2] = d2\}$$

Combination of features

Combining state features into pairs and generating new features.

$$\mathcal{F}^2(s) =$$
$$\{\langle v_1, d_1, v_2, d_2\rangle \mid v_1, v_2 \in \mathcal{V} \wedge d_1, d_2 \in \mathcal{D}_v \wedge s[v_1] = d_1 \wedge s[v_2] = d2\}$$

Outcome:

- Number of new features $\binom{n}{2} - \frac{n}{2}$
- More accurate action-values but worse result
- More exploitation
- Less trials

Results

Parameters:

- Heuristic Function – **IDS** $\sim$ 75 and **UCT*** $\sim$ 35
- Data set size – **Bigger data set is better**
- Gradient Descent Types – **Best choice depends on the task**
- Learning rate – **Best choice depends on the task**
- Number of epochs – **Best choice depends on the task**
- Momentum and learning rate decay – **No momentum and small learning rate decay**
- Combination of features – **Without expected improvement**

Results

Final result

No single parameter configuration which outperforms the baseline algorithms.

Results

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No single parameter configuration which outperforms the baseline algorithms.

There is a *Set of Best Parameter Configurations* for which the baseline algorithms are outperformed.

Results

	DP-UCT	IPPC 2011	IPPC 2014	SBPC OH
Wildfire	0.79	0.83	0.69	**0.86**
Triangle	0.42	0.39	**0.88**	0.7
Academic	**0.68**	0.3	0.31	0.6
Elevators	0.59	**0.96**	**0.96**	0.9
Tamarisk	0.64	**0.9**	0.88	**0.9**
Sysadmin	0.61	0.74	0.8	**0.99**
Recon	0.58	**0.98**	0.94	**0.98**
Game	0.71	0.91	**0.97**	0.96
Traffic	0.86	0.93	**0.98**	0.7
Crossing	0.42	0.81	**0.99**	0.83
Skill	0.93	0.93	0.94	**1.0**
Navigation	0.67	0.58	0.92	**1.0**
Total	0.66	0.77	0.86	**0.87**

Conclusion

"All models are wrong, but some models are useful." - George Box

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Conclusion:

- No single parameter configuration for learning an offline heuristic in domain independent probabilistic planning.
- Many parameters which demand a large number of experiments and fine tuning.
- A set of parameter configurations was found that outperforms the baseline algorithms for 0.01.

Future Work

Future work:

- Other Gradient descent methods
- More domain knowledge for better analysis of the data set
- Investigating the underperformance of combined features approach
- Multiple iterations of learning
- Online learning

Questions?

Additional slides

Policy

Definition (Value functions)

Let $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R}, H, s_0 \rangle$ be a MDP, $s \in \mathcal{S}$ a state and π a policy. The **state-value** $V_\pi(s, d)$ of s under π with d steps to go is defined as

$$V_\pi(s, d) = Q_\pi(s, \pi(s, d))$$

where the **action-value** $Q_\pi(s, \pi(s, d))$ under π is defined as

$$Q_\pi(s, a) = \begin{cases} \mathcal{R}(s, a) + \sum_{s' \in \mathcal{S}} (\mathbb{P}_\mathcal{T}[s' | s, a] \cdot V_\pi(s', d - 1)) & , d > 0 \\ \mathcal{R}(s, a) & , d = 0 \end{cases}$$

for all state-action pairs (s, a).

Policy

Definition (Optimal policy)

Let the Bellman optimality equation for a state $s \in \mathcal{S}$ be a set of equations that describe $V^*(s, d)$, where

$$V^*(s, d) = \max_{a \in \mathcal{A}} Q^*(s, d, a)$$

$$Q^*(s, d, a) = \begin{cases} \mathcal{R}(s, a) + \sum_{s' \in \mathcal{S}} (\mathbb{P}_{\mathcal{T}}[s' | s, a] \cdot V^*(s', d - 1)) & , d > 0 \\ \mathcal{R}(s, a) & , d = 0 \end{cases}$$

A policy π^* is an **optimal policy** if $\pi^* \in \arg\max_{a \in \mathcal{A}} Q^*(s, d, a)$ for all $s \in \mathcal{S}$.

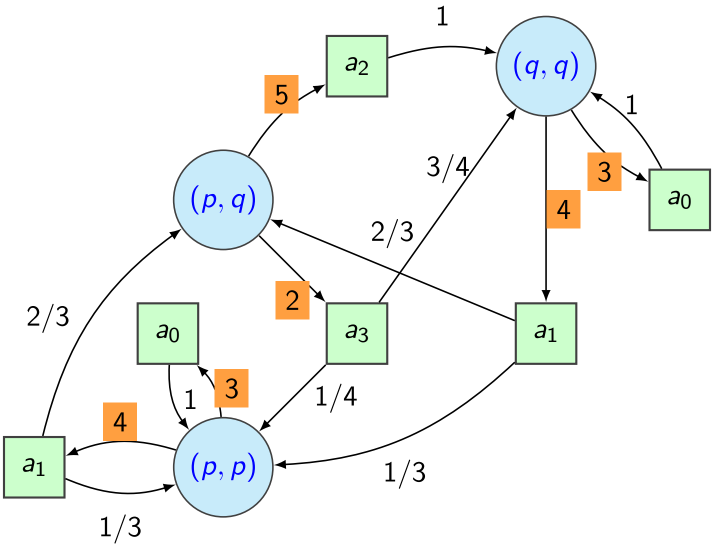
Planning task

Definition (Planning task)

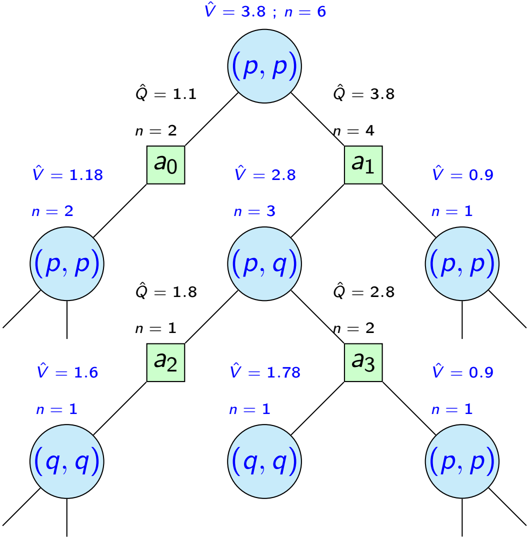
A **planning task** is a 4-tuple $T = \langle \mathcal{V}, \mathcal{A}, H, s_0 \rangle$ where:

- $\mathcal{V}$ – the finite set **of finite-domain variables** v with domain $\mathcal{D}_v$
- $\mathcal{A}$ – the finite set of **actions** $\langle effect_a, reward_a \rangle = a \in \mathcal{A}$ where
  - $effect_a$ – is a probability distribution over partial variable assignment $\{(p_i^a, e_i^a)\}_{i=1}^n$ where p_i^a is a probability, e_i^a is a partial variable assignment and $\sum_{i=1}^{n} p_i^a = 1$
  - $reward_a$ – the reward of applying action a
- $H \in \mathbb{N}$ – the **finite horizon**
- $s_0 \in \mathcal{V}$ – the **initial state**

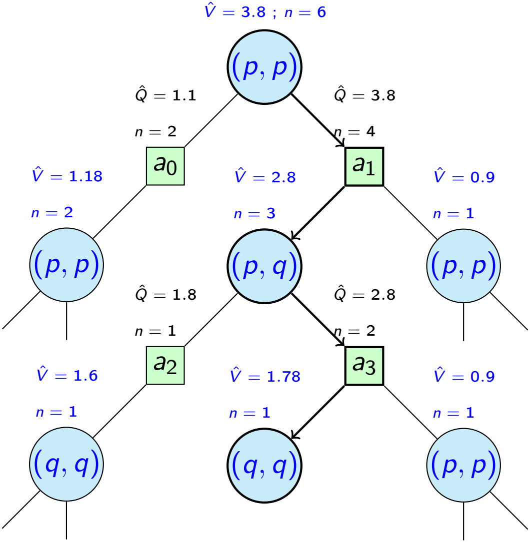
Example of a factored MDP



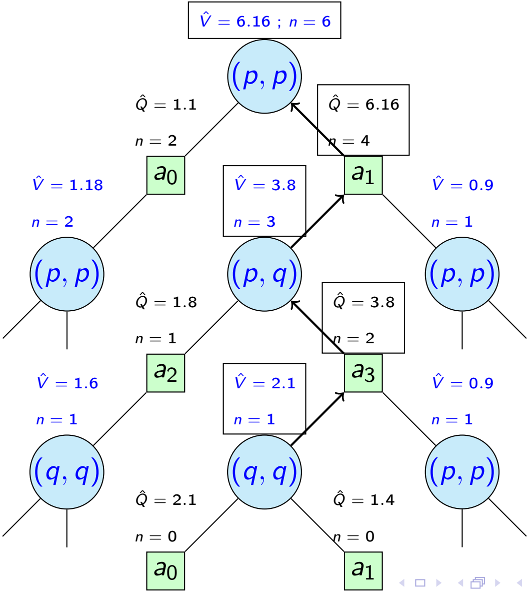
UCT* - Action Selection



UCT* - Action Selection



UCT* - Backup


$\hat{V} = 6.16$; $n = 6$
(p, p)
$\hat{Q} = 1.1$
$n = 2$
$\hat{Q} = 6.16$
$n = 4$
a_0
a_1
$\hat{V} = 1.18$
$n = 2$
$\hat{V} = 3.8$
$n = 3$
$\hat{V} = 0.9$
$n = 1$
(p, p)
(p, q)
(p, p)
$\hat{Q} = 1.8$
$n = 1$
$\hat{Q} = 3.8$
$n = 2$
a_2
a_3
$\hat{V} = 1.6$
$n = 1$
$\hat{V} = 2.1$
$n = 1$
$\hat{V} = 0.9$
$n = 1$
(q, q)
(q, q)
(p, p)
$\hat{Q} = 2.1$
$n = 0$
$\hat{Q} = 1.4$
$n = 0$
a_0
a_1

Gradient Descent Methods

Gradient descent methods:

$$w_f := w_f - \alpha \frac{\partial J(\mathcal{W}, s)}{\partial w_f}, \forall w_f \in \mathcal{W}$$

Where we have the error function over the data set $\mathfrak{D}$':

$$J(\mathcal{W}, s) = \frac{1}{2m} \sum_{(s,\hat{Q}) \in \mathfrak{D}} (\hat{h}(s) - \hat{Q})^2$$

and by deriving we get:

$$\frac{\partial}{\partial w_f} J(\mathcal{W}, s) = \frac{1}{m} \sum_{(s,\hat{Q}) \in \mathfrak{D}} (\hat{h}(s) - \hat{Q}) \cdot \frac{\partial}{\partial w_f} \hat{h}(s), \forall w_f \in \mathcal{W}$$

Gradient Descent Methods

Based on the portion of $\mathfrak{D}$ we have three gradient descent types:

Gradient Descent Methods

Based on the portion of $\mathfrak{D}$ we have three gradient descent types:

- Stochastic Gradient Descent

$$\frac{\partial}{\partial w_f} J(\mathcal{W}, s) = (\hat{h}(s) - \hat{Q}) \cdot \frac{\partial}{\partial w_f} \hat{h}(s), \forall w_f \in \mathcal{W}$$

- Batch Gradient Descent

$$\frac{\partial}{\partial w_f} J(\mathcal{W}, s) = \frac{1}{m} \sum_{(s,\hat{Q}) \in \mathfrak{D}} (\hat{h}(s) - \hat{Q}) \cdot \frac{\partial}{\partial w_f} \hat{h}(s), \forall w_f \in \mathcal{W}$$

- Mini Batch Gradient Descent

$$\frac{\partial}{\partial w_f} J(\mathcal{W}, s) = \frac{1}{k - j} \sum_{(s,\hat{Q}) \in \mathfrak{B}} (\hat{h}(s) - \hat{Q}) \cdot \frac{\partial}{\partial w_f} \hat{h}(s), \forall w_f \in \mathcal{W}$$