University of Basel

Computational Complexity of Classical Planning Domains Based on Grids

Travis Rivera Petit <travis.riverapetit@unibas.ch>

Philosophisch-Naturwissenschaftlichen Fakultät, University of Basel

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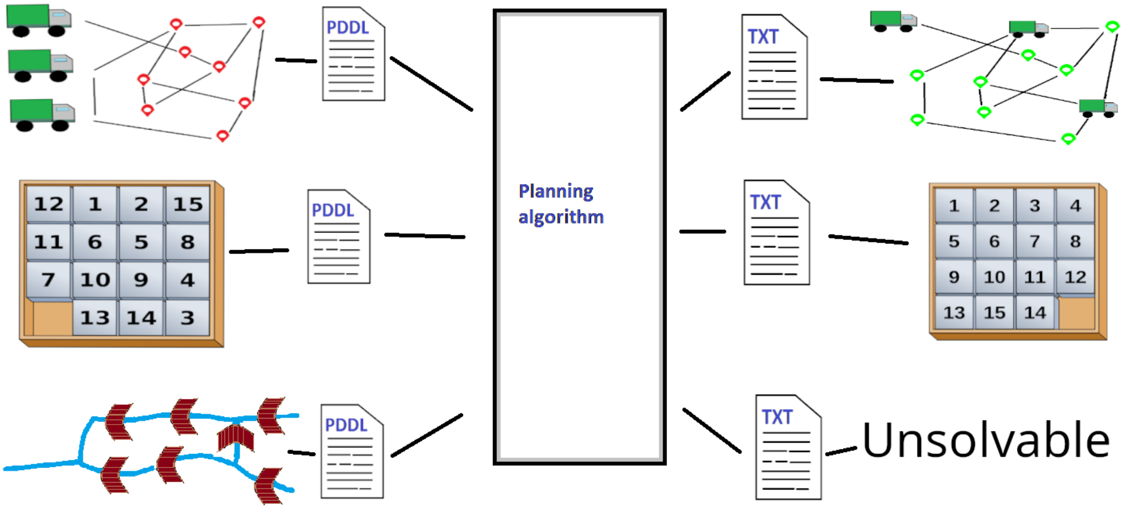
Roadmap

Roadmap

Classical Planning

- Branch of AI
- Studies domain-independent, fully observable, discrete planning tasks


PDDL
PDDL
PDDL
Planning algorithm
TXT
TXT
TXT
Unsolvable
12 1 2 15
11 6 5 8
7 10 9 4
13 14 3
1 2 3 4
5 6 7 8
9 10 11 12
13 15 14

International Planning Competitions (IPCs)

- Planning algorithms are tested at the International Planning Competitions
- **Goal of the thesis:** How hard are these domains?
- This thesis: domains based on grids
- Measure hardness with computational complexity

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Computational Complexity

- $\#$ of resources needed to solve problems **regardless of technology**
- **P** $= \{\Sigma : \Sigma$ is decidable in polynomial time$\}$
- **NP** $= \{\Sigma : \Sigma$ is decidable in non-deterministic polynomial time$\}$
- **PSPACE** $= \{\Sigma : \Sigma$ is decidable using polynomial memory$\}$
- **NPSPACE** $= \{\Sigma : \Sigma$ is decidable using non-deterministic polynomial memory$\}$

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE}$$

Computational Complexity ∩ Classical Planning

PE-$\mathcal{D}$
INPUT: A planning task Π of a domain $\mathcal{D}$.
QUESTION: Does there exist a plan for Π?

BPE-$\mathcal{D}$
INPUT: A planning task Π of a domain $\mathcal{D}$ and a positive integer K.
QUESTION: Does there exist a plan for Π of length $\leq K$?

If $\mathcal{D}$ is arbitrary, then both are **PSPACE**-complete (Bylander, 1994).

Before the thesis

	PlanEx	Bounded PlanEx
VisitAll	?	?
TERMES	?	?
Nurikabe	?	?
Tidybot	?	?
Floortile	?	?

Roadmap

1. Motivation
2. Domains
   - 2.1 **VisitAll**
   - 2.2 TERMES
   - 2.3 Nurikabe
   - 2.4 Tidybot & Floortile
3. Future Work

VisitAll

- An agent in the middle of a square grid must visit a subset of the cells
- Two domains: `VISIT-ALL` and `VISIT-SOME`
- Demo: `https://editor.p5js.org/trp/full/vB0bLv29p`
- Plan existence is trivial: the answer is always YES

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Hamiltonian paths

Definition

A **Hamiltonian path** is a simple path that visits all of the vertices in a graph.

Not all graphs have Hamiltonian paths.

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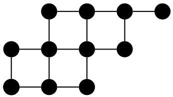
Types of grids

Definition

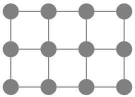
A **grid graph** is a vertex induced, finite, connected subgraph of the integer lattice.

Definition

A **rectangular graph** is a grid graph whose vertex set is $\{1, \ldots, n\} \times \{1, \ldots, m\}$. It is **squared** if $n = m$.



A grid graph



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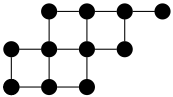

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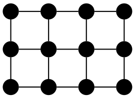

A rectangular graph

VISIT-ALL: Bounded plan existence

Lemma (Follows from Itai. et. al, 1982)

Let $G = (V_0 \dot{\cup} V_1, E)$ be a squared graph of side length $n \geq 2$ with $|V_0| = |V_1| + 1$.

- $s \in V_1 \wedge n$ is odd $\implies$ the shortest path in G that starts in s and visits all vertices has length $|V| + 1$.
- Otherwise, G has a HP starting at s. Always the case if s is in the middle!

Let $K > 0$ and I be an instance of VISIT-ALL. Then

- If $K < |V| - 1$, I cannot be solved in K steps or less.
- If $K \geq |V| - 1$, I can be solved in at most K steps

$\implies$ BPE-VISITALL $\in$ **P**

VISIT-ALL: Bounded plan existence

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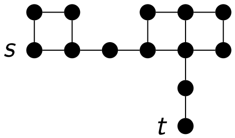
VISIT-SOME: Bounded plan existence

Lemma

The s-t HP problem on grid graphs is **NP**-complete even if $\deg(t) = 1$.

Then



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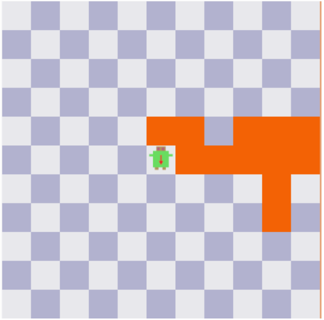


$K = |V| - 1$

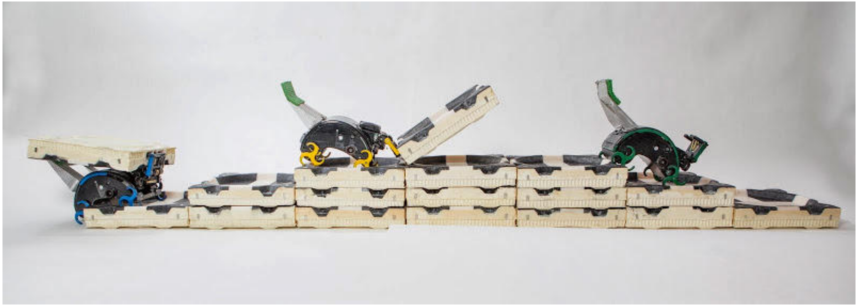
	PlanEx	Bounded PlanEx
VisitAll	P	P[a] NP-complete[b]
TERMES	?	?
Nurikabe	?	?
Tidybot	?	?
Floortile	?	?

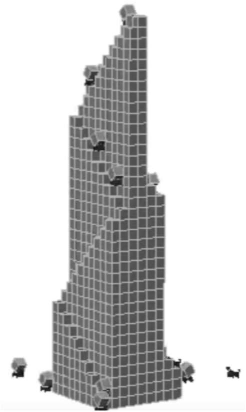
[a] If the goal is to visit all the cells in the grid.
[b] In the general case, where the goal can be any subset of cells.

Roadmap

TERMES

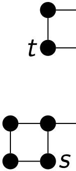
Variant of termite-inspired Harvard TERMES robots.



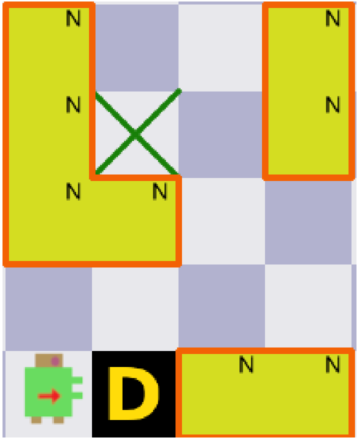


Demo: `https://editor.p5js.org/trp/full/CN4Bf3bDb`

TERMES plan existence



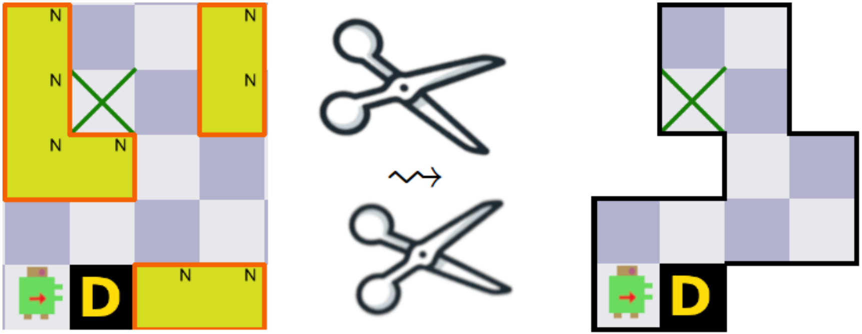
$\rightsquigarrow$



Goal: Put $|V| - 1$ blocks in X

TERMES plan existence

If $\texttt{num-blocks}_s(u) = N$, then the robot cannot interact with that cell:



Robot must build a ramp of height $|V| - 1$ by following an s-t HP in the grid graph.
Then the robot must destroy the ramp leading to t.

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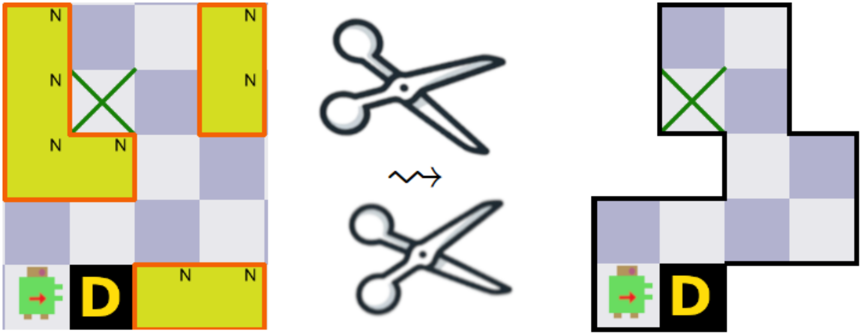

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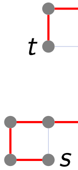
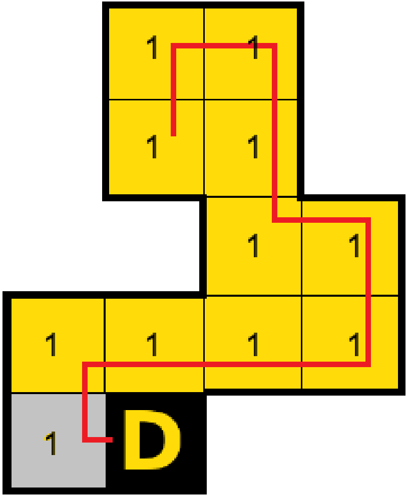
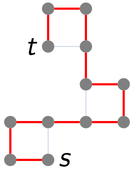
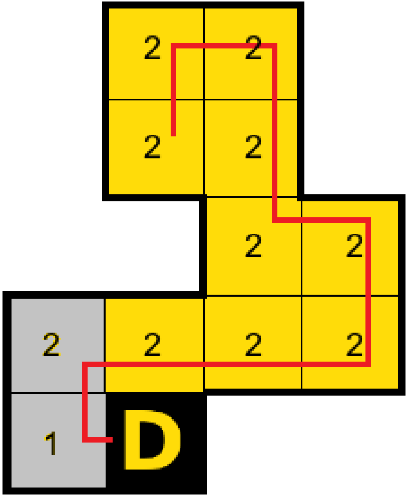




TERMES plan existence





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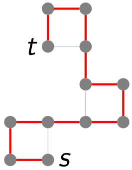

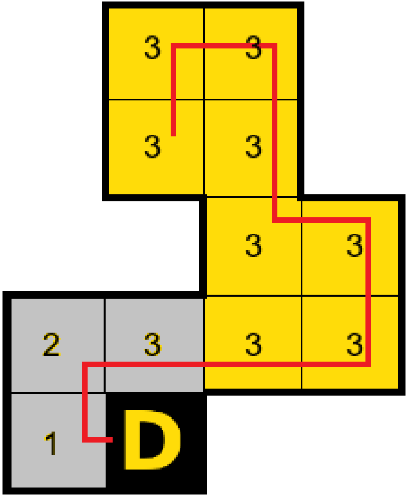

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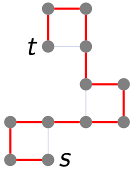

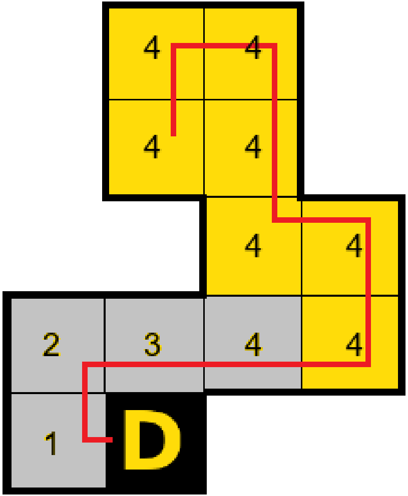

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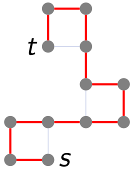

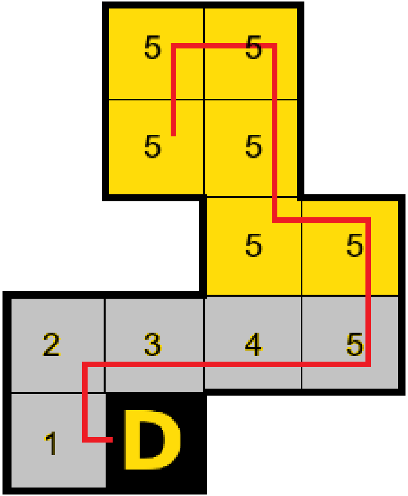

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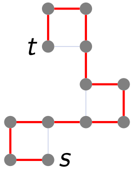
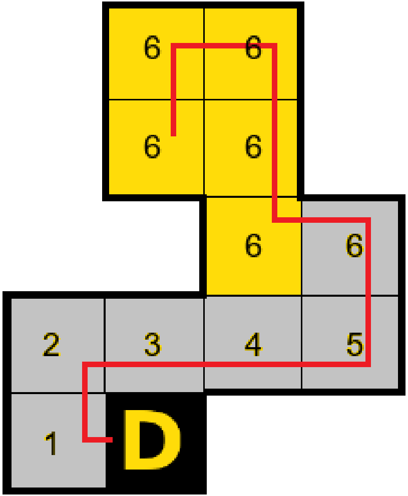
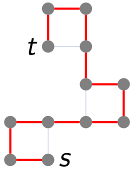
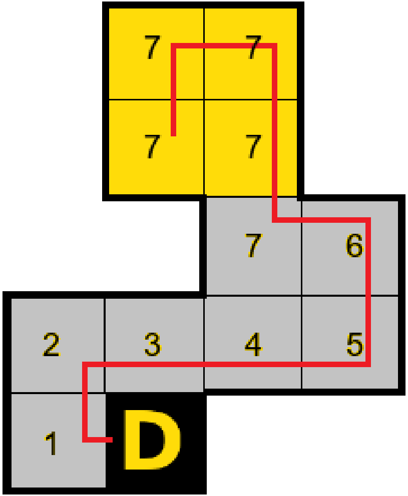




TERMES plan existence





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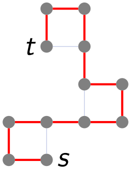

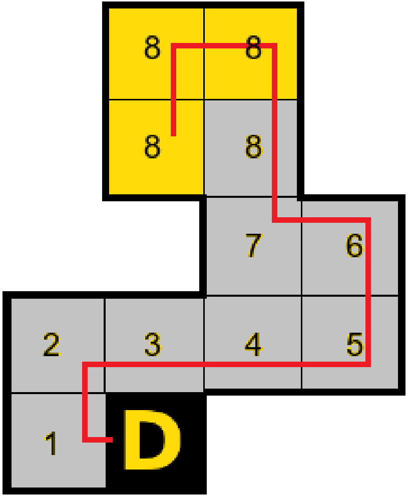

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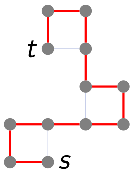

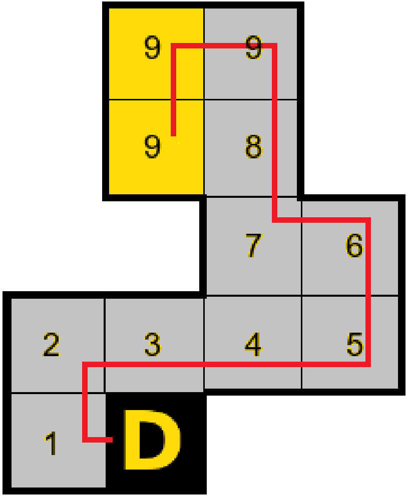

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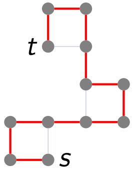

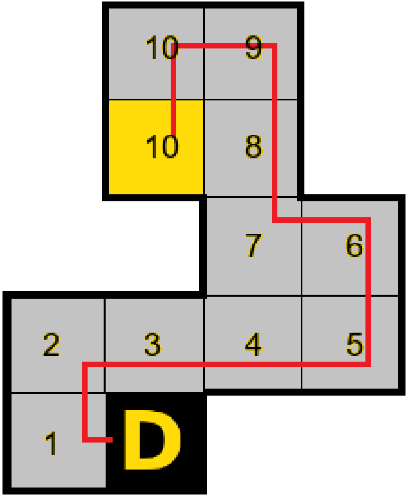

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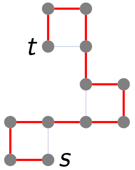
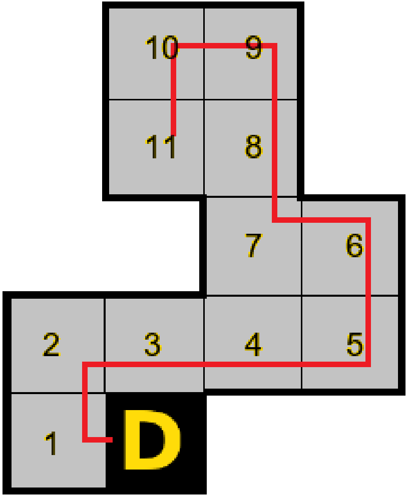




TERMES plan existence

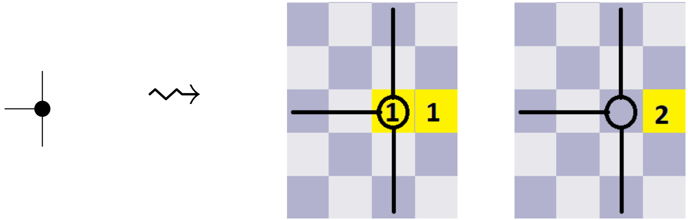
$\Longrightarrow$ PE-TERMES is **NP**-hard

TERMES bounded plan existence

Lemma (Papadimitriou et. al., 1984)

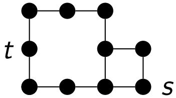
The s-t HP problem for grid graphs remains **NP**-complete even if $\deg(u) \leq 3 \ \forall u \in V$

Idea



s_0 **goal**

TERMES bounded plan existence



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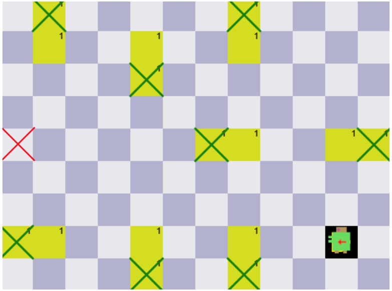

Goal: 2 in X and 1 in X

$K = 5 \cdot |V| - 5$

Demo: `https://editor.p5js.org/trp/full/0egSR9T3Z`

	PlanEx	**Bounded PlanEx**
VisitAll	**P**	**P**[a] **NP**-complete[b]
TERMES	**NP**-hard[c] ?[d]	**NP**-hard[c] ?[d]
Nurikabe	?	?
Tidybot	?	?
Floortile	?	?

[c] If the initial state is allowed to contain blocks.
[d] If the initial state is empty.

Roadmap

Nurikabe

- Japanese logic puzzle
- `IPC-NURIKABE`: robot moves around the grid and paints cells individually
- Demo: `https://editor.p5js.org/trp/full/NumhOFmqC`

2									2
						2			
	2			7					
						3		3	
		2					3		
2			4						
	1					2		4	

Nurikabe

Lemma

All solvable instances of `IPC-NURIKABE` have polynomial length plans.

Proof idea: Given a plan π, create a new plan π' with $\|\pi'\| = O\big(|V|^2\big)$

$\Longrightarrow$ PlanEx and BoundedPlanEx $\in$ **NP**

Nurikabe

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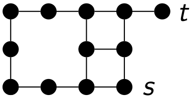
Bounded Plan Existence



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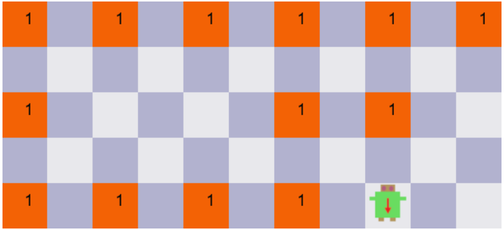
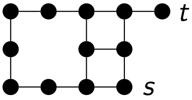


$K = 4 \cdot |V| - 2$

Demo: `https://editor.p5js.org/trp/full/VTm_o5Mt5`

$\Longrightarrow$ BPE-IPC-NURIKABE is **NP**-complete

Bounded Plan Existence



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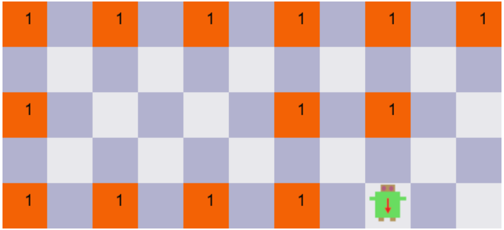


$K = 4 \cdot |V| - 2$

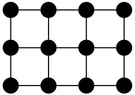
Demo: `https://editor.p5js.org/trp/full/VTm_o5Mt5`

$\Longrightarrow$ BPE-IPC-NURIKABE is **NP**-complete

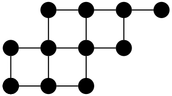
Plan existence

Definition

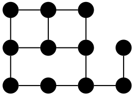
A **subgrid graph** is an edge induced, connected, finite subgraph of the integer lattice.



Rectangular graph



Grid graph



Subgrid graph

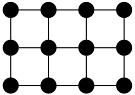
All grid graphs are subgrid graphs.

Not all subgrid graphs are grid graphs.

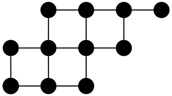
Plan existence

Definition

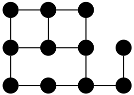
A **subgrid graph** is an edge induced, connected, finite subgraph of the integer lattice.



Rectangular graph



Grid graph



Subgrid graph

All grid graphs are subgrid graphs.

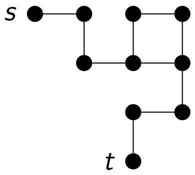
Not all subgrid graphs are grid graphs.

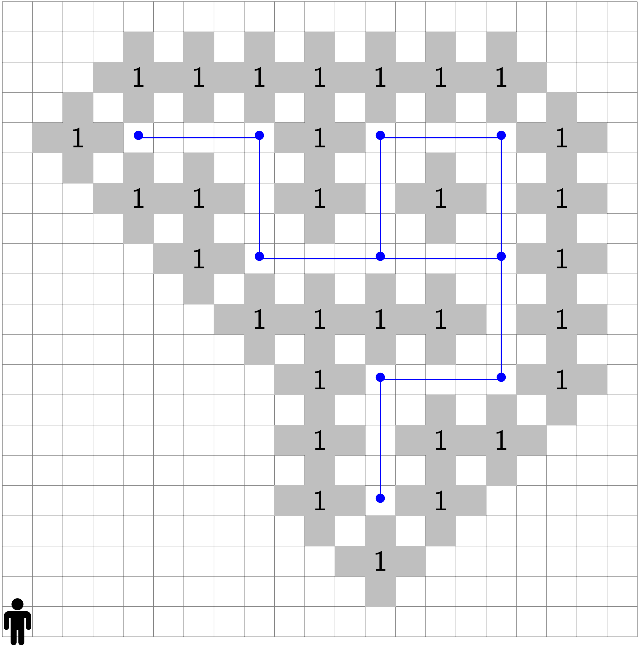
Plan existence

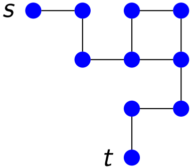
Lemma

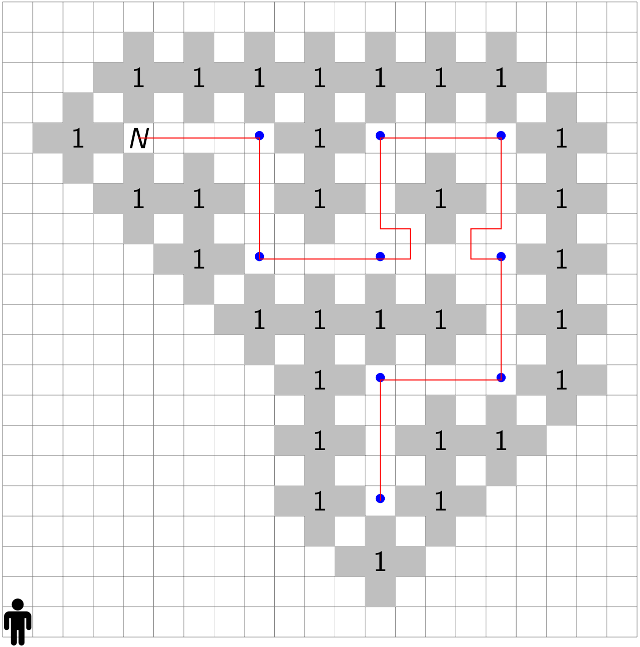
The s-t HP problem for subgrid graphs is **NP**-complete even if

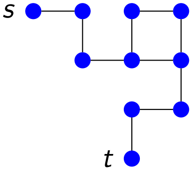
- $\deg(u) \leq 3 \; \forall u$
- $\deg(s) = 1$
- $\deg(t) = 1$
- The degree of t's neighbor is 2






s
t





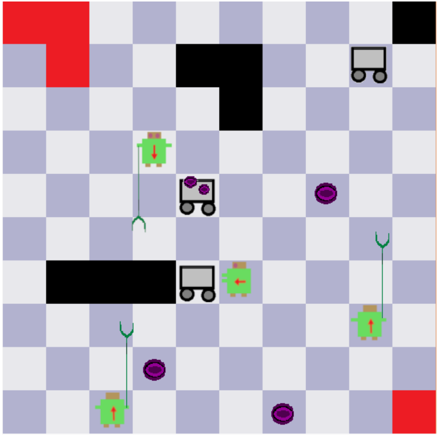
$$N = 4 \cdot |V| + 2 \cdot D_G^3(V) - 3$$

	PlanEx	Bounded PlanEx
VisitAll	P	P[a] NP-complete[b]
TERMES	NP-hard[c] ?[d]	NP-hard[c] ?[d]
Nurikabe	NP-complete	NP-complete
Tidybot	?	?
Floortile	?	?

Roadmap

Tidybot

- Multi-agent logistics transportation problem
- In IPC: one robot, one cart, gripper radius = 1



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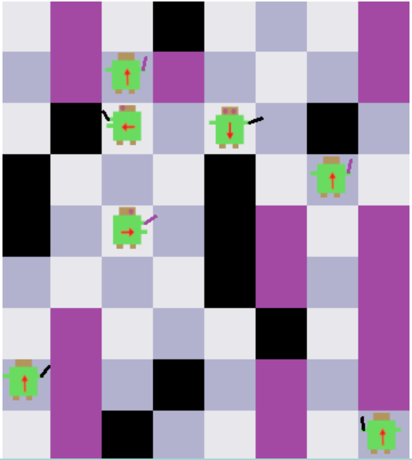
VisitAll	P	P[a] NP-complete[b]
TERMES	NP-hard[c] ?[d]	NP-hard[c] ?[d]
Nurikabe	NP-complete	NP-complete
Tidybot	P[e] PSPACE-complete[f]	NP-complete[e] PSPACE-complete[f]
Floortile	?	?

[e] If there is only one cart, one robot, and no obstacles.
[f] In the general case.

Floortile

- Robots paint patterns on grids
- Non-uniform action costs



	PlanEx	Bounded PlanEx
VisitAll	**P**	**P**[a] **NP**-complete[b]
TERMES	**NP**-hard[c] ?[d]	**NP**-hard[c] ?[d]
Nurikabe	**NP**-complete	**NP**-complete
Tidybot	**P**[e] **PSPACE**-complete[f]	**NP**-complete[e] **PSPACE**-complete[f]
Floortile	**P**	?

Roadmap

Future work

More domains!

- Labyrinth
- Ricochet Robots
- Protein Folding
- Spanner
- Slitherlink
- Snake

More classes!

- **L**
- **NL**
- ...

Open questions

- TERMES on empty grid
- TERMES $\in$ **NP**?
- Floortile Bounded PlanEx

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- TERMES on empty grid
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- Floortile Bounded PlanEx

Contributions: before

	PlanEx	Bounded PlanEx
VisitAll	?	?
TERMES	?	?
Tidybot	?	?
Floortile	?	?
Nurikabe	?	?

Contributions: after

	PlanEx	Bounded PlanEx
VisitAll	**P**	**P**[a] **NP**-complete[b]
TERMES	**NP**-hard[c] ?[d]	**NP**-hard[c] ?[d]
Nurikabe	**NP**-complete	**NP**-complete
Tidybot	**P**[e] **PSPACE**-complete[f]	**NP**-complete[e] **PSPACE**-complete[f]
Floortile	**P**	?

- a If the goal is to visit all the cells in the grid.
- b In the general case, where the goal can be any subset of cells.
- c If the initial state is allowed to contain blocks.
- d If the initial state is empty.
- e If there is only one cart, one robot, and no obstacles.
- f In the general case.