Integrating Partial Order Reduction and Symmetry Elimination for Cost-Optimal Classical Planning: Additional Examples

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This technical report contains additional examples for the paper "Integrating Partial Order Reduction and Symmetry Elimination for Cost-Optimal Classical Planning" [Wehrle *et al.*, 2015]. In the following, we will denote partial states as sets of variable/value pairs.

The Relative Pruning Power

The pruning power of strong stubborn sets and symmetry elimination is orthogonal. To see this, we first show that symmetry elimination can prune more than strong stubborn sets.

Example 1 *Let Π_1 be the planning task with binary variables $\mathcal{V} = \{a, b, c, d, g\}$ and uniform-cost operators $\mathcal{O} = \{o_a, o_b, o_c, o_d\}$ with*

- *$pre(o_a) = \{\langle a,1\rangle\}$, $eff(o_a) = \{\langle c,1\rangle, \langle a,0\rangle, \langle d,0\rangle\}$,*
- *$pre(o_b) = \{\langle b,1\rangle\}$, $eff(o_b) = \{\langle d,1\rangle, \langle b,0\rangle, \langle c,0\rangle\}$,*
- *$pre(o_c) = \{\langle c,1\rangle, \langle b,0\rangle\}$, $eff(o_c) = \{\langle g,1\rangle, \langle c,0\rangle\}$, and*
- *$pre(o_d) = \{\langle d,1\rangle, \langle a,0\rangle\}$, $eff(o_d) = \{\langle g,1\rangle, \langle d,0\rangle\}$.*

Let $s_0 = \{\langle a,1\rangle, \langle b,1\rangle, \langle c,0\rangle, \langle d,0\rangle, \langle g,0\rangle\}$ and $s_\star = \{\langle g,1\rangle\}$. The state transition graph of Π_1 is depicted in Figure 1. States are denoted by variables with value 1, and the initial and goal states are indicated as dashed and double circled locations, respectively.

We observe that, e. g., there is a structural symmetry σ that maps operators o_a to o_b, o_c to o_d, and stabilizes the initial state. More generally, there is a canonical operator labeling CL_s induced by symmetries that stabilize $s = s_0$ such that $CL_s[o_a] = CL_s[o_b]$ and $CL_s[o_c] = CL_s[o_d]$. Hence, the induced symmetric operator pruning function prunes either o_a or o_b in s_0. In contrast, as o_a and o_b interfere in s_0, strong stubborn sets will necessarily include both o_a and o_b.

For the other direction, consider the following example, showing that generalized strong stubborn sets can obtain state space reductions where no structural symmetries exist.

Example 2 *Let Π_2 be the planning task with binary variables $\mathcal{V} = \{a, b, c\}$ and uniform-cost operators $\mathcal{O} = \{o_a, o_b\}$ with*

- *$pre(o_a) = \{\langle a,0\rangle\}$, $eff(o_a) = \{\langle a,1\rangle\}$, and*
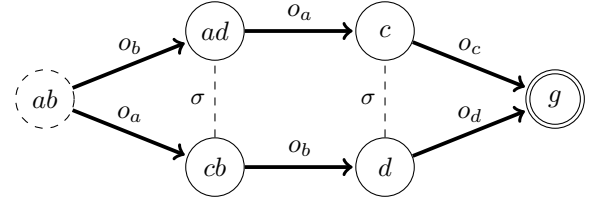- *$pre(o_b) = \{\langle b,0\rangle\}$, $eff(o_b) = \{\langle b,1\rangle, \langle c,1\rangle\}$.*

Figure 1: State transition graph and symmetries of the example task Π_1.

Let $s_0 = \{\langle a,0\rangle, \langle b,0\rangle, \langle c,0\rangle\}$ and $s_\star = \{\langle a,1\rangle, \langle b,1\rangle\}$. The state transition graph of Π_2 is depicted in Figure 2. States are denoted by variables with value 1, and again, the initial and goal states are indicated as dashed and double circled locations, respectively.

We observe that there are no non-trivial structural symmetries in the task: Non-trivial structural symmetries would necessarily map o_a to o_b, which is not possible due to different sizes of their effects. In contrast, there are strong stubborn sets in s_0 that only contain one of o_a and o_b because both are applicable, but do not interfere in s_0. Hence both $\{o_a\}$ and $\{o_b\}$ are strong stubborn sets in s_0.
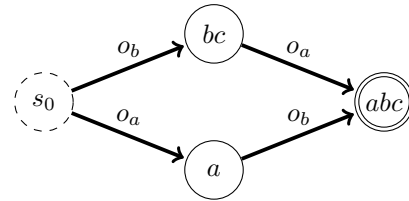

Figure 2: State transition graph of the example task Π_2.

Symmetrical Operator Pruning

Structural symmetries σ that do not stabilize the current state s (i. e., $\sigma(s) \neq s$) are not guaranteed to yield safe successor pruning functions in general. To see this, consider the following example.

Example 3 *Let Π_3 be a planning task with variables $\mathcal{V} = \{a, b, c\}$ and uniform cost operators $\mathcal{O} = \{o_b, o_c\}$ with*

- $pre(o_b) = \{\langle a,0\rangle\}$, $eff(o_b) = \{\langle a,1\rangle, \langle b,0\rangle\}$, and
- $pre(o_c) = \{\langle a,0\rangle\}$, $eff(o_c) = \{\langle a,1\rangle, \langle c,0\rangle\}$.

Let $s_0 = \{\langle a,0\rangle, \langle b,1\rangle, \langle c,0\rangle\}$ and $s_\star = \{\langle b,0\rangle, \langle c,0\rangle\}$. The state transition graph of Π_3 is depicted in Figure 3. States are denoted by variables with value 1, the initial state is indicated as dashed, and the goal state is indicated as double circled, respectively.

There is a structural symmetry σ that maps operator o_b to o_c, variable b to variable c, and stabilizes variable a. Clearly, σ does not stabilize s_0. Assume the canonical operator of o_b is o_c. We observe that applying o_b in s_0 achieves the goal, whereas applying o_c does not.
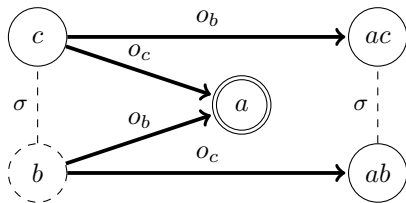

Figure 3: State transition graph and symmetries of the example task Π_3.

References

[Wehrle *et al.*, 2015] Martin Wehrle, Malte Helmert, Alexander Shleyfman, and Michael Katz. Integrating partial order reduction and symmetry elimination for cost-optimal classical planning. In *Proc. IJCAI 2015*, 2015.