

The AME2012 atomic mass evaluation *

(II). Tables, graphs and references

M. Wang^{1,2,3}, G. Audi^{2,§}, A.H. Wapstra^{4,†}, F.G. Kondev⁵, M. MacCormick⁶, X. Xu^{1,7}, and B. Pfeiffer^{8,‡}

¹ Institute of Modern Physics, CAS, 509 Nanchang Rd., Lanzhou 730000, China

² Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse, CSNSM, CNRS/IN2P3, Université Paris-Sud, Bât. 108, F-91405 Orsay Campus, France

³ Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany

⁴ National Institute of Nuclear Physics and High-Energy Physics, NIKHEF, 1009DB Amsterdam, The Netherlands

⁵ Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA

⁶ Institut de Physique Nucléaire, CNRS/IN2P3, Université Paris-Sud, F-91406 Orsay cedex, France

⁷ Graduate University of Chinese Academy of Sciences, Beijing, 100049, People's Republic of China

⁸ GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, D-64291 Darmstadt, Germany

Abstract This paper is the second part of the new evaluation of atomic masses, AME2012. From the results of a least-squares calculation, described in Part I, for all accepted experimental data, we derive here tables and graphs to replace those of AME2003. The first table lists atomic masses. It is followed by a table of the influences of data on primary nuclides, a table of separation energies and reaction energies, and finally, a series of graphs of separation and decay energies. The last section in this paper lists all references to the input data used in Part I of this AME2012 and also to the data included in the NUBASE2012 evaluation (first paper in this issue).

AMDC: <http://amdc.in2p3.fr/> and <http://amdc.impcas.ac.cn/>

1 Introduction

The description of the general procedures and policies are given in Part I of this series of two papers (p. 1287 of the present issue), where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I.

Firstly, we present the table of atomic masses (Table I) expressed as mass excess in energy unit, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass unit.

The second table is the table of *influences* on primary nuclides (Table II). For each of the *primary* nuclides entering this evaluation, we give the three main data and their influences on the mass of this nuclide (see the definitions in Part I, Section 5.1, p. 1306).

Thirdly, we give a table for values and their uncertainties for the separation energies and reaction energies for twelve carefully selected combinations of nuclides. This selection, together with the β -decay energies in Table I,

yields all differences in masses between any pair of nuclei differing at most by 2 units in Z and N . A method is indicated in which many more reaction energy values can be derived from the present table.

The following series of graphs are then presented: two-neutron separation energies and α -decay energies as a function of neutron number, two-proton separation energies as a function of proton number and double β -decay energies as a function of mass number which are considered as the most illustrative ones for representing the regular trends in the mass surface and useful to derive estimates for unknown masses.

Finally, references to the input data used in Part I of AME2012 and in NUBASE2012, the first paper of this issue, are given in the last section of this paper.

2 The atomic mass table

As in our previous works AME2003 [1, 2], AME'95 [3], AME'93 [4, 5, 6, 7], ... the tables presented in this

* This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUNAMCO).

§ Corresponding author. *E-mail address*: amdc.audi@gmail.com (G. Audi).

† Deceased, December 2006.

‡ Present address: II. Physikalisches Institut, Justus-Liebig-Universität, Heinrich-Buff-Ring 16, D-35392 Gießen.

©2012 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

work give atomic masses and derived quantities. With very few exceptions, experimental data on masses of nuclei refer to “atomic” masses or to masses of singly ionized atoms. In this last case the ionization energy is generally (much) smaller than the error on the mass, and, for the small number of very precise mass measurements, corrections for the first -and second- ionization potentials could be applied without much loss of accuracy. The same is true for the electron mass M_e involved, see Table A in Part I, p. 1290. This is the reason for the decision to present, in our evaluations, atomic rather than nuclear masses.

Nuclear masses can be calculated from atomic ones by using the formula:

$$M_N(A, Z) = M_A(A, Z) - Z \times M_e + B_e(Z) \quad (1)$$

Nowadays, several mass measurements are made on fully or almost fully ionized atoms. Then, a correction must be made for the total binding energy of all removed electrons $B_e(Z)$. They can be found in the table for calculated total atomic binding energy of all electrons of Huang et al. [8]. Unfortunately, the precision of the calculated

values $B_e(Z)$ is not clear; this quantity (up to 760 keV for ${}_{92}\text{U}$) cannot be measured easily. Very probably, its precision for ${}_{92}\text{U}$ is rather better than the 2 keV accuracy with which the mass of, e.g., ${}^{238}\text{U}$ is known. A simple formula, approximating the results of [8], is given in the review of Lunney, Pearson and Thibault [9]:

$$B_{el}(Z) = 14.4381 Z^{2.39} + 1.55468 \times 10^{-6} Z^{5.35} \text{ eV} \quad (2)$$

The atomic masses are given in mass units and the derived quantities in energy units. For the atomic mass unit we use the “unified atomic mass unit,” symbol “u”, defined as 1/12 of the atomic mass of one ${}^{12}\text{C}$ atom in its electronic and nuclear ground states and in its rest coordinate system. In our work energy values are expressed as electron-volt, using the *maintained* volt V_{90} . For a discussion see Part I, Section 2, p. 1291.

Due to the drastic increase in the precision of the mass values of the very light nuclei, the printing format of the mass table is not adequate. Table A gives, for the most precise among them, values of mass excesses and atomic masses.

Table A. The most precisely known masses.

| | Mass excess (keV ₉₀) | | Atomic mass (μu) | |
|-------------------------|----------------------------------|-----------|-------------------------------|-----------|
| ${}^1_0\text{n}$ | 8 071.317 144 | 0.000 458 | 1 008 664.915 850 | 0.000 491 |
| ${}^1_1\text{H}$ | 7 288.970 591 | 0.000 087 | 1 007 825.032 231 | 0.000 093 |
| ${}^2_1\text{H}$ | 13 135.721 741 | 0.000 114 | 2 014 101.778 120 | 0.000 122 |
| ${}^4_2\text{He}$ | 2 424.915 609 | 0.000 059 | 4 002 603.254 130 | 0.000 063 |
| ${}^{13}_6\text{C}$ | 3 125.008 747 | 0.000 211 | 13 003 354.835 071 | 0.000 227 |
| ${}^{14}_7\text{N}$ | 2 863.416 686 | 0.000 188 | 14 003 074.004 426 | 0.000 202 |
| ${}^{15}_7\text{N}$ | 101.438 657 | 0.000 599 | 15 000 108.898 884 | 0.000 643 |
| ${}^{16}_8\text{O}$ | -4 737.001 374 | 0.000 160 | 15 994 914.619 566 | 0.000 172 |
| ${}^{17}_8\text{O}$ | -808.763 613 | 0.000 644 | 16 999 131.756 500 | 0.000 692 |
| ${}^{18}_8\text{O}$ | -782.815 582 | 0.000 707 | 17 999 159.612 858 | 0.000 758 |
| ${}^{19}_9\text{F}$ | -1 487.444 343 | 0.000 860 | 18 998 403.162 727 | 0.000 923 |
| ${}^{28}_{14}\text{Si}$ | -21 492.794 586 | 0.000 410 | 27 976 926.534 649 | 0.000 440 |
| ${}^{29}_{14}\text{Si}$ | -21 895.078 667 | 0.000 487 | 28 976 494.664 901 | 0.000 523 |
| ${}^{31}_{15}\text{P}$ | -24 440.541 107 | 0.000 655 | 30 973 761.998 417 | 0.000 702 |
| ${}^{32}_{16}\text{S}$ | -26 015.533 531 | 0.001 316 | 31 972 071.174 408 | 0.001 412 |

Mass excesses expressed in keV, of practical use, are also given. Conversion of the errors from μu to keV were obtained by:

$$\sigma_{M_{\text{keV}}}^2 = (\sigma_{M_u} \times u)^2 + (M_u \times \sigma_u)^2 \quad (3)$$

where M_u is the mass excess in μu , and σ_u the error of u expressed in eV₉₀. The part dependent on M_u is only important for very few nuclides.

As mentioned in Part I, since AME2003 we don't give any more values for the binding energies, $ZM_H + NM_n -$

Table B. Correlation matrices for the most precisely known very light nuclei (in squared nano atomic mass units).

| | n | H | D | ⁴ He | ¹³ C | ¹⁴ N | ¹⁵ N | ¹⁶ O | ²⁸ Si |
|------------------|-----------|-----------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|
| n | 0.241418 | | | | | | | | |
| H | -0.006094 | 0.008708 | | | | | | | |
| D | 0.012274 | 0.002612 | 0.014891 | | | | | | |
| ⁴ He | 0.000000 | 0.000000 | 0.000000 | 0.004011 | | | | | |
| ¹³ C | 0.004869 | -0.006656 | -0.001785 | 0.000000 | 0.051325 | | | | |
| ¹⁴ N | -0.001079 | 0.001890 | 0.000811 | 0.000000 | 0.037139 | 0.040928 | | | |
| ¹⁵ N | -0.000790 | 0.013496 | 0.012707 | 0.000000 | -0.005664 | 0.006929 | 0.413749 | | |
| ¹⁶ O | -0.000749 | 0.002027 | 0.001279 | 0.000000 | 0.010791 | 0.013155 | 0.005573 | 0.029467 | |
| ²⁸ Si | -0.003321 | 0.006777 | 0.003455 | 0.000000 | 0.028736 | 0.030353 | 0.036443 | 0.016784 | 0.193341 |

| | n | H | D | ³ H | ³ He | ¹⁶ O | ²⁰ Ne | ²³ Na | ²⁸ Si |
|------------------|-----------|----------|----------|----------------|-----------------|-----------------|------------------|------------------|------------------|
| n | 0.241418 | | | | | | | | |
| H | -0.006094 | 0.008708 | | | | | | | |
| D | 0.012274 | 0.002612 | 0.014891 | | | | | | |
| ³ H | 0.000000 | 0.000000 | 0.000000 | 5.632850 | | | | | |
| ³ He | 0.000000 | 0.000000 | 0.000000 | 5.553019 | 6.271468 | | | | |
| ¹⁶ O | -0.000749 | 0.002027 | 0.001279 | 0.000000 | 0.000000 | 0.029467 | | | |
| ²⁰ Ne | 0.027477 | 0.012383 | 0.039872 | 0.000001 | 0.000001 | 0.005511 | 2.830803 | | |
| ²³ Na | -0.000001 | 0.000003 | 0.000001 | 0.000000 | 0.000000 | 0.000001 | 0.000024 | 3.782120 | |
| ²⁸ Si | -0.003321 | 0.006777 | 0.003455 | 0.000000 | 0.000000 | 0.016784 | 0.014612 | 0.000011 | 0.193341 |

M , as we used to until and including in AME1993. Otherwise than before, its error equals that in the value of the mass excess, which makes its use unnecessary. Since AME2003 we give instead the binding energy per nucleon, which is of educational interest, connected to the Aston curve and the maximum stability around the ‘iron-peak’ of importance in astrophysics. (*Note*: the highest binding energy per nucleon is actually observed for ⁶²Ni followed by ⁵⁸Fe and only then by ⁵⁶Fe).

3 Influences on primary nuclides

Table II presents a list of all primary nuclides, and for each of them the main data contributing to its mass determination (up to the three most important ones) and the *influences* of these data on this nuclide.

This Table II complements the information given in the main table (Part I, Table I) where we display the *significance* (total flux) and the main *flux* of each datum. In other words, the flow-of-information matrix \mathbf{F} , defined in Part I, Section 5.1, is (partly) displayed once along lines and once along columns.

4 Nuclear-reaction and separation energies

The result of the least-squares adjustment of experimental data (reaction and decay energies and mass-spectrometric data) determining atomic masses of nuclides, as described in Part I, is not represented completely by the adjusted values of the input data given there and the resulting values of the atomic masses given in the Table I. A complete representation would require reproduction of a matrix of correlation coefficients. Since this matrix contains $\frac{1}{2}N(N+1)$ elements in which $N =$

1176, this is not very attractive. For the first time in this AME2012, and following the suggestion of B.N. Taylor we made available at the AMDC Web-site in directory masstables/Amc2012 a full list of correlation coefficients [10].

The main use of the correlation matrix is in obtaining errors in linear combinations of atomic masses. In practice, the correlations are important only for combinations involving two neighbouring nuclides with small differences in mass number and particles such as n, p, d, t, ³He and α . Such combinations, consisting of various kinds of decay and binding energies of particles or groups of particles, are important for studies of the trends in the nuclear energy surface and for Q-values of frequently studied reactions. As before [5], we present in Table III values for 12 such combinations and their standard errors. The β -decay energies are given in Table I.

With the help of the instructions given in the ‘Explanation of Table’, values for 28 additional reactions and their standard errors can be derived. The derived values will be correct, but in a few cases (of reactions on very light nuclei measured with extreme precision) the errors will be slightly larger than would follow from a calculation including correlations.

The precision (standard error) in the value of any combination of the most precise mass values, for very light nuclei, can be obtained with the help of the correlation coefficients given in Table B. When doing this, one should use mass values and their uncertainties as given in the mass table (Table I) in μu , and not the mass-excesses (in keV).

We have also prepared a table of neutron, proton and deuteron pairing energies, available from the AMDC [11], defined as:

$$\begin{aligned}
 P_n(A, Z) &= \frac{1}{4}(-1)^{A-Z+1}[S_n(A+1, Z) - 2S_n(A, Z) + S_n(A-1, Z)] \\
 P_p(A, Z) &= \frac{1}{4}(-1)^{Z+1}[S_p(A+1, Z+1) - 2S_p(A, Z) + S_p(A-1, Z-1)] \\
 P_d(A, Z) &= \frac{1}{4}(-1)^{Z+1}[S_d(A+2, Z+1) - 2S_d(A, Z) + S_d(A-2, Z-1)]
 \end{aligned}$$

S_n , S_p , and S_d are the neutron, proton and deuteron separation energies, the latter being defined as

$$S_d(A, Z) = -M(A, Z) + M(A-2, Z-1) + M(d) = -Q(\gamma, d),$$

the quantities S_n , and S_p are defined below in the Explanation of Table III (p. 1696), and $Q(\gamma, d)$ can be calculated as indicated also there.

Remark: P_n is also sometimes written as:

$$P_n(A, Z) = \frac{1}{4}(-1)^{A-Z+1}[-M(A+1, Z) + 3M(A, Z) - 3M(A-1, Z) + M(A-2, Z)]$$

displaying thus more clearly the combination of the involved masses. And similarly for P_p and P_d .

5 Graphs of trends in the mass surface

All the information contained in the mass table (Table I) and in the nuclear reaction and separation energy table (Table III) can in principle be displayed in a plot of the binding energy or the mass versus Z , N , or A . Such a plot, in which the binding energies vary rapidly, is complicated by the fact that there are four sheets, corresponding to the four possible combinations of parity for Z and N . These sheets are nearly parallel almost everywhere in this three dimensional space and have remarkably regular trends, as one may convince oneself by making various cuts (e.g. Z or N or A constant). Any derivative of the binding energies also defines four sheets. In the present context, *derivative* means a specified difference between the masses of two nearby nuclei. They are also smooth and have the advantage of displaying much smaller variations (see also Part I, Section 4). For a derivative specified in such a way that differences are between nuclides in the same mass sheet, the nearly parallelism of these leads to an (almost) unique surface for the derivative, allowing thus a single display. Therefore, in order to illustrate the regular trends in the mass surface, four derivatives of this last type were chosen:

1. the two-neutron separation energies versus N , with lines connecting the isotopes of a given element (Figs. 1–9);
2. the two-proton separation energies versus Z , with lines connecting the isotones (the same number of neutrons) (Figs. 10–17);
3. the α -decay energies versus N , with lines connecting the isotopes of a given element (Figs. 18–26);

4. the double β -decay energies versus A , with lines connecting the isotopes and the isotones (Figs. 27–36).

These graphs of regular trends supersede earlier graphs [2].

Other various representations are possible (e.g. separately for odd and even nuclei: one-neutron separation energies versus N , one-proton separation energy versus Z , β -decay energy versus A , ...); they can all be built starting from the values in Table III. They cannot all be given in the present printed version, but they are retrievable from the *Web* distribution [11].

Clearly showing the trends in the mass surface, these graphs can be quite useful for checking the quality of any interpolation or extrapolation (if not too far) and provide an excellent testground for theoretical mass models. When some masses in a defined region deviate from the regular trends in the mass-surface TMS, almost always there is a serious physical cause, like a shell or subshell closure or an onset of deformation. But, if only one mass exhibits an irregular pattern, violating the general trends, then one may seriously question the correctness of the related data. See the discussion in Part I, Section 4, p. 1303).

6 List of references for the NUBASE2012 and AME2012 evaluations

Full references related to all the input data used in the present AME2012 evaluation, as well as in the NUBASE2012 evaluation (first article in this issue), are listed in a special table, at the end of this paper.

A list of identifiers for journals, books, conferences ... is given first, as much as possible in the CODEN-style

(see [12]). With one exception though, for the *Eur. Phys. Journal* for which we preferred the ‘EPJAA’ identifier, that we think more practical to use, than the ‘ZAANE’ identifier as adopted by the NSR.

In both evaluations, the references for the regular journals were quoted in the NSR [12] key number style, where available. They are listed in this Section by year of publication and first author name.

References

- [1] A.H. Wapstra, G. Audi and C. Thibault, Nucl. Phys. **A 729** (2003) 129; <http://amdc.in2p3.fr/masstables/Ame2003/Ame2003a.pdf>
- [2] G. Audi, A.H. Wapstra and C. Thibault, Nucl. Phys. **A 729** (2003) 337; <http://amdc.in2p3.fr/masstables/Ame2003/Ame2003b.pdf>
- [3] G. Audi and A.H. Wapstra, Nucl. Phys. **A 595** (1995) 409; <http://amdc.in2p3.fr/masstables/Ame1995/>
- [4] G. Audi and A.H. Wapstra, Nucl. Phys. **A 565** (1993) 1; <http://amdc.in2p3.fr/masstables/Ame1993/>
- [5] G. Audi and A.H. Wapstra, Nucl. Phys. **A 565** (1993) 66.
- [6] C. Borcea, G. Audi, A.H. Wapstra and P. Favaron, Nucl. Phys. **A 565** (1993) 158.
- [7] G. Audi, A.H. Wapstra and M. Dedieu, Nucl. Phys. **A 565** (1993) 193.
- [8] K.-N. Huang, M. Aoyagi, M.H. Chen, B. Crasemann and H. Mark, At. Nucl. Data Tables **18** (1976) 243.
- [9] D. Lunney, J.M. Pearson and C. Thibault, Rev. Mod. Phys. **75** (2003) 1021.
- [10] The full list of correlation coefficients can be downloaded from: <http://amdc.in2p3.fr/masstables/Ame2012/a0p5gqddq.zip> or <http://amdc.impcas.ac.cn/masstables/Ame2012/a0p5gqddq.zip>.
- [11] The AME2012 files in the electronic distribution and complementary documents can be retrieved from the Atomic Mass Data Center (AMDC) through the *Web*: <http://amdc.in2p3.fr/> and <http://amdc.impcas.ac.cn/>
- [12] Nuclear Structure Reference (NSR): a computer file of indexed references maintained by NNDC, Brookhaven National Laboratory; B. Pritychenko, E. Běták, M.A. Kellett, B. Singh and J. Totans, Nucl. Instrum. Methods Phys. Res. **A640** (2011) 213; <http://www.nndc.bnl.gov/nsr/>