GAUNTLET

Gauntlet Research Report

Market Risk Assessment

An analysis of the financial risk to participants in the Compound protocol.

Compound

February 2020

Contents

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Part I

Background

Compound allows participants to trustlessly supply and borrow Ethereum assets, providing appealing interest rates for borrowers and passive income for suppliers. By using collateral and amortizing risk across individual suppliers in a liquidity pool, Compound's Ethereum smart contract has been a profitable place to supply crypto since its inception in 2018. The protocol implemented in Compound's smart contract is detailed in the Compound whitepaper.

However, despite the fact that Compound has grown well past nine figures (of USD value) without any suppliers losing money, it is still technically possible, under extreme conditions, for borrowers to default on their borrowed assets and suppliers to lose their principal. Understanding when this failure condition can happen boils down to understanding various types of risks associated with the protocol, including protocol security risk,[1] governance risk,[2] and market risk. This report focuses on evaluating market risk — the risk of a user experiencing losses due to market fluctuations external to the smart contract itself.

We use a rigorous definition of market risks to construct simulation-based stress tests that evaluate the economic security of the Compound protocol as it scales to underwriting billions of dollars of borrowed assets. These stress tests are trained on historical data and put through a battery of scenarios that represent the expected and worst case economic outcomes for the protocol. Our stress tests are constructed analogously to how transaction-level backtesting is done in high-frequency and algorithmic trading. These techniques are used to estimate the market risk of a systematic trading strategy before it is deployed to the market. As there are over $1 trillion US dollars of assets managed by funds that use these techniques to provide daily actuarial analyses to risk managers, we believe that these are the best methodologies for evaluating market risk.[3] By modifying these techniques to handle the idiosyncrasies of cryptocurrencies, we are able to provide similar statistical power in these actuarial analyses.

The first portion of this report will define the set of market risks that users of the Compound protocol face, breaking them down into their principal quantitative components. Subsequently, we will describe the incentives behind the mechanism that the Compound protocol uses to ensure that it is solvent — liquidations. Finally, we will conclude by detailing how liquidators are similar to trading strategies and detail the market impact models that are used to analyze their incentives and expected returns.

The second portion will focus on methodology and results from agent-based simulations of the Compound smart contract. Our methodology utilizes careful simulation to closely replicate

[1] Examined by independent smart contract auditors: Certora, OpenZeppelin, and Trail of Bits

[2] More broadly, this refers to things like administrator mismanagement, voter participation, etc.

[3] Arnott et al. (2005); Curcuru et al. (2014); Hsu (2004)

the live environment that users interact with in the Compound protocol. This approach and some of our novel technologies, such as a custom Ethereum virtual machine, ensure that our results replicate reality with high fidelity. We conclude by detailing the results of these simulations, providing actuarial assurances for the conditions under which the Compound protocol is insolvent.

Our conclusions show that **the Compound protocol can scale to a larger size and handle high volatility scenarios for a variety of collateral types**. In particular, we find statistically significant evidence that even when Ether (ETH) realizes it's maximum historical volatility, the Compound system is able to grow total borrowed value by more than 10x while having a sub-1% chance of default.[4] Note that in this report we will refer to the protocol being in 'default' as equivalent to being under-collateralized. Moreover, we find that the system stays significantly over-collateralized in extreme scenarios and that current liquidation incentives are sufficient for more liquid collateral types, such as ETH. We also note that for collateral that realizes super-linear slippage (e.g. trading costs per unit quantity increase with larger liquidation sizes), one needs to be more aggressive with both liquidation incentives and collateralization ratios. The same techniques used to justify these conclusions also provide guidance on how to set protocol parameters for new collateral types that are added in the future. Finally, we note that more detailed descriptions of our simulation methodology and a glossary of terms utilized throughout this report can be found in Appendix 9.

[4] This 10x is relative to the size of the Ether market. In the case where that grows a commensurate amount, as it easily could, then Compound could grow even larger.

Part II
Defining Market Risks

1 Market Risks

The decentralized nature of the Compound protocol renders risk assessment both more complex and crucial than similar assessments in traditional markets. The main causes for this increase in complexity are the multitude of participant behaviors in the Compound protocol as well as their interactions with exogenous markets, such as centralized cryptocurrency trading venues. Unlike formal verification and smart contract auditing, which focus on *endogenous* risks within a smart contract, economic analysis of protocols focuses on how *exogenous* shocks affect participant behavior. As the Compound protocol uses a deterministic function of liquidity supply and borrowing demand to determine the interest rates that suppliers and borrowers receive, one need only consider market prices, supplier supply behavior, and borrowing demand to accurately model exogenous risk (see Appendix 10). More specifically, the primary sources of exogenous risk stem from the following components:

1. Shocks to market prices of collateral that cause the contract to become insolvent due to under-collateralization
2. Loss of liquidity in an external market place, leading to a liquidator being disincentivized to liquidate defaulted collateral
3. Cascades of liquidations impacting external market prices which in turn lead to further liquidations (i.e. a deflationary spiral)

In order to quantify the effects of these risk components, we first need to delve into the notions of assets and liabilities within the Compound protocol.

1.1 Assets and Liabilities

In the Compound protocol, the main assets are the collateral tokens that suppliers have committed to liquidity pools, whereas the main liabilities are the outstanding borrowed assets. Token holders contribute their ERC-20 assets to a liquidity pool, and are in turn paid a yield on their supplied tokens. Borrowers borrow an asset by first committing collateral before withdrawing up to a certain amount from the liquidity pool. This amount is controlled by the *collateral factor*,[5] which is the ratio of the maximum outstanding debt to collateral. The system

[5] You can find more information on the collateral factor in the Compound developer documentation.

forces borrowers to over-collateralize their borrowed assets (e.g. a fully-secured credit facility), thus enforcing the invariant that assets must always be greater than liabilities. For instance, one can deposit $100 of ETH[6] into the contract and withdraw $75 if the contract has a collateral factor of 75%. The borrower's collateral requirement is the value of outstanding debt divided by the collateral factor. When the value of the borrower's collateral asset falls below the collateral requirement, the collateral position becomes liquidatable.

The *net liabilities* of Compound are defined as the asset values less liabilities, so that the system is deemed solvent when the net liabilities are positive. As a decentralized protocol, Compound utilizes a series of economic incentives to ensure that net liabilities are always positive. When the market value of the collateral backing a lien falls below the collateral requirement, the protocol sells the collateral at a discount to a liquidator. This discount, termed the *liquidation incentive*, provides a liquidator with financial incentive to buy the collateral from the protocol, effectively repaying the borrowed asset on behalf of the borrower. With liquidation, the protocol acts much like a bank selling a defaulted asset at a foreclosure auction to increase their net liabilities. In particular, the liquidator acts analogously to the foreclosure auction winner, who is usually able to claim the defaulted asset at a discount.

As an oversimplified example, suppose that the Compound protocol has an ETH borrow position that is in default, with the current collateral amount equal to $100. If the liquidation incentive is 105% (5% extra bonus), then the liquidator would pay the Compound Smart Contract $95 for the ETH collateral. Moreover, if the liquidator has low time preference (Appendix 11.6.2), then they will sell the collateral as soon as possible. In practice, the Compound protocol only lets liquidators liquidate a portion of the borrow amount, and they receive collateral equal to 105% of the borrow value repaid. This has the benefit of increasing the collateralization ratio on the remaining portion of the borrowed asset, while avoiding complicated mechanics of completely closing borrow positions.[7] In this sense, liquidation in Compound resembles an algorithmic trading strategy, as there is a race to be the first liquidator to claim portions of the collateral and sell it on the market with minimal transaction and slippage costs.

1.1.1 Synthetic Assets: cTokens

There is a slight nuance in how assets and liabilities are treated — technically, the assets that suppliers and borrowers interact with are cTokens. These tokens, which wrap standard ERC-20 assets, serve as contingent claims on assets and earned interest. Suppliers supply assets

[6] In this stylized example, we use US Dollars as a numéraire, whereas in reality, one would have to execute this transaction in the Compound protocol against a USD stablecoin. Stablecoins are digital representations of US dollars, with some backed by bank deposits (USDC, TUSD) and others backed by digital collateral (DAI).

[7] Contrast this with the model MakerDAO uses, where there are auctions to liquidate the entire borrowed asset. This can create a delay which adds to market risk as well as unnecessarily closes borrow positions which could be merely reduced to a safe level.

as ERC-20 tokens and are returned cTokens, whereas borrowers supply collateral, which is converted to a cToken and used to make outstanding interest payments. Unlike traditional assets, cTokens immediately realize earned interest as payments are paid pro rata to holders on every block update.

Technically, there is a security risk that a cToken cannot be converted back to the underlying asset if the contract has many outstanding borrowed assets that are not being repaid as collateral is redeemed. This would mean that the contract is illiquid, but not necessarily insolvent. This report focuses on solvency, and liquidity will be considered more deeply in future analysis.

1.2 Risk Sensitive Parameters of the Protocol

The main levers protocol designers can wield in Compound to reduce risk are the collateral factor and liquidation incentive. However, these two levers impact the incentives of the protocol in different ways. The collateral factor controls the riskiness of borrowers — the closer it is to 100%, the more likely risky borrowers will default by borrowing USD stablecoin against collateral that is rapidly decaying in value. On the other hand, the liquidation incentive controls how likely liquidators are to take liabilities off of the smart contract's balance sheet. The higher the liquidation incentive, the less time a defaulted borrowed asset will be a liability on the Compound protocol. If we dissect how the three risk components of §1 connect to these two parameters, we find the following:

- The risk inherent in the collateral factor is connected to the nature of shocks to the market price of the collateral
- The risks that liquidators with low time preference face is connected to the loss of liquidity in an external market place
- Cascading liquidations affect both the collateral factor and the liquidation incentive because they create a feedback loop between price shocks and a loss of liquidity

This implies that under normal market conditions, when liquidations are independently distributed (e.g. uncorrelated), the collateral factor and liquidation incentive control borrower risk and supplier's ability to recoup losses, respectively. However, in situations when liquidations have a 'knock-on' effect and are correlated, these parameters affect both borrower and supplier behavior. Therefore, to study the true market risk of the system, we need to sample a variety of market and liquidity conditions in order to stress test these scenarios.

2 Liquidation

Akin to foreclosure sale participants in traditional finance, liquidators can repay the outstanding debt with discounts in exchange for the borrower's cToken collateral. In both foreclosure sales and in Compound liquidations, discounts are used to incentivize purchases of defaulted collateral. The Compound protocol provides a discount by giving liquidators additional collateral as the liquidation incentive to perform liquidation. However, unlike the all-or-nothing transactions of foreclosure sales, an individual liquidator can only repay a portion of the debt. The *close factor* is the protocol parameter that specifies the proportion eligible to be liquidated by any individual liquidator. When a liquidator finds a profitable trade, she repays a portion of the outstanding debt (determined by the close factor) in return for the borrower's collateral. Depending on a liquidator's risk preference, she may sell the collateral immediately to protect against price-fluctuation risk or just hold the received collateral.

Liquidation incentives create an arbitrage opportunity or a price discount for the liquidator in exchange for the reduction of Compound's risk exposure. The higher the liquidation incentive is, the more liquidators will participate in the liquidation process as they get steeper discounts relative to market prices. In other words, tuning the liquidation incentive is one of the most effective ways to adjust the protocol's safety boundary. The liquidation incentive also has an influence on a borrower's decision to borrow asset within the protocol. When a borrower's lien is liquidated, the liquidation incentive can be viewed as a bonus amount of a borrower's collateral that is given to the liquidator to compensate for the risk they engender while taking a liability off of the protocol's balance sheet. If the liquidation incentive is too high, a borrower may be unwilling to borrow assets from Compound in the first place, or she may open a borrowing position and maintain a high collateral factor. In general, one expects that increased liquidation incentives negatively impact borrowing demand.

The collateral factor defines a maximum borrowing capacity for each asset enabled within the protocol. Borrowers must manage their own debt and keep their liens over-collateralized to ensure a certain margin of safety with respect to the maximum borrowing capacity. This margin of safety fluctuates with market conditions and depends on the borrowers' own risk profile. When the market volatility is high, risk-averse borrowers maintain a high margin of safety to avoid their collateral being liquidated. In contrast, risk-seeking borrowers maintain a low margin of safety and actively refinance their debt to optimize their usage of borrowed capital. Understanding the interaction between collateral factor and the safety margin requires studying the influence of psychology on the participant's behavior. Randomized controlled trials and other experimental methods are designed to understand this type of causal relationship.

Rational liquidators with short time preference are defined to be participants who purchase collateral from the Compound smart contract and immediately sell it on a centralized venue (e.g. have low risk tolerance). For brevity, we will refer to rational liquidators with short time preference as *greedy liquidators*. To simplify the analysis and simulate the worst-case scenario for Compound, we assume that all liquidators are greedy and sell the collateral immediately to a market, instead of having liquidators that repay the outstanding debt and hold the collateral. This focus on greedy liquidators emulates the worst-case protocol behavior as adverse market and liquidity conditions can cause cascading defaults. Greedy liquidators tend to inflame cascading defaults as they create sell pressure and can cause a deleveraging spiral.[8] The main source of loss for greedy liquidators is the loss due to price impact, or *slippage*, that is caused by selling a large quantity of an asset. Given that greedy liquidators immediately sell, they must optimize the quantity that they are willing to liquidate based on market prices and expectations of slippage.

3 Slippage

Slippage refers to the expected change in a tradeable asset's price p due to a matched order of size q and is mathematically denoted $\Delta p(q)$. Formally, $\Delta p(q)$ is defined to be the difference between the market midpoint price and the actual average execution price when a market participant executes a trade. Slippage inevitably happens on every trade, and this effect tends to be magnified in thin or high volatility markets. For a liquidation opportunity, slippage is the only cost that can be partially controlled by the liquidator, whereas trading fees and smart contract transaction fees are usually external restrictions. Therefore, slippage is one of the major factors that influence a liquidator's decision-making.

Market impact, which is a synonym for slippage, has been studied extensively in traditional finance.[9] Many market impact models have been proposed and tested for solving optimal order execution problems. In traditional markets, the marginal increase in price impact is usually observed to decrease as a function of trade quantity, which formally corresponds to $\Delta p(q)$ being a concave function.[10] However, this appears to not be true for cryptocurrency markets, where empirical data suggests that $\Delta p(q)$ is linear or even convex (e.g. the marginal cost *increases* with quantity).[11] Despite each type of model having different underlying assumptions and functional forms, a majority of the models comprise trade volume-to-market size, volatility and time variables. Analyzing trade size, volatility and how these variables interact with liquidation is the primary focus of this analysis. The analysis in this report only considers greedy liquidators that sell repossessed collateral on centralized exchanges with order books, such as Coinbase and Binance. As decentralized exchanges and automated market makers, such as

[8] See Klages-Mundt and Minca (2019) for an in-depth discussion of this in MakerDAO

[9] Tóth et al. (2011); Gatheral and Schied (2011)

[10] Eisler et al. (2012); Gatheral (2010)

[11] Makarov and Schoar (2019); Wei (2018)

Uniswap,[12] provide an alternative source of liquidity, one might ask why this assumption was enforced. The reasons for this choice are two-fold:

- Order book depth on centralized exchanges is order of magnitudes greater than that of decentralized exchanges for most assets[13]
- Slippage in automated market makers is usually designed to be small for small trades and expensive for large quantities, so greedy liquidators would likely end up going to a centralized exchange during the most volatile times to stay profitable

We will break up the dominant features of slippage into *market variables* that are exogenous to the Compound smart contract state and *protocol variables*.

3.1 Key Market Variables

3.1.1 Outstanding Debt

The total traded quantity that the protocol will need liquidated in times of net negative liabilities will be a function of the total outstanding debt in the system. Since this quantity is the input to the slippage function $\Delta p(q)$, it is clear that the choice of slippage model needs to be cognizant of the amount of outstanding debt. We will define the amount of outstanding debt in this analysis to be the sum of all the borrowers' total outstanding debt value normalized by the average daily trading volume of underlying collateral. This metric captures the size of debt relative to the underlying liquidity, and gives readers a good intuition around how big Compound's market can grow safely relative to the trading markets. Since the trading volume of different assets varies, using unitless metrics (such as the amount of outstanding debt) provides a more intuitive comparison between different assets. The simulation in this report assumes borrowers borrowing USD stablecoin backed by ETH, as this is the most common use case in the Compound protocol. As an example, suppose that the ETH daily trading volume is 100 million USD, 0.5 total outstanding debt is equivalent to 50 million USD of total outstanding debt value.

Estimating the average daily trading volume of cryptocurrencies is difficult, as wash trading and other market manipulation practices are known issues in the cryptocurrency market.[14] Numerous studies have concluded that the reported volume from various cryptocurrency exchanges may be unrepresentative of the assets' underlying liquidity. For this reason, we aggregated the average daily trading volume from the top 10 exchanges with well-functioning

[12] Angeris et al. (2019)

[13] We do note that this is not true for assets such as MKR and SNX, as their primary market is Uniswap. However, for the larger assets that are listed on Compound such as ETH, DAI, and REP, there is far more centralized exchange liquidity.

[14] Alameda Research (2019)

markets identified by Bitwise Investments.[15] This indexing methodology has been adopted as the de facto industry standard, with major brokers and the Securities and Exchange Commission utilizing the Bitwise index for volume estimation.[16]

3.1.2 Asset Volatility

Volatility measures the degree of variation of asset price changes over a given time interval. Historically, it is traditionally defined as the standard deviation of logarithmic returns and is usually denoted σ.[17] Research studies show that volatility is typically a linear coefficient in a market impact model.[18] Given that asset volatility changes over time and is affected by market microstructure, it's equally important to understand how liquidator behavior changes when the market volatility changes. We assess this by sweeping through a variety of different volatility levels to ensure that we emulate how greedy liquidators interact with a plethora of market environments. Note that we normalize our volatility calculation in a manner akin to what is used by exchanges such as BitMEX.[19]

3.2 Key Protocol Variables

3.2.1 Liquidation Incentive

The liquidation incentive is the main driver for liquidators to repay borrowers' outstanding debt. If liquidation incentives don't exist, no rational liquidator will be willing to reduce the borrower's risk exposure during the collateral price drop. The size of liquidation incentive has a substantial influence on the liquidators' decision-making process. A greedy liquidator adjusts their strategy to ensure that she receives positive returns on every liquidation opportunity. Borrowers' outstanding debt will no longer get repaid if the liquidation incentive is too low and can't cover the cost of arbitrage, which includes slippage, trading fees, and transaction fees. Note that latency costs and smart contract front-running[20] are intentionally left out of this analysis because these introduce additional complexity. In particular, we use constant gas costs throughout all simulations detailed in this report. We can adjust our simulations to handle front-running, should there be further empirical evidence that liquidations costs are dominated

[15] Hougan et al. (2019); Bitwise Asset Management (2019)

[16] Securities and Exchange Commission (2019)

[17] Hull (1991)

[18] Mathematically, this means that there exists a function $f : [0, \infty) \to \mathbb{R}$ such that $\Delta p(q) = \sigma f(q) + o(1)$; see Hull (1991); Almgren et al. (2005) for theoretical and empirical evidence of this. In particular, note that this appears to hold for many markets in terms of *permanent* impact cost, whereas instantanous impact cost tends to depend much more on an asset's microstructure details.

[19] BitMEX (2020)

[20] Daian et al. (2020)

by losses due to such behaviors. However, it appears that there currently is more front-running on exchanges than there is for liquidations, guiding our decision to elide an transaction delay model.[21] Furthermore, we sweep a wide range of slippage conditions in our analysis, and delays in order execution have a very similar effect as increased slippage for liquidators.

3.2.2 Collateral Factor

In the ideal scenario, if the liquidation incentive is enough to motivate liquidators to close out liquidatable borrowings instantly all the time, there's no need to require the over-collateralization of borrowed assets. In reality, due to transaction latency or the absence of sophisticated liquidators, a borrowed asset may not be closed out at the time it becomes liquidatable. Over-collateralization is required to protect a decentralized interest rate protocol from default. The collateral factor specifies a borrower's minimum collateral requirement. It acts as an additional buffer to prevent the system from becoming under-collateralized if the debt cannot be liquidated in time.

The collateral factor setting is essentially dependent on the liquidation incentive. Given that the liquidation incentive is paid in the form of the collateral asset, a high liquidation incentive means that more collateral will get sold during liquidations. If the collateral factor is too large (the collateral requirement is too low) to account for the high liquidation incentive, the liquidator may be unable to get their full value to close the entire position. For this reason, when one sets the liquidation incentive and collateral factor for a new asset, she needs to enforce the following constraint:

$$\frac{1}{\text{collateral factor}} \geq \text{liquidation incentive}$$

Note that it is unclear how borrowers will manage their debt and collateral according to various collateral factor settings, and validating an assumption made for borrower's behavior is nontrivial due to the lack of empirical data. In our simulation, we rescale the borrower's collateral-to-debt ratio based on the ratio between the simulation input collateral factor and Compound's default ETH collateral factor. The assumption we made for borrower's behavior is our best guess, and the real-world borrower behavior may deviate from the simulation.

3.2.3 Close Factor

A liquidator is only allowed to liquidate a portion of the borrowed asset in each transaction, and the maximum fraction is specified by the close factor. Compound protocol designers face the trade-off between user adoption and protocol safety when determining the close factor:

[21] Zhang (2020); Schmidt (2020)

a small close factor can prevent a substantial amount of a borrower's borrowed asset from being liquidated and hence increases borrowing demand, while a large close factor enables liquidators to instantly reduce the system's risk exposure. During an extreme price shock, the design of close factors forces liquidators to break down a large repayment into multiple value decaying repayments, and delays the liquidations in consequence. Properly simulating the effect of the close factor requires precise modeling of the liquidator's and borrower's response time to liquidation events. Although the close factor is a part of the simulation process, we will not further examine the effect of changing the close factor in this report.

Part III

Simulated Stress Tests

4 Background on Simulation

4.1 Agent-Based Simulation

The main tool that we use to perform simulation-based stress tests on Compound's Ethereum smart contracts is agent-based simulation (ABS). ABS has been used in a variety of stress test contexts, including to estimate censorship in cryptocurrency protocols,[22] detect fraudulent trading activity in CFTC exchanges,[23] and in stress testing frameworks from the European Central Bank[24] and the Federal Reserve.[25] These simulations, while powerful, can be difficult to make both useful and accurate as model complexity can make it hard to match experimental results.[26] Careful design, tuning, and infrastructure architecture can help avoid these pitfalls and has made ABS invaluable in industries such as algorithmic trading and self-driving car deployment.

In such industries, one takes care to ensure that the simulation environment replicates the live environment as closely as possible. This is enforced by having the agent models interact with the same code that is deployed in a live environment in order to minimize errors due to mistranslations or missing minutae. While the infrastructure overhead of simulating users interacting with a piece of complex software can be heavy, it ensures that errors are limited to those in models of agents as opposed to errors in the models of system dynamics.

As an example, the Compound interest rate curve (Appendix 10) is described via a simple mathematical formula. One can simulate agents directly interacting with this formula, without needing to host the Ethereum environment and having the agents generate transactions. However, Ethereum's 256-bit numerical system and precision differences between different ERC-20 contracts can often lead to disastrous losses due to numerical errors. These cannot be probed without running simulations directly against the Ethereum smart contract and generating the exact same transactions that an agent would if they were a liquidator interacting with the live contract.

[22] Chitra et al. (2019)
[23] Yang et al. (2012)
[24] Halaj (2018); Liu et al. (2017)
[25] Geanakoplos et al. (2012); Bookstaber et al. (2018)
[26] Fagiolo and Roventini (2016)

4.2 Gauntlet Simulation Environment

The Gauntlet platform, which was used for all simulations and results in this report, provides a modular, generic ABS interface for running simulations directly against Ethereum smart contracts. In this system, the agent models are specified via a Python domain-specific language (DSL), akin to Facebook's PyTorch,[27] and interact with a custom-built Ethereum virtual machine that is written in C++. Agents can also interact with non-blockchain modules, such as historical or synthetic market data and/or other off-chain systems. Gauntlet has made significant performance optimizations for interacting with the EVM in Python, resulting in performance gains of 50-100x over the stock tooling. The DSL hides the blockchain-level details from the analyst, allowing the end-user to develop strategies that can migrate from one smart contract to another, should they have similar interfaces. Most of the platform's design is inspired by similar platforms in algorithmic trading that allow for quantitative researchers to develop strategies that execute over multiple exchanges (with varying order books, wire protocols, slippage models, etc.) without having to know these low-level details. Moreover, the non-blockchain portions of the simulation are analogous to trading back-testing environments,[28] so that agents are interacting with realistic order books and financial data. It should be noted that the strategies emit valid EVM transactions and can be deployed to Ethereum mainnet using the same code path.

4.3 Compound Simulation Overview

For the simulations in this report, we deployed the Compound contracts within the Gauntlet platform and also set up a variety of slippage models and synthetic price trajectories (see Appendix 11.2). We implemented liquidator strategies in our DSL, which allowed for a variety of liquidators with different risk and time preferences to interact directly with the Compound contracts and with simulated order books (see Appendix 11.6). These strategies also include optimization components so that liquidators can optimize the amount of collateral purchased based on their slippage estimates (see Appendices 11.3 and 11.6.1) We also wrote strategies for borrowers in the Compound protocol using the DSL and fit their risk preferences based on historical data (see Appendix 11.4). For further details on simulation methodology, please consult Appendix 11.

5 Questions Addressed in Stress Tests

From a liquidity supplier's perspective, the protocol is safe only if the supplied assets can be safely withdrawn. A functioning liquidation mechanism is critical to the safe operation of the

[27] Paszke et al. (2019)
[28] Nystrup et al. (2019)

Compound market. When an asset price drops and no liquidators have an incentive to repay the borrower's outstanding debt, the system fails and some suppliers cannot withdraw their assets. Recall that a rational liquidator's goal is to make a profit in each liquidation opportunity, which depends on the liquidation incentive and slippage (this is dependent on the trade size and volatility). In light of this, the main questions that we focus on answering are the following:

- Is the protocol safe when the total outstanding debt is high?
- Is the protocol safe under volatile market conditions?
- If Compound wants to support a new asset, how should one set the liquidation incentive and collateral factor so that the system will have a large enough margin of safety?

We will first define some metrics that will help us answer these questions in a quantitative manner. An *under-collateralized run* is a simulation run that ends with $>$1% of the value of the market's total outstanding debt that is under-collateralized. Let the *under-collateralized run percentage* be defined as the percentage of simulation runs that are under-collateralized runs. This metric is used to quantify the safety of the system, as the system will be at risk if borrowers with a large amount of outstanding debt are under-collateralized. As we want to ensure that the system is never under-collateralized, we use a strict 1% debt threshold to define the failure criteria.

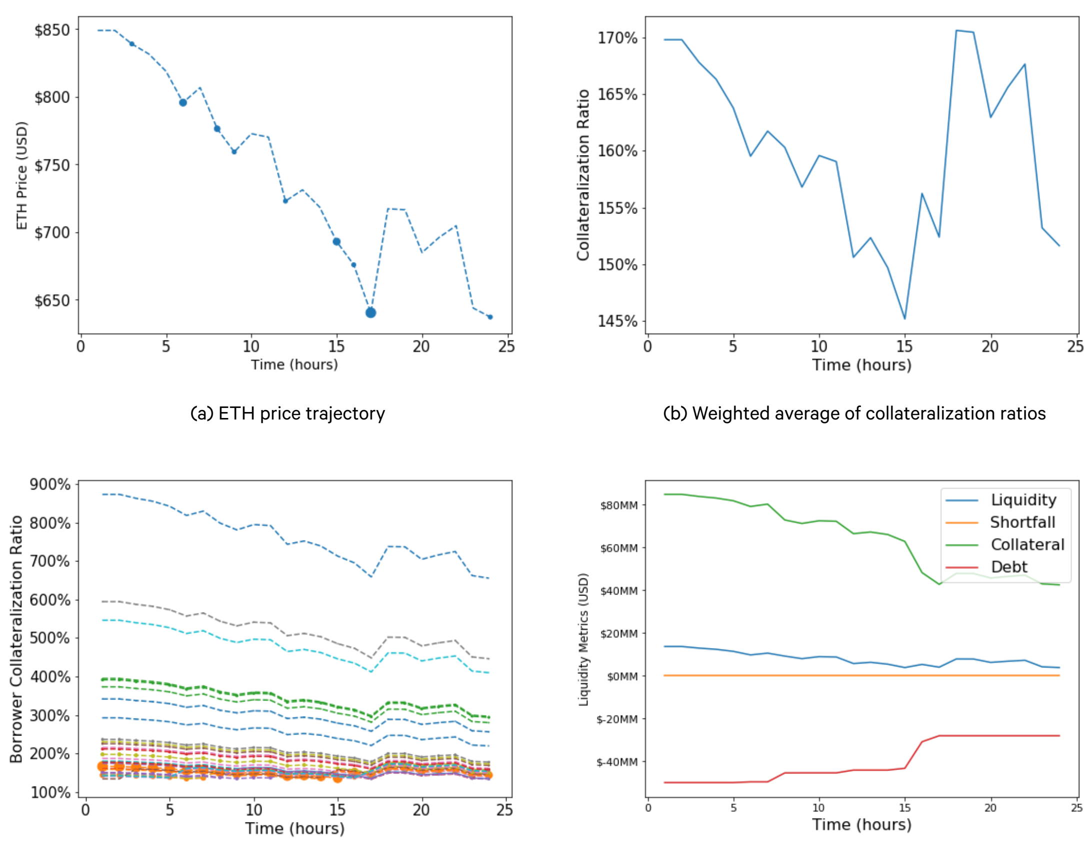
The simulation assumes borrowers use ETH as collateral and are borrowing the stablecoin DAI from the Compound protocol. Each liquidator evaluates all borrower's debt-to-collateral ratio and repays DAI on the borrower's behalf if there's an arbitrage opportunity. We stress tested a wide range of market conditions and analyzed the simulation outcomes. The test scenarios include:

- Modeling historical data.
- Simulating various total outstanding debt and asset volatility.
- Simulating various total outstanding debt and liquidation incentives.
- Simulating various total outstanding debt and collateral factors.

6 Results

6.1 Historical Data

This section gives a brief overview of the liquidation mechanism and demonstrates how the key metrics change over time. We replayed the price trajectory of the worst day in Ethereum



(a) ETH price trajectory

(b) Weighted average of collateralization ratios

(c) Borrowers' collateralization ratios

(d) Liquidity metrics

Figure 1: Liquidation mechanism simulation with the price trajectory of the worst day in Ethereum history (2018-02-05). The simulation assumes $100MM USD of ETH daily trading volume and the sum of the total outstanding debt value is $50MM USD.

history and simulated the liquidation mechanism. This day includes a major price crash, where the ETH price dropped 26%. The simulation results are in Figure 1, which will give readers an idea of what is involved in the simulation. In figure 1a, the size of the dot represents the number of liquidations. As the price dropped, borrowers with a low collateralization ratio got liquidated first. When the price bottomed out at about $640, a large portion of the borrowing positions got liquidated.

Figure 1b shows the simulated liquidity pool's weighted average of collateralization ratios. In this report, borrowers use ETH as collateral and withdraw stablecoin DAI from the liquidity pool. Assuming that the price of DAI is stable, the collateralization ratio changes are mainly impacted by two factors: ETH price and liquidation. ETH price change affects the collateral value, whereas liquidations reduce the quantity of DAI and ETH issued in the liquidity pool. We randomly sample the borrowers' initial collateralization ratios from Compound's real-world distribution (see Appendix 11.4). The initial collateralization ratio starts at around 170% and decreases as the price declines. The liquidator holds the collateralization ratio above 133% ($= 1/0.75$ default collateral factor) minimum collateralization ratio by liquidating the risky debts, which proves that the liquidation mechanism is functioning in this scenario.

In figure 1c, each line represents an individual borrower's collateralization ratio and the dot size represents the borrower's outstanding debt value. All the lines roughly follow a similar trajectory, which is driven by the ETH price change. When a borrower gets liquidated, a portion of the borrower's collateral and debt will get reduced, which results in an increase of the collateralization ratio. Similar to the previous chart, the individual borrower's collateralization ratio should never go below 133% minimum collateralization ratio with a well-functioning liquidation mechanism. If a borrower does not get liquidated in the first place, there's an extra 33% buffer (relative to the borrower's outstanding debt value) to prevent a borrower from being under-collateralized.

The liquidity metrics can be shown in figure 1d. Liquidity/shortfall is defined as

$$\text{collateral value} * \text{collateral factor} - \text{outstanding debt value} \tag{1}$$

Either liquidity or shortfall is non-zero, depending on how the borrowing capacity compares to the outstanding debt value. As the ETH price dips, the collateral value and the liquidity both trend down. The shortfall remains at $0 since all the risky borrowing positions are being liquidated in time.

6.2 Total Outstanding Debt vs. Asset Volatility

As was discussed in section 3.1, the collateral asset's quantity to be traded and the asset's volatility are two major market variables causing slippage, and slippage is one of the main factors influencing a liquidator's behavior. This suggests that the protocol's safety heavily depends on the total outstanding debt and the collateral asset's volatility.

In our simulation, the total outstanding debt is defined as the asset pool's total outstanding stablecoin debt value normalized by the collateral asset's daily trading volume. Considering that different collateral assets have different orders of magnitude of trading volume, normalizing the total outstanding debt enables us to intuitively compare the debt (relative to the collateral asset's liquidity) between different collateral assets. The simulation time duration is a day, hence we use daily volatility instead of commonly used annualized volatility to make it straightforward to understand.

There is not strong agreement on the daily trading volume of most crypto tokens. Centralized exchanges are susceptible to wash trading, and decentralized exchanges are dwarfed by their centralized counterparts. As the ability to sell collateral quickly is one of the driving factors of safety, this creates an uncertainty that is addressed via simulation. By varying the ratio of outstanding debt to market size widely in our simulations, we cover a broad swath of scenarios that you might see in the practice. If you have very conservative assumptions on the total market depth of the collateral order book, you can assume a higher ratio of debt ratio, examined in the top of the heatmap. Our assumptions on ETH market size are fairly conservative ($100mm), falling on the lower end of Messari's daily trading volumes for the beginning of 2020. [29].

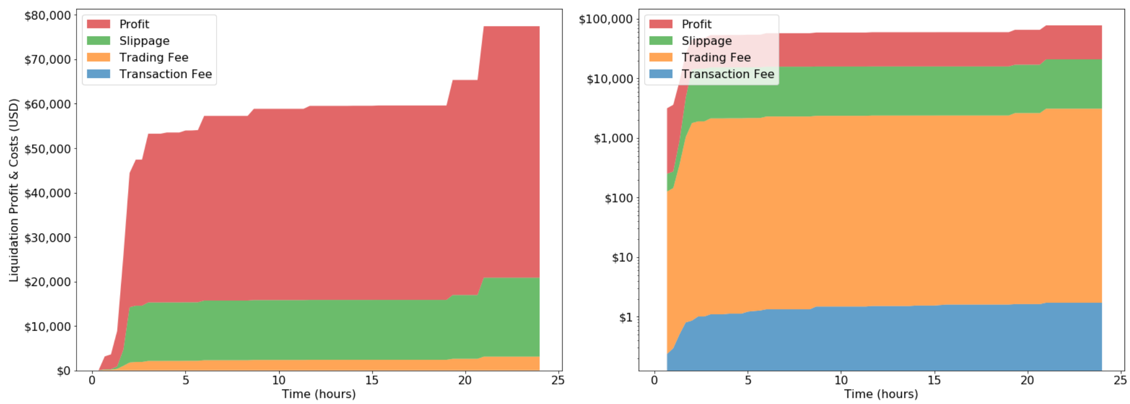
Although the protocol provides liquidation incentives to the liquidator, there are still unavoidable costs for a liquidator to arbitrage, including slippage, transaction fee, and trading fee. Transaction fees are the gas fees paid to an Ethereum miner for executing on-chain transactions. When a liquidator sells his received collateral, he needs to pay the trading fee to the exchange.

In Figure 2, we see liquidator profit and loss charts broken up into transaction fee, trading fee, slippage, and profit. There are more arbitrage opportunities when the asset's volatility is high (figure 2b), and subsequently the liquidator's total revenue (the sum of profit and costs) is higher than the revenue in the low volatility regime (figure 2a). Our simulation uses a linear slippage model, which means that the price slippage is proportional to the asset's volatility (see Appendix 11.6.1 for the empirical rationale). The chart demonstrates that price slippage is the major cost of arbitrage. In the high volatility scenario, slippage represents more than 30% of the liquidator's revenue. Even in the low volatility scenario, the liquidator still has to pay more than 10% of the revenue for slippage. In both scenarios, the trading fee takes a fixed percentage of revenue and the on-chain transaction fee is insignificant at this level of the total outstanding debt.

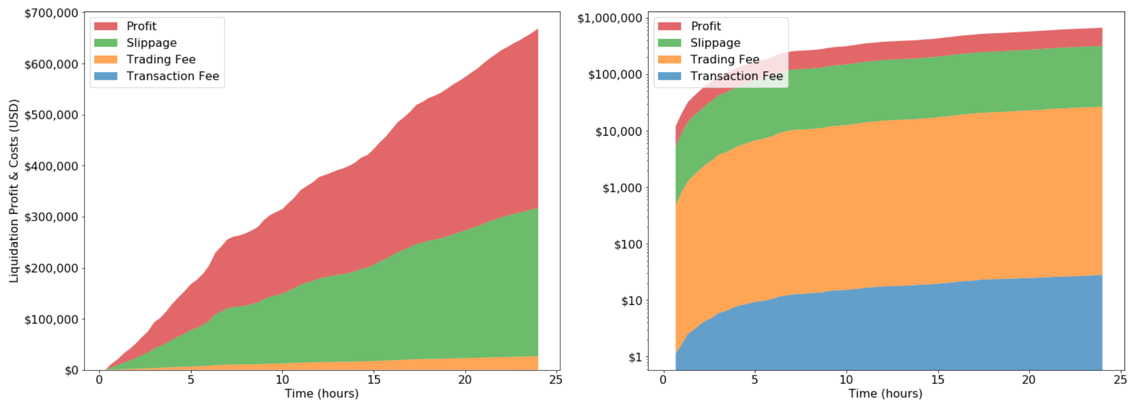
Figure 3 shows the total liquidated debt amount with different initial total outstanding debt and ETH volatility. The results match our intuition: the total liquidated debt amount is proportional to both total outstanding debt and volatility. In the high volatility scenario, a borrower's collateral value has a high chance to fall below the collateral requirement and, as a consequence, the collateral will get liquidated. Though liquidations are a necessary part of the Compound protocol, they can also serve as a leading indicator of under-collateralization.

Figure 4 demonstrates the liquidatable and under-collateralized run percentages heatmap. Recall that an account becomes liquidatable if the collateral value falls below the collateral requirement and the collateral is available to be liquidated. When the price of the collateral asset drops further and the collateral value is below the outstanding debt value, an account becomes under-collateralized. Here we set a strict 1% debt threshold to define the failure

[29] Messari Historical Ethereum Data



(a) 3% daily volatility



(b) 50% daily volatility

Figure 2: Mean aggregate liquidator profit and costs over 30 simulation runs. Note that the y-axis on the left-hand side is using a linear scale (in dollars), whereas the right-hand side is using a logarithmic scale. The simulation assumes $100MM USD of ETH daily trading volume and the sum of the total outstanding debt value is $50MM USD.

criteria, i.e., the simulation run fails when over 1% of the outstanding debt is liquidatable/under-collateralized. For each data point in the heatmap, we aggregate 30 simulation runs with the same market variables and calculate the percentage of the runs that fail. The lighter the data point is, the fewer simulation runs fail. If the data point is white, the protocol is safe and none of the simulation runs have more than 1% of the under-collateralized debt. We use this metric to quantify the safety of the protocol.

The heatmaps demonstrate how large the protocol can scale under a reasonable volatility assumption. The BitMEX weekly historical ETH volatility index reports that the highest ETH

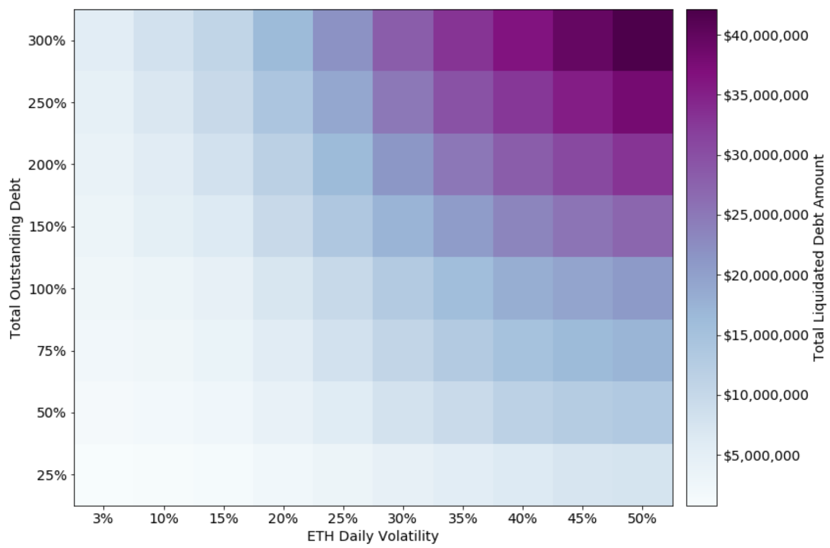


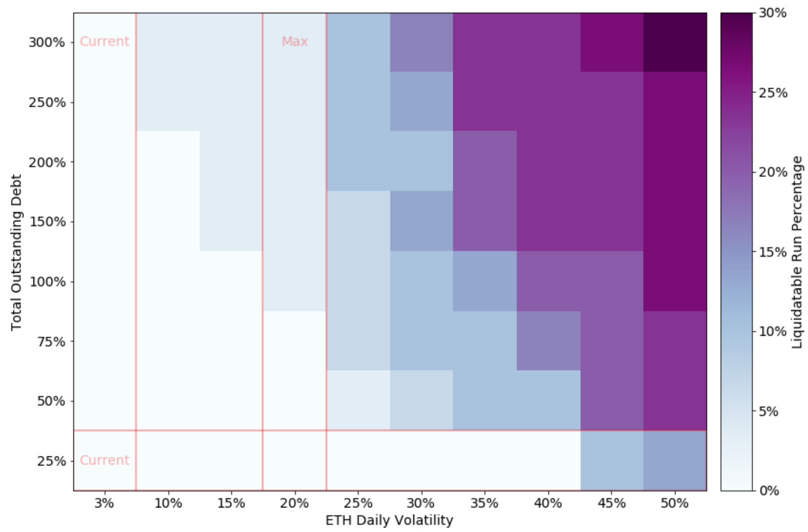
Figure 3: Total liquidated debt amounts over 24 hour period. The simulation assumes $100MM USD of ETH daily trading volume. A 75% total outstanding debt is equivalent to $75MM USD worth of the total outstanding debt value. To explain an example cell here: $20mm in liquidations corresponds to 20% of the Total Outstanding Debt in the 100% case, which is intuitively a worrying number liquidations. However, this does match intuition, because you only see this happen when ETH has a worrying level volatility (close to 50%)

weekly volatility in history happened in August 2017 and peaked at around 20%[30] daily volatility[31]. Assuming that the ETH market capitalization will grow over time and the volatility will decrease, we consider daily volatility < 20% as reasonable. Figure 4a shows that when the daily volatility is 20% and the total outstanding debt is greater than or equal to $100 MM USD, a few risky borrowing positions will not be fully liquidated at the end of the simulation runs. However, with the same 20% daily volatility assumption, none of the borrowers are under-collateralized, and the protocol can scale to at least 10x of the current borrow size, as shown in figure 4b.

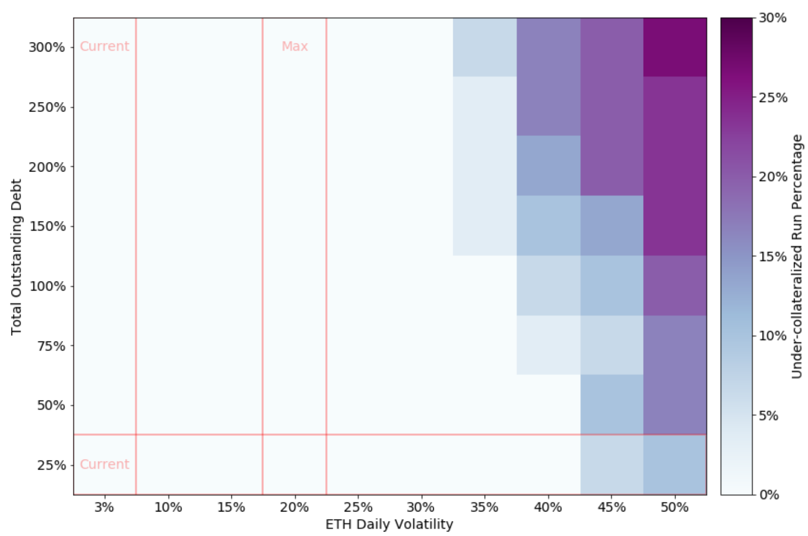
Figure 4b highlights the safe operating space of the protocol. The protocol is safe when the volatility is below 35% and the liquidity pool's total outstanding debt value is below ETH's daily trading volume. As the volatility reaches 45%, some suppliers may be unable to withdraw their supplied assets.

[30] BitMEX (2020)

[31] The daily volatility is converted from realized weekly volatility.



(a) Percentages of simulation runs that end with $> 1\%$ of liquidatable debt



(b) Percentages of simulation runs that end with $> 1\%$ of under-collateralized debt

Figure 4: The Compound contracts are deployed with the default parameters. According to BitMEX weekly historical ETH volatility index, the current daily volatility is around 3% and the highest historical daily volatility is around 20%. The simulation assumes $100MM USD of ETH daily trading volume. Compound's current total outstanding stablecoin debt value is around $25MM USD. The current total outstanding debt is around 25%, which is the total outstanding debt value normalized by the daily trading volume of the collateral asset.

6.3 Total Outstanding Debt vs. Liquidation Incentive

In this section, we examine how changes in liquidation incentive can affect the protocol's safety. ETH's daily volatility in 2019 ranges between 1% and 4%. If we use 4% daily volatility to generate synthetic price trajectories and adjust liquidation incentives to evaluate the market risk, none of the simulation runs ends with a liquidatable or under-collateralized borrower.

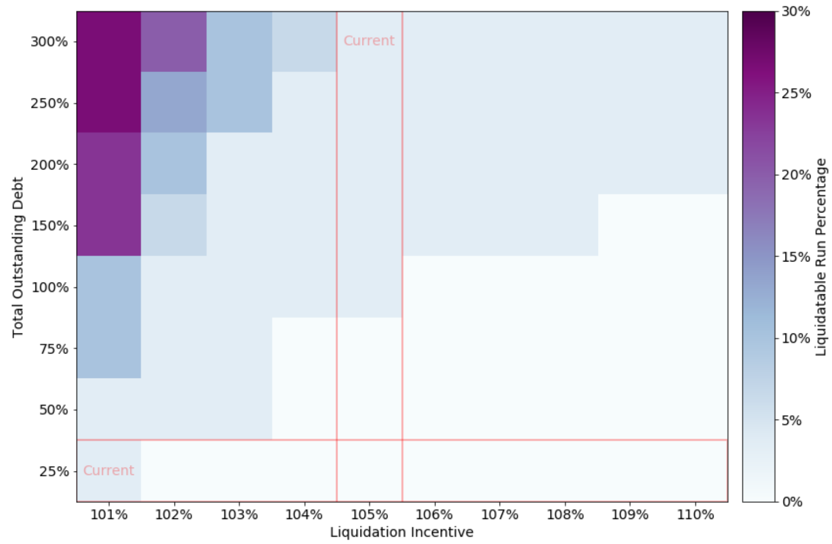


Figure 5: Percentages of simulation runs that end with $> 1\%$ of liquidatable debt. The current default liquidation incentive is 105%. The simulation assumes 20% ETH daily volatility and $100MM USD of ETH daily trading volume. Compound's current total outstanding stablecoin debt value is around $25MM USD. The current total outstanding debt is around 25%, which is the total outstanding debt value normalized by the daily trading volume of the collateral asset.

To better assess the system's safety with various liquidation incentives, we use 20% daily volatility as an assumption. With the default 105% liquidation incentive, some borrowers cannot get liquidated when the total outstanding debt value is above the ETH daily trading volume, as can be seen in figure 5. Despite this, when we evaluate the under-collateralization risk with the same range of parameters, all the simulation runs end with over-collateralized positions. Given that a low liquidation incentive is ineffective in attracting liquidators to arbitrage, one may wonder why there's no under-collateralization risk in this scenario. A possible explanation for this is that the default 75% collateral factor sets a high collateral requirement, such that the price changes of 20% volatility is insufficient to move the collateral value below the outstanding debt value.

6.4 Total Outstanding Debt vs. Collateral Factor

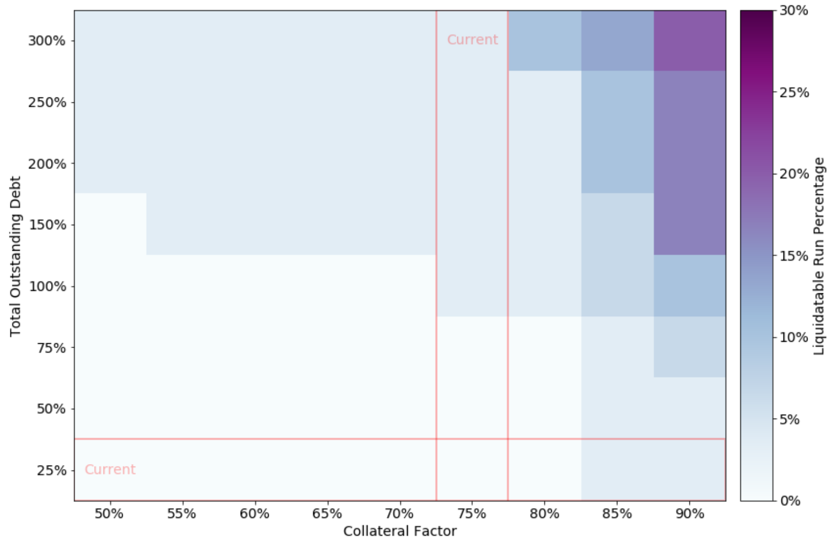
Figure 6 compares the system's safety under different collateral factors and total outstanding debt. As mentioned before, the collateral factor controls a borrower's minimum collateral requirement. The closer the collateral factor is to one, the less collateral a borrower needs to maintain a borrowing position; consequently, borrowers are more likely to default. As illustrated in figure 6b, the system is risky in the high collateral factor and large outstanding debt region. Ideally, collateral factor setting should be a function of the collateral asset's volatility. Assuming a liquidator doesn't exist and the collateral factor is 80%, a borrower would only default if the collateral asset price experienced greater than a 20% price decline. Figure 6b suggests that the protocol is safe when the collateral factor is below 80%, which implies that it's rare to find a $>$20% price decline with the given volatility assumption.

7 Conclusions

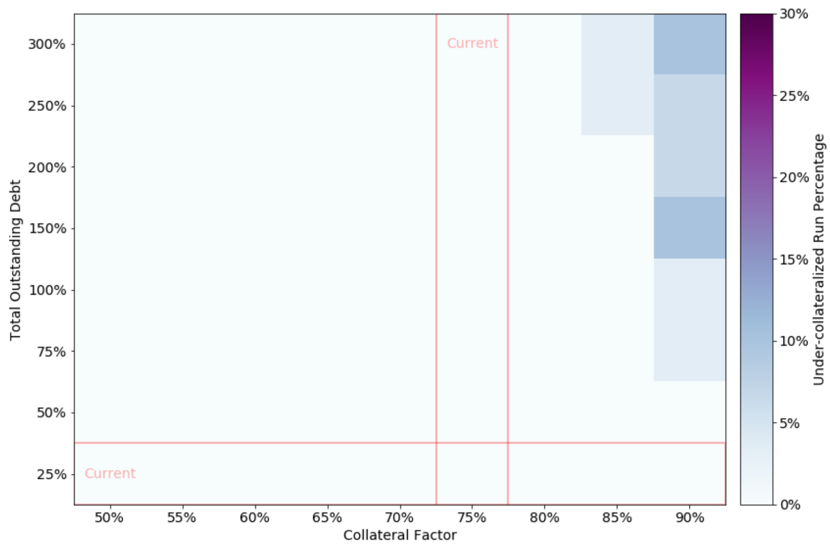
In this report we conducted a market-risk assessment of the Compound protocol via agent-based simulations run against the Compound contracts. We stress-tested the liquidation mechanism under a wide range of market volatility and sizing scenarios to ensure that the protocol can prevent borrowers from becoming under-collateralized in most of these cases. We also used historical market data from centralized cryptocurrency exchanges to ensure that assumptions about volatility and slippage are representative of real-world conditions.

We found that the protocol, as currently parameterized, should be robust enough to scale to at least 3x the current borrow size as long as ETH price volatility does not exceed historical highs. We also analyzed the effectiveness of the liquidation incentive and collateral factor, the two primary risk levers the Compound protocol employs, to navigate the trade-off between safety and capital efficiency.

Our methodology can also be applied to other collateral types on Compound with significantly different liquidity profiles, such as REP. This work informs the Compound community on how to choose collateral factors and liquidation incentives for new assets as they are added to the protocol.



(a) Percentages of simulation runs that end with $> 1\%$ of liquidatable debt



(b) Percentages of simulation runs that end with $> 1\%$ of under-collateralized debt

Figure 6: The current default collateral factor is 75%. The simulation assumes 20% ETH daily volatility and $100MM USD of ETH daily trading volume. Compound's current total outstanding stablecoin debt value is around $25MM USD. The current total outstanding debt is around 25%, which is the total outstanding debt value normalized by the daily trading volume of the collateral asset.

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Part IV
Appendix

9 Glossary

- Debt: Amount of asset borrowed from an asset pool.
- Under-collateralized: An account is under-collateralized if the value of an account's debt exceeds the value of the collateral.
- Collateral factor: Maximum debt-to-collateral ratio of an asset a user may borrow. When the debt-to-collateral ratio exceeds the collateral factor, the collateral is available to be liquidated.
- Collateralization ratio: The ratio of collateral-to-debt, usually reported in percentage points. For instance, a collateralization ratio of 200% means that one needs two times as much collateral deposited into the contract as the maximum borrow quantity. Concretely, this would mean that one must deposit $200 worth of ETH in order to borrow $100 of a stablecoin.
- Borrowing capacity: Current value of collateral deposited into the contract multiplied by the collateral factor.

- Collateral requirement: Value of debt divided by the collateral factor.
- Liquidatable: An account is liquidatable if the account's value of debt exceeds its borrowing capacity. In other words, an account is liquidatable if the account's collateral value falls below the collateral requirement.
- Slippage: The amount of price impact that a liquidator engenders when trying to sell collateral. Slippage is denoted $\Delta p(q)$ and is formally defined as the difference between the midpoint price at time t, $p_{\text{mid}}(t)$ and the execution price, $p_{\text{exec}}(q,t)$ for a traded quantity q at time t, $\Delta p(q,t) = p_{\text{mid}}(t) - p_{\text{exec}}(q,t)$. This quantity is usually a function of other variables, such as implied and realized volatilities. Slippage is also known as market impact within academic literature.

10 Interest Rate Curves

Within the cryptocurrency space, *bonding curves* are deterministic functions of smart contract state that determine bid and ask spreads. Bonding curves are known as *pricing rules* within the algorithmic game theory literature and were first introduced by Hansen[32] in the study of automated market makers.[33] These were first introduced to Ethereum smart contracts by de la Rouviere[34] as a way to create tokenized markets whose buy and sell prices were determined algorithmically. Instead of using a bonding curve to provide bids and offers, the Compound protocol utilizes a bonding curve to compute the spread between the supply and borrowing interest rates. One that think of this as an analogue of the traditional yield curve from finance, albeit computed in a different manner.

The contract also uses the bonding curve to enforce the no-arbitrage condition that the supply interest rate must be strictly lower than the borrowing interest rate. If this were not true, then an arbitrageur could break the system by borrowing cTokens from the contract and adding liened tokens to the liquidity supply, leading to net negative liabilities. Moreover the contract also enforces softer constraints that control the difference between the supply and borrowing interest rates. The main idea behind the curve used in Compound is that if there is more liquidity supply than borrowing demand, then the interest rate to supply liquidity should be significantly lower than the interest rate to borrow.

Formally, the Compound V2 smart contract[35] constructs the bonding curve as a function of the *utilization rate* at block height t, $U_t \in [0,1]$. If we denote the borrowing demand at

[32] Hanson (2003)
[33] Othman et al. (2013)
[34] Rouviere (2017)
[35] Leshner and Hayes (2019)

height t (in tokens) as B_t and the liquidity supply at height t as L_t, then the utilization rate is defined as

$$U_t = \frac{B_t}{L_t + B_t}$$

We compute the borrowing interest rate, β_t and the supply interest rate, ℓ_t, using the following formulas, where $\beta_0, \beta_1 \in (0,1)$ are interest-rate parameters and $\gamma_0 \in (0,1)$ is a measure of the spread between supply and borrowing (i.e. $1 - \gamma_0$ is the relative spread).

$$\beta_t = U_t(\beta_0 + \beta_1 U_t) \tag{2}$$

$$\ell_t = (1 - \gamma_0)\beta_t \tag{3}$$

For reference, the Compound V2 contract uses the values $\beta_0 = 5\%$ and $\beta_1 = 45\%$. The choice of quadratic bonding curve has a variety of benefits that have been profiled in a number of articles and papers.[36]

11 Simulation Details

11.1 Environment and Sampling

The simulation environment allows for configuration of the Compound network's state, including agent distribution, agent behavior, and smart contract parameters. The simulation environment directly interacts with the Compound smart contract deployed on Gauntlet's simulated Ethereum virtual machine. At each time step, agents observe the state of environment variables and on-chain contracts. The policy defines the agent's behavior at a given time and state. When each agent performs an action, the simulation environment submits transactions to the blockchain and updates the state of the smart contract.

For each set of parameters, we run simulation 30 times to make sure that the sample size is large enough to cover a wide range of borrowing distributions.

11.1.1 Simulation Duration

If the simulation time duration is too short, the price movement will be insufficient to affect the overall agent behavior. On the other hand, even if we assume that most borrowers are passive participants and won't often change their debt position, they may still adjust their position when the time duration is long. To balance both of these factors, we use one day as the time duration to run simulations.

[36] Chitra (2019); Obadia (2019)

11.1.2 Agents

There are four types of agents in the simulation setup: supplier, borrower, liquidator, and oracle. In the initialization phase, a supplier supplies DAI to the Compound smart contract, which mints and returns cDAI. Borrowers supply ETH as collateral and borrow DAI from the contract. During each time step, liquidator looks for under-collateralized borrowers and repays DAI to close out the resulting ETH collateral position. The price oracle updates the on-chain token price based on the GBM model or the price trajectories from historical data.

11.2 Price Trajectory

We use a standard Geometric Brownian motion (GBM) to simulate price trajectories. This stochastic process obeys the Îto stochastic differential equation, $dX_t = \mu S_t dt + \sigma S_t dW_t$, where dW_t is the standard Wiener measure on $\mathbb{R}$. GBM is also equivalent to the exponential of a randomly varying quantity follows a Brownian motion, e.g. $X_t = X_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$. The graph below shows the ETH price trajectories with a $200 initial price and various daily volatilities. For each volatility, we generate 30 different price paths to cover a wide range of variation.

We also simulated other Lévy processes, including processes with jumps to emulate flash crashes.[37] However, if the jump parameter (e.g. the mean of a particular Poisson process) led to the average jump being larger than the collateral factor, then the system halted. When the jump parameter is smaller than the collateral factor, there were few differences between the simulation results with and without jumps. Future work will investigate jump processes with memory (such as Hawkes processes[38]), which could have qualitatively different behavior.

11.3 Slippage

The square-root model is the most popular market impact model in traditional finance.[39] To validate whether the same assumption holds in cryptocurrency trading, we pulled the hourly exchange order book snapshot between 12/1 to 12/7 from Coinbase Pro. Since including orders with extremely low prices will bias the slippage estimation, orders with a price below 70% of the market mid-price were filtered out prior to fitting our model. For a given trade size, the slippage is estimated by the market mid-price minus the volume-weighted average price. We

[37] Note that for (semi)-martingales and non-predictable processes, Lévy processes are the broadest parametrizable family of processes that can be written as the sum of a drift-diffusion process and an independent jump process. See the Lévy-Îto decomposition theorem in Lawler (2010) for a detailed discussion.

[38] Hawkes and Oakes (1974)

[39] Almgren et al. (2005); Gatheral and Schied (2011)

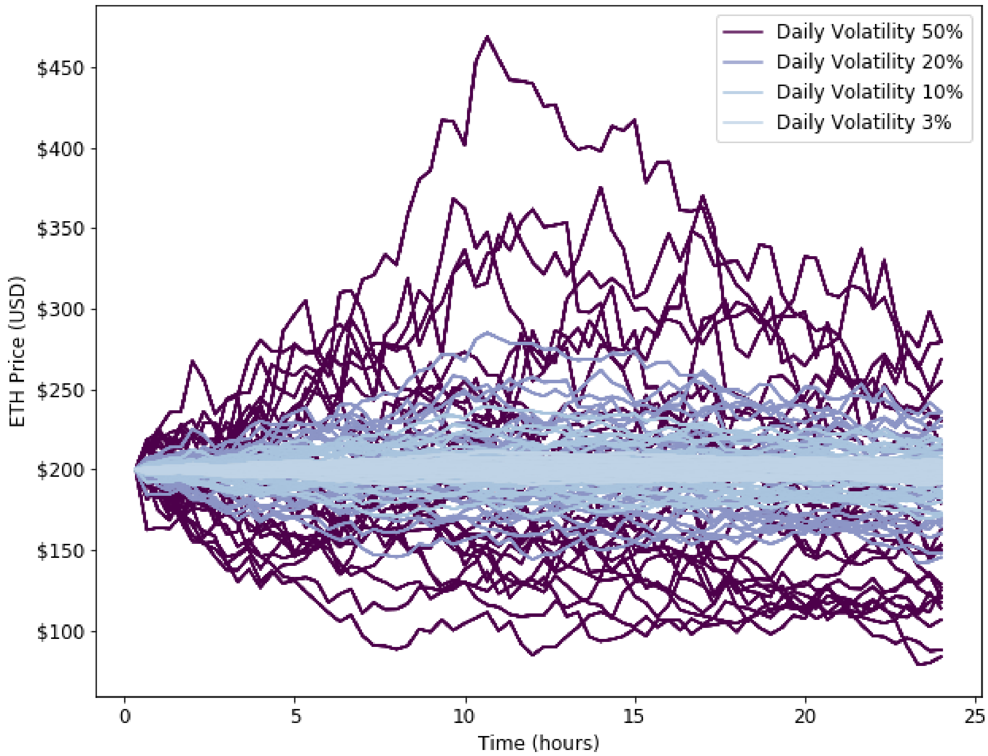

Figure 7: Simulated price trajectories under different volatilities

define the slippage ratio to be the slippage normalized by the market mid-price. Note that the slippage estimate here is modeling the worst-case scenario as we assume that all liquidators are greedy. In practice, a trader with a longer time preference can sell their order through the OTC market or split one order into multiple suborders/exchanges to reduce the slippage.

We use a non-linear least squares method[40] to fit our model and choose the following basis functions to represent our slippage model:

- Square-root: price slippage is proportional to the square root of the quantity traded, i.e. $\Delta p(q) = I\sigma\sqrt{\underline{q}}$
- Linear: price slippage is proportional to the quantity traded, i.e. $\Delta p(q) = I\sigma \underline{q}$

[40] Atkinson and Han (2005)

- Quadratic: price slippage is proportional to the square of quantity traded, i.e. $\Delta p(q) = I\sigma \underline{q}^2$

where the intensity I is a constant determined by the model and $\underline{q}$ is the order size normalized by the average daily trading volume. Theoretically, a more complicated basis function can be chosen to reduce the model's error. For the sake of interpretability and preventing overfitting, we chose simple functional forms for this analysis.

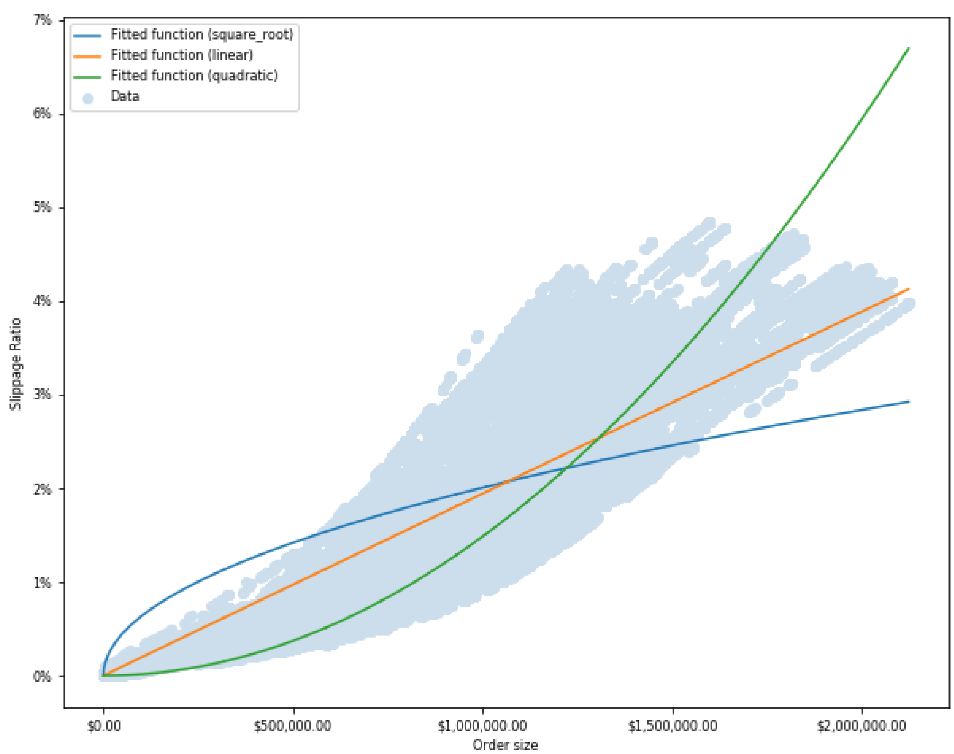

Figure 8: Historical order size versus estimated price slippage data and slippage models

Our results from fitting show that the linear model ($R^2 = 0.76$) fit the data better than the square-root model ($R^2 = 0.56$), so we picked the linear market impact model for our simulation. Moreover, the linear slippage model fits well for sizes below $\$500,000$, as illustrated in Figure 8. One reason for why the square-root model does not fit well with existing data is a dearth of liquidity. Cryptocurrency exchanges' liquidity is relatively small compared to the liquidity that exists on exchanges in traditional finance. In traditional finance, the ratio of change in slippage with respect to change in order size decreased when the order size increased (e.g. $\Delta p(q)$ is concave). However, when there's not enough liquidity, the slippage will be magnified when the order size is large.

11.4 Borrower

The borrower's collateralization ratio (value of collateral / value of outstanding debt) and the total collateral value are two influential parameters on the system dynamics. We visualize real-world Compound borrower data to better understand how borrowers behave. The scatter plot in Figure 9 shows the joint distribution of the logarithm of the borrower's collateral value and collateralization ratio. Again, the collateral factor represents the percentage of the supplied value that can be borrowed, so that the inverse of collateral factor is the minimum collateralization ratio that a borrower will not get liquidated.

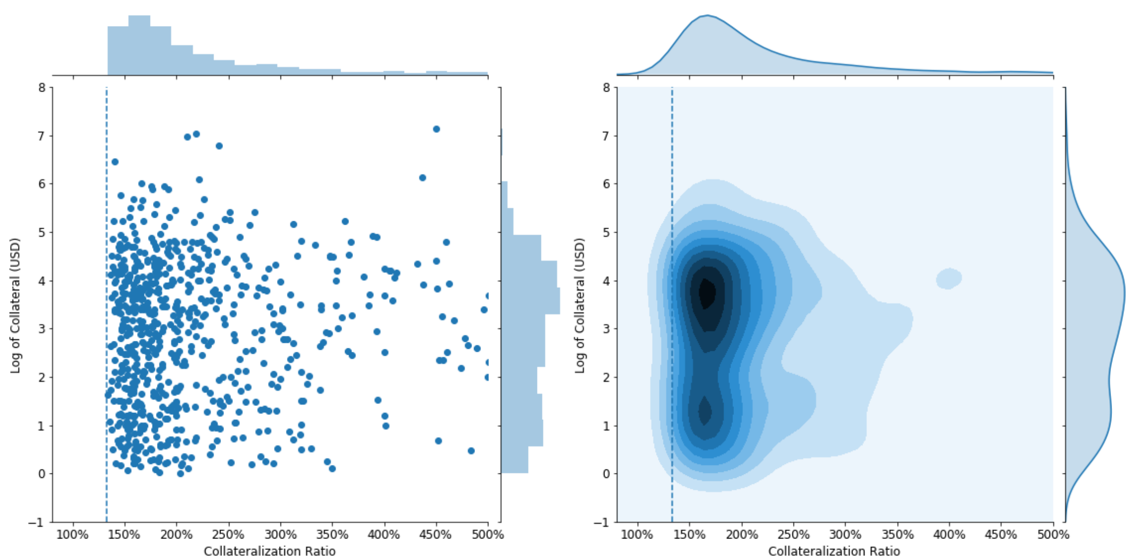

Figure 9: Compound borrower data from contract inception on mainnet to October 19, 2019. The dashed line represents the liquidation threshold (e.g. the minimum allowable inverse collateral factor).

To close the gap between simulation and reality, we have sampled the borrower's collateral value and collateralization ratio from the real-world distribution. The detailed steps for generating a representative sampling of borrowers from historical data are:

- Load Compound's borrower data.

- Use multivariate kernel density estimation to fit the borrower data. The probability density contour plot is on the above right-hand side.
- Sample the collateral value and collateralization ratio from the probability density contour plot.
- During sampling, it's possible that some sampled data points are under-collateralized. For those data points, increase the collateralization ratio to the minimum value satisfying collateral factor constraint to avoid smart contract initialization failure.
- Re-scale the samples such that the sum of the total outstanding debt value equals the simulation input and uses the processed sample data for simulation.

11.5 Liquidity Supply

In the simulation, a supplier supplies DAI to the protocol, which begins accumulating interest rate. The supplier receives a quantity of cDAI equal to the underlying DAI supplied, divided by the exchange rate. The purpose of having a supplier is to provide liquidity to the system and enabling borrowers to withdraw DAI from the liquidity pool. The DAI supplied amount in the simulation equals the sum of all the borrowers' total outstanding debt + 1.

11.6 Liquidator

A rational liquidator's main goal is to maximize profits, by ensuring that the revenue received from the liquidation incentive outweighs the costs. The slippage model assumes that a liquidator will submit a market order on a single exchange, so the cost is the worst-case estimate. Let the liquidation incentive be denoted by η, the trading fee denoted by γ, and the transaction fee denoted by α. We can then formulate the profit $\mathfrak{p}$ of each trade as

$$\mathfrak{p} = \max((\eta - \Delta p(q) - \gamma)\underline{q} - \alpha, 0)$$

For each liquidation opportunity, the liquidator repays the minimum value among

- Maximum repay value: borrower's outstanding debt $\times$ close factor
- Value of borrower's liquidatable collateral
- Liquidator agent's perceived optimal repay amount

The Compound protocol defines maximum repayment amount for liquidating a borrower. If the collateral price drops too fast during periods of extreme volatility and falls below the maximum repay value, a liquidator can only repay up to the value of the borrower's liquidatable collateral. A liquidator also estimates the perceived slippage and calculates the optimal repay amount to maximize the profit. The optimal repayment amount calculation is discussed in the next section. After the liquidator acquires the collateral from the Compound protocol, she immediately sells all received ETH in exchange for USD on an open exchange to realize profit.

11.6.1 Optimal Liquidation Amount

To derive the optimal repayment amount to maximize liquidator profit, we first plug the slippage model into the profit function. For instance, in the linear slippage case, we have the following:

$$\mathfrak{p} = (\eta - I\sigma\underline{q} - \gamma)\underline{q} - \alpha$$

To maximize profit, we find the derivative of profit with respect to the normalized order size $\underline{q}$ and find a value $\underline{q}^*$ such that $\partial_{\underline{q}}\mathfrak{p}(\underline{q}^*) = 0$. By construction, the value $\underline{q}^*$ is the optimal order size that maximizes the net profit. Performing this calculation yields that the optimal value of $\underline{q}$ to maximize profit is,

$$\underline{q}^* = \frac{\eta - \gamma}{I\sigma}$$

Figure 10 shows liquidation incentive, profit curve and the optimal repay value under various volatility assumptions. The area enclosed by rectangle with vertices $(0, 0)$, $(\mathfrak{p}(\underline{q}), 0)$, $(\underline{q}, \mathfrak{p}(\underline{q}))$, $(0, \mathfrak{p}(\underline{q})) \in \mathbb{R}^2$ is the liquidator's net profit. Based on the above derivation, we see that a liquidator has no incentive to liquidate any collateral larger than $\underline{q}^*$. If the value of the borrower's liquidatable collateral is less than $\underline{q}^*$, the rational strategy for a liquidator is to liquidate maximum liquidatable collateral.

11.6.2 Compound liquidator time preferences

To be safe, we always assume that liquidators immediately sell their collateral. This may seem like a presumptuous simplification, but it is the expected outcome of any profit-maximizing strategy, regardless of your valuation of the collateral asset.

Let's say that all participants in Compound have a valuation of the collateral asset v. We'll denote the liquidation incentive as $l = 1.05$. Consider a case where there is a collateral position with \$75 of DAI borrowed against \$101 of ETH. The collateral factor for ETH is .75, so if the price drops by more than \$1, the position can be liquidated. The liquidatable position can be then claimed for \$95. There are three cases for the liquidator here:

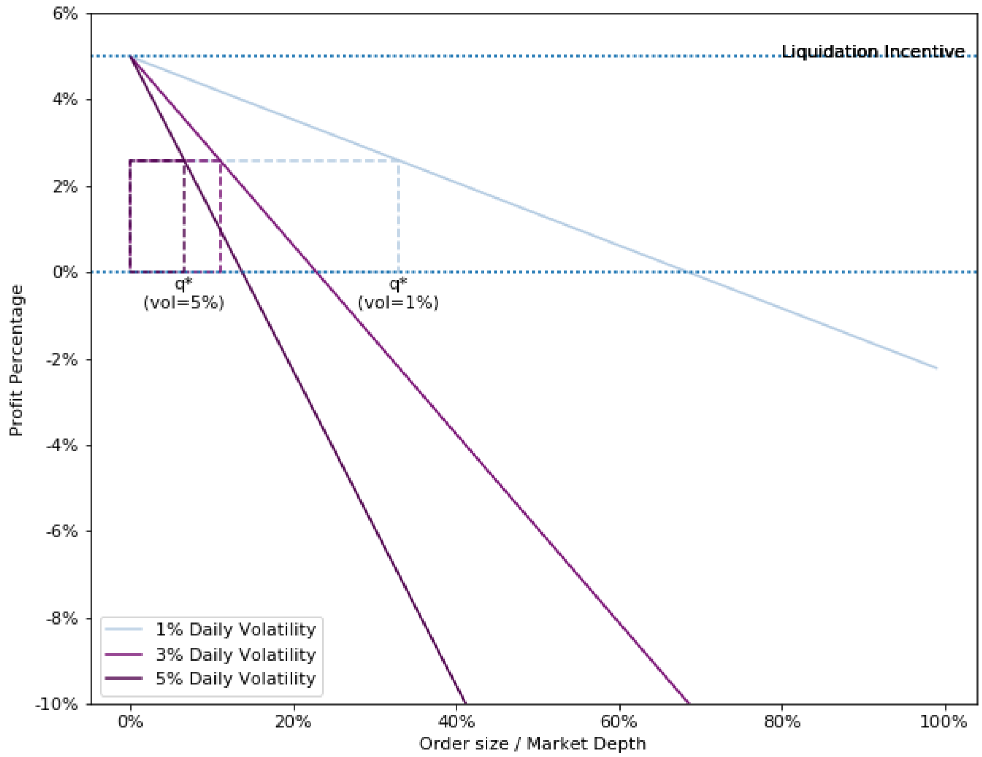

Figure 10: Profit curve and optimal liquidation amount with the assumption of linear slippage model

- **Case 1:** $v \geq \$100$
  In this case, this participant would have bought ETH already, regardless of the liquidation. Assuming there is a large, liquid market for ETH, they will have no more assets to allocate to ETH before the liquidation even occurs.

- **Case 2:** v is between $\$95$ and $\$100$
  In this case, the participant will by the ETH from the liquidation. However, since the market price is greater than v, they will also sell immediately.

- **Case 3:** $v < \$95$
  This participant will still not want to buy the ETH from the liquidation, since the liquidation incentive discount still doesn't give them an opportunity to buy at a price below

their validation.

This simple analysis excludes transaction costs, and the market impact of buying and selling the collateral, but it serves to show that the assumption that liquidators always sell the collateral immediately isn't a glib one. The only case where a rational liquidator would hang on to the collateral is in Case 1, but this implies that they acted irrationally in the past and didn't buy the asset on the open market when they had the opportunity to do so for less than their intrinsic valuation.

11.7 Raw data

Total Outstanding Debt \ ETH Daily Volatility	0.03	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
3.0	0.0	0.03	0.03	0.03	0.1	0.17	0.23	0.23	0.27	0.3
2.5	0.0	0.03	0.03	0.03	0.1	0.13	0.23	0.23	0.23	0.27
2.0	0.0	0.0	0.03	0.03	0.1	0.1	0.2	0.23	0.23	0.27
1.5	0.0	0.0	0.03	0.03	0.07	0.13	0.2	0.23	0.23	0.27
1.0	0.0	0.0	0.0	0.03	0.07	0.1	0.13	0.2	0.2	0.27
0.75	0.0	0.0	0.0	0.0	0.07	0.1	0.1	0.17	0.2	0.23
0.5	0.0	0.0	0.0	0.0	0.03	0.07	0.1	0.1	0.2	0.23
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.13

Table 1: Raw data of figure 4a

Total Outstanding Debt \ ETH Daily Volatility	0.03	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.07	0.17	0.2	0.27
2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.17	0.2	0.23
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.13	0.2	0.23
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.1	0.13	0.23
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.07	0.1	0.2
0.75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.07	0.17
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.17
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.07	0.1

Table 2: Raw data of figure 4b

Total Outstanding Debt \ Liquidation Incentive	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.1
3.0	0.27	0.2	0.1	0.07	0.03	0.03	0.03	0.03	0.03	0.03
2.5	0.27	0.13	0.1	0.03	0.03	0.03	0.03	0.03	0.03	0.03
2.0	0.23	0.1	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
1.5	0.23	0.07	0.03	0.03	0.03	0.03	0.03	0.03	0.0	0.0
1.0	0.1	0.03	0.03	0.03	0.03	0.0	0.0	0.0	0.0	0.0
0.75	0.1	0.03	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.03	0.03	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3: Raw data of figure 5

Total Outstanding Debt \ Collateral Factor	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
3.0	0.03	0.03	0.03	0.03	0.03	0.03	0.1	0.13	0.2
2.5	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.1	0.17
2.0	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.1	0.17
1.5	0.0	0.03	0.03	0.03	0.03	0.03	0.03	0.07	0.17
1.0	0.0	0.0	0.0	0.0	0.0	0.03	0.03	0.07	0.1
0.75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.07
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.03
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.03

Table 4: Raw data of figure 6a

Total Outstanding Debt \ Collateral Factor	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.1
2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.07
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.07
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03
0.75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 5: Raw data of figure 6b

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