The big ideas of the mathematics framework

This document is a compilation of the big ideas associated with the signposts of the mathematics framework. It is important that these are used alongside the sets of illustrations, which support a deeper understanding of what developing expertise looks like by highlighting how students use their knowledge and skills to respond to a specific problem. If the big ideas are used without the illustrations there is a reasonable possibility that teachers will interpret them in a variety of different ways.

Additive thinking

This progression is similar to the additive domain of the Number Framework in that it focuses on the increasingly sophisticated and flexible addition and subtraction strategies that students develop to solve increasingly complex problems. However, the sets of exemplars are not a direct match to the stages of the Number Framework. For example, imaging (stage 3 in the additive domain) is not identified by a discrete set of exemplars, and the higher stages of the domain are represented by more than one set of exemplars.

Set 1

The students compare quantities using informal language. They know some number names and parts of the number-word sequence, and they may subitise (instantly recognise) small quantities.

Set 2

The students use one-to-one correspondence, their knowledge of the number-word sequence, cardinality, and ordinality as they count sets of objects.

Set 3

The students count all of the objects to solve simple addition or subtraction problems. They do this with real objects or by imaging the objects.

Set 4

The students solve problems involving the addition or subtraction of single-digit numbers by counting on or back from the larger number. The language of the problem guides the student to the operation of addition or subtraction.

Set 5

The students have begun to recognise that numbers are abstract units that can be either treated as wholes or partitioned and recombined. This is called part-whole thinking. Students partition single-digit numbers to form “tidy numbers” or use known addition facts to ten to solve problems. The language of the problem guides the student to the operation of addition or subtraction.

Set 6

The students solve problems involving two- and three-digit numbers, in which the mathematical operation is transparent in the wording, by applying a strategy from a limited rehearsed repertoire. The strategies are most likely to involve place-value partitioning, or compensation when the number is close to a tidy number.

Set 7

The students respond flexibly to addition and subtraction problems involving whole numbers and simple decimals by applying and explaining a range of strategies, including the use of inverse operations, as they seek the most efficient method.

Set 8

The students demonstrate flexibility, a strong number sense, and an ability to carry out multiple steps as they estimate and solve complex problems that involve adding and subtracting whole numbers, decimals, fractions, and integers.

Multiplicative thinking

This progression combines elements from both the multiplicative and proportional domains of the Number Framework. However, as with additive thinking, the sets of exemplars are not a direct match to the stages of the Number Framework. This progression focuses on students' ability to think multiplicatively as they solve multiplication, division, and proportional problems involving an extended range of whole numbers, decimals, fractions, ratios, and percentages, and in a range of contexts.

Set 1

The students demonstrate early multiplicative thinking as they understand the concept of "same" and recognise equal groups. However, they are unable to quantify the objects to solve multiplication problems.

Set 2

The students solve multiplication problems by considering each object one at a time and by counting from one to find the total number of objects in equal sets. The student solves simple division problems by equally distributing objects one at a time into sets, and by counting from one to check that the sets are of the same size.

Set 3

The students solve single-digit multiplication and division problems by using additive thinking strategies, including skip counting, repeated addition, and repeated subtraction.

Set 4

The students solve single-digit, single-step multiplication problems by applying known multiplication facts or by using known facts to derive unknown ones. The students solve simple division problems by using known multiplication facts or by trial and error with different-sized groups.

Set 5

The students use their known multiplication basic facts and place-value knowledge to solve multiplication and division problems involving single-digit multipliers or divisors.

Set 6

The students solve multi-digit problems in which the mathematical operation is transparent in the wording. They do this by applying a multiplicative strategy from a limited rehearsed repertoire that is likely to include place value, or compensation when the number is close to a tidy number.

Set 7

The students respond flexibly to multiplication and division problems involving whole numbers, common fractions, decimals, and percentages by selecting from a range of strategies based on the type of problem and numbers involved.

Set 8

The students demonstrate flexibility and a strong number sense as they estimate and solve complex problems that involve whole numbers, decimals, fractions, and integers.

Set 9

The students apply proportional thinking to solve real problems involving percentages, rates, and ratios, and form one- and two-step algebraic equations to solve for an unknown.

Patterns and relationships

This algebraic thinking progression develops students’ understanding of the structure of and the relationships within numbers, shapes, and measures. In exploring patterns of increasing complexity, students develop the ability to recognise, explain, and generalise relationships between quantities and objects.

Set 1

The students copy or create simple patterns with a unit repeat of two, but they are unable to discuss or explain their thinking.

Set 2

The students create, continue, and describe repeating patterns to solve problems.

Set 3

The students continue and describe growing sequential spatial and number patterns to solve problems.

Set 4

The students connect the elements of sequential patterns with their ordinal position to solve problems.

Set 5

The students use tables, graphs, and rules to solve problems that involve sequential patterns.

Set 6

The students use diagrams, tables, graphs, and equations to solve problems that involve linear relationships.

Set 7

The students graph linear relationships, demonstrate understanding of a standard parabola, and form equations to describe relational representations, and they can apply these to solve real problems.

Using symbols and expressions to think mathematically

This algebraic thinking progression is fundamental to all other aspects of mathematics. It focuses on the ways in which symbols, expressions, and equations are used to communicate mathematical ideas. In solving problems in a range of contexts, students must make sense of the symbols they read and must be able to express their understanding of a problem, using the symbolic language of mathematics.

Set 1

The students recognise some numeral and operation symbols and know what these stand for.

Set 2

The students record equations for simple addition and subtraction problems and explain their thinking. They are able to read simple additive equations and suggest appropriate problem contexts.

Set 3

The students record equality and simple inequality statements for additive and multiplicative problems and can explain their thinking. They are able to read equality and simple inequality statements and can suggest appropriate problem contexts.

Set 4

The students read and record additive and multiplicative equality and inequality equations for problems involving unknowns. They are able to suggest appropriate problem contexts for equations involving unknowns, and they recognise the need to use an inverse operation to find an unknown.

Set 5

The students use the algebraic convention of letter symbols and the concept of equivalence to solve simple proportional problems.

Set 6

The students write equality statements of problem situations involving four or more terms or factors, including one unknown. They use formal operations to solve equations involving real numbers by operating equally on both sides.

Set 7

The students use a systematic approach and algebraic notation, including quadratic equations, to model real world situations and to solve problems.

Geometric thinking

As students make sense of and navigate their spatial world, they come to recognise, describe, and use the properties and symmetries of shapes, and to describe movement and position with increasing accuracy.

Set 1

The students are becoming aware of space and shape as they interact with and navigate their immediate environment.

Set 2

The students notice shapes and use their appearance to solve problems. They flip, turn, and slide shapes, and describe their location in relation to other things in their immediate environment.

Set 3

The students notice and use features of shapes and transformations to solve single-step problems. They describe features and explain in their own words why shapes and transformations are the same or different. They are able to see things from a perspective other than their own and are learning to use geometric terms appropriately.

Set 4

The students understand and use the features of shapes and transformations to solve multi-step problems. They are able to view and draw shapes from multiple perspectives. The students use the language of geometry accurately as they measure and quantify space and shape, and they demonstrate precision in their solutions.

Set 5

The students understand, use, and generalise the properties and classes of shapes and transformations to solve problems. They are able to reason and justify their solutions and use formal geometric language in a clear and precise way.

Set 6

The students understand and can use a scale factor of enlargement, the angle properties of polygons, and trigonometric approaches to solve real-world problems. They are able to describe a locus defined by a set of points, and they can explain their mathematical reasoning within a practical situation.

Measurement sense

Understanding what a measurable attribute is and becoming familiar with the units, systems, and processes that are used in measuring attributes is the focus of this progression. Progression in the understanding of measurement is determined by increased sophistication in the measurable attributes of objects and the complexity of the attribute being measured.

Set 1

The students physically manipulate objects to directly compare them.

Set 2

The students make multiple comparisons between objects and order the objects according to the attribute being measured. They use an indirect or intermediary measure to compare the length of two objects that cannot be directly compared.

Set 3

The students use numbers and a non-standard unit to quantify the attribute that is being measured, for example, five rods for length or nine scoops for capacity. They understand that, when comparing objects, the unit being used to measure must not change. They also understand that, when measuring length or area, they cannot leave gaps or create overlaps as they repeatedly place the measuring unit to mark out the total length or area.

Set 4

The students estimate and measure using the commonly used standard metric units of measure: centimetre, metre, litre, and kilogram. They are developing an understanding of the concept of a scale; for example by constructing a measuring device and accurately marking the device with numbers for all units in that measure, including the positioning of zero. They are able to partition a unit of measure and measure a half unit.

Set 5

The students measure length, capacity, and weight by selecting an appropriate measuring device. They demonstrate greater precision in their measurements by reading scales to marked and unmarked intervals. The students have knowledge of the standard metric units and are able to name and record these units in abbreviated forms (mm, cm, m, km, g, kg, ml, l). They use benchmarks to help them estimate measurements. The students' understanding of how an array is structured supports their calculation of area and volume.

Set 6

The students derive measurements of area and of the volume of cuboids from measurements of length. For example, they find the area of shapes by separating or reconstructing the shapes into rectangles. They understand the relationship between related units in the metric system, and they multiply and divide by powers of 10 to convert into those related units. They understand that the attributes of area and perimeter are not dependent on each other.

Set 7

The students deduce and use formulae to measure perimeter, area, and volume. They are able to separate a two-dimensional shape into familiar shapes to calculate area. They understand that the volume of a prism is calculated by measuring the area of the base and by multiplying that area by the prism's height. They calculate circumference from the measurement of diameter or radius, and they understand that the relationship between circumference and diameter (or radius) is a constant ratio. They understand the metric relationship of: 1 g = 1 ml= 1 cm^3.

Set 8

The students can recognise the complexity of the attribute being measured and use correct units with precision. They can solve measurement problems by splitting shapes into component parts, applying formulae, and using measurement ratios and trigonometric approaches appropriately.

Statistical investigations

The statistical investigations progression is based on the students' development of an increasingly sophisticated implementation of the statistical inquiry cycle that includes posing investigative questions, collecting data, displaying data, and discussing results.

Set 1

The students participate in a teacher-led investigation that involves the collection and display of category data in order to answer an investigative question. However, the students are unable to explain their reasoning.

Set 2

The students participate in a teacher-led investigation that involves the collection and display of category data in order to answer an investigative question, and they can explain and make sense of the findings.

Set 3

The students conduct a statistical investigation that involves the collection, display, and discussion of category and whole-number data in order to answer an investigative question posed by their teacher. They draw an appropriate conclusion and communicate their findings in context.

Set 4

The students pose summary-type investigative questions. They collect and display whole-number data in different ways and communicate their findings in context.

Set 5

The students pose questions to compare or investigate relationships between variables of interest. They collect multivariate category, measurement, or time series data, and they can choose the appropriate displays to analyse and communicate their findings in context.

Set 6

The students pose investigative questions about a wider population. They collect samples of multivariate data and analyse these using displays to find patterns within, between, and beyond the data, and to notice unusual values. They recognise whether data need to be cleaned and can discuss appropriate reasons for variations in the data. The students communicate their findings with appropriate displays and generalise in context.

Interpreting statistical and chance situations

As students are exposed to the statistical evidence presented by others, they need to be able to interpret and gain information from what they see, and critically evaluate both the quality of the evidence and the arguments being presented on the basis of that evidence.

Set 1

The students participate in class statistical investigations and recognise certainty and uncertainty in simple chance situations. They can acknowledge an incorrect statement, but they are unable to explain their reasoning.

Set 2

The students can explain whether statements about data displays are correct with reference to the data. They can identify different outcomes in simple chance situations, and they are able to explain their thinking.

Set 3

The students identify and explain errors in statements and/or data displays of others' statistical investigations. They identify whether an outcome is more likely by systematically recording the results of chance experiments.

Set 4

The students examine and question the steps in the process and results of others' statistical investigations in order to evaluate the appropriateness of the methods and conclusions. They investigate chance situations using experiments and simple models.

Set 5

The students examine the process and results of others' statistical investigations in order to evaluate whether or not their claims are believable and reasonable. They understand and explain why statistical investigations might be presented in particular ways. The students investigate chance situations by making connections between experimental results and theoretical models.