Shape-from-copies

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Abstract

It is shown how multiple occurrences of a planar shape in a single plane or parallel planes can be used to extract information about their common orientation and to consequently deproject the image. This special case of shape-from-contour is coined "shape-from-copies". In particular, pseudo-orthographic projections of planar contour segments related by a similarity are considered. Skewed rotational and mirror symmetries are special cases. One of the major outcomes is that in general considering similarities with an even number of reflections is preferable, in contrast to the popular use of skewed mirror symmetry.

Keywords: deprojection, shape-from-contour, skewed symmetry, invariant

1 Introduction

When assuming orthographic projection, skewed mirror symmetry yields a 1-parameter family of slant-tilt combinations in order for the deprojected shape to have mirror symmetry [6, 1, 2, 4]. The perspective case yields stronger constraints [11, 3], but is not considered here. In this paper, the skewed symmetry instance of what has become known as the "non-accidentalness" approach [9] is generalized towards the deprojection of parallel occurrences of a planar shape. This shape might be unknown, i.e. no fronto-parallel view of the shape has to be given, or, equivalently, no view of known orientation. Such knowledge would yield much stronger information of course [7, 5]. Assuming many occurrences in all kinds of directions to be present, Naito and Rosenfeld [10] suggested techniques for the selection of a fronto-parallel view, thereby bypassing the need for such model to be given. Here it is shown that a few parallel copies of a further unknown planar shape might strongly constrain their planar orientation.

2 Parallel copies of a planar shape

In this section the theoretical underpinning for the deprojection algorithm in the next section is presented. As explained before, the problem under consideration is the following: an image (oblique view) is given of a scene in which several parallel instances of the same planar shape are identified. The question then is: find out the relative position (slant and tilt) of the plane with respect to the camera and consequently derive an orthogonal view of the object plane.

First, we will more rigorously define what "copy" is supposed to mean. Then, deformations by oblique viewing are discussed.

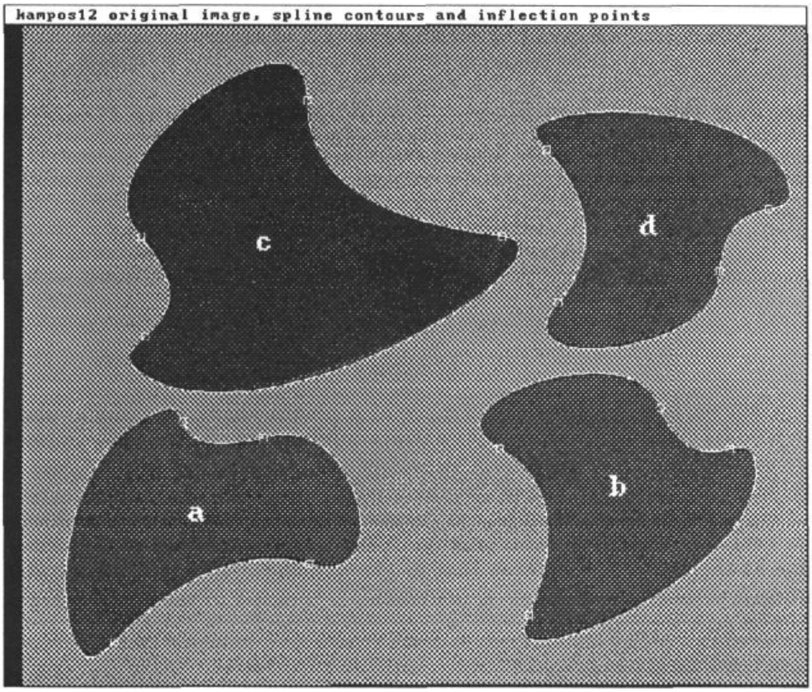

Figure 1: Original image named kampos12 with spline contours and inflection points.

2.1 Definition of copy

Consider a perpendicular view of two parallel, planar shapes. We say that one shape is a *copy* of the other if there exists *a similarity transformation* mapping one onto the other. In particular, we will consider the contours of the objects. "Shapes" can then be complete outlines or contour segments. Choosing a reference frame, a similarity transformation sends a point $\mathbf{x} = (x, y)^t$ to a point $\mathbf{x}' = (x', y')^t$ as follows:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ \pm s_2 & \mp s_1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \tag{1}$$

with scaling factor $s = \sqrt{s_1^2 + s_2^2} \neq 0$. The group of similarity transformations includes rotations, translations, scalings, mirror reflections, and any combination thereof. They come in two categories: those comprising an even number of reflections + scaling, and those with an odd number of reflections + scaling. We'll refer to these types as *even and odd similarities*, resp. Note that even similarities have a positive determinant, whereas odd similarities have a negative one. Shapes that can be matched by such transformations are called *even copies and odd copies*.

Fig. 1 shows an oblique camera image of four objects with equal shapes. Objects a, b and d have the same size and c is twice as large (in area). Shapes a, c and d are even copies of each other, whereas a and b, b and c, and b and d make odd copy pairs. The extracted contours and their inflections are highlighted, since they are used for the detection of the similarities.

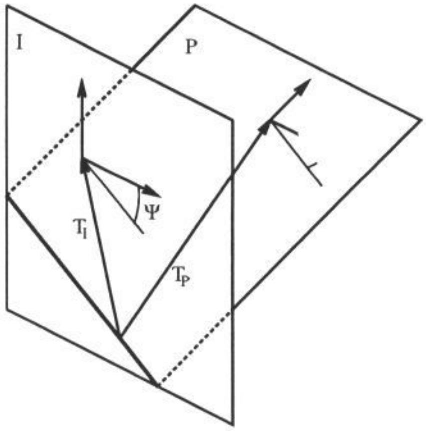

Figure 2: Relation between contour plane and image plane coordinates (see text).

2.2 Copies under oblique viewing conditions

When copies are viewed obliquely, they are no longer mathematically similar in the image. In this section we investigate the type of transformations that emerge.

It will be assumed that the objects are rather small compared to their distance to the camera. In that case, the *pseudo-orthographic projection model*, i.e. orthographic projection followed by scaling to mimic the perspective effect of distance, can be used. Assuming the same scale factor for the whole scene and appropriate choices for the world coordinate system, the transition between 3-D coordinates $(X, Y, Z)^t$ to image coordinates $(x, y)^t$ reduces to $x = \lambda X$ and $y = \lambda Y$ with λ a positive constant.

Suppose that in the plane of the contour segments, the similarity

$$\begin{pmatrix} x'_P \\ y'_P \end{pmatrix} = S \begin{pmatrix} x_P \\ y_P \end{pmatrix} + T \tag{2}$$

holds between two contours, where coordinates (x_P, y_P) are taken with respect to some Euclidean reference frame in the contour plane P; and with S and T as in equation (1).

If the plane P containing the contour segments is not parallel to the image plane I, then these two planes intersect along a straight line. Assume that this line makes an angle θ with the Cartesian coordinate frame in P and an angle ψ with the Cartesian coordinate frame in I, as is illustrated in fig. 2. If both these frames would be rotated and translated so as to align their x-axis with the line of intersection and to make their origin coincide in an (arbitrary) point on this line, then going from contour plane coordinates to image plane coordinates under orthographic projection is a matter of simply scaling the y-contour plane coordinates while maintaining the x-coordinates unchanged. Put differently, the effect of the contour plane's slant is a compression along the new y-direction. Image coordinates are then obtained in a 4-step process:

step 1 Rotate the contour plane frame over the angle θ and translate the rotated frame to the new origin. This yields the multiplication of the contour plane coordinates by the matrix R_P^{-1} with $R_P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, followed by a translation over the vector T_P (represented by a column vector whose components are the coordinates of the old origin with respect to the reference frame with the line of intersection as x-axis). The new x_P-axis is now aligned with the line of intersection.

step 2 Perform a compression C along the new y_P-axis, with $C = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$.

step 3 Translate over the vector $-T_I$ (where T_I is the column vector expressing the coordinates of the origin of the image reference frame with respect to the frame in I with the line of intersection as x-axis) so as to bring the temporary origin on the intersection line to the origin of the image plane frame. Then rotate back over the angle $-\psi$ so as to align the x-axis with the x_I-axis of the original image plane frame, i.e. apply R_I with $R_I = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix}$.

step 4 Finally scale with the pseudo-orthographic factor λ.

Together this yields

$$\begin{pmatrix} x_I \\ y_I \end{pmatrix} = \lambda(R_I C R_P^{-1} \begin{pmatrix} x_P \\ y_P \end{pmatrix} + R_I(CT_P - T_I)) \ . \tag{3}$$

Note that there exist simple relationships between k and ψ on the one hand, and σ and τ — the so-called *slant* and *tilt* — on the other hand[1].

Now let us assume that the original contour segments are related by the similarity (2). Then the image coordinates are related by

$$\begin{pmatrix} x'_I \\ y'_I \end{pmatrix} = A^* \begin{pmatrix} x_I \\ y_I \end{pmatrix} + T^* \ , \tag{4}$$

where

$$A^* = R_I \, C \, R_P^{-1} \, S \, R_P \, C^{-1} \, R_I^{-1} \text{ and} \tag{5}$$

$$T^* = \lambda R_I \, C \, R_P^{-1}((S - I)R_P(C^{-1}T_i - T_P) + T) \ , \tag{6}$$

with primes indicating coordinates for the second contour segment. Transformations of this type form a group when fixing the plane P, which is isomorphic to the group of 2D similarities (this subgroup of the 2D affine group is actually conjugated to the group of similarity transformations). We'll refer to these transformations as *skewed similarities*.

Note also that the determinant of the linear part of the transformation (4) equals the determinant of the matrix S of the corresponding similarity:

$$\det A^* = \det\left(R_I C R_P^{-1} S R_P C^{-1} R_I^{-1}\right) = \det S \ . \tag{7}$$

Consequently, the scaling factor between the original shapes and the even or odd character of the similarity present transpires through the value and the sign of the determinant of the resulting affine transformation.

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[1] The slant is the angle which the planes I and P make, the tilt is the angle between the normal to the line of intersection lying in I and the image x axis. Then the normal to P has (X, Y, Z) components $(\sin\sigma\cos\tau, \sin\sigma\sin\tau, \cos\sigma)$. Taking the vector product of this normal and the normal $(0, 0, 1)$ to I yields as direction for the line of intersection $(\sin\sigma\sin\tau, -\sin\sigma\cos\tau, 0)$. ψ is given as the negative angle between the projection of this direction and the x-axis and is easily found to be $\frac{\pi}{2} - \tau$. k is immediately found as $\cos\sigma$.

3 Skewed similarities and deprojection

If the skewed similarity (4) relating the image contours of two copies is known, it can be used to extract information about the orientation of the plane P containing these contours. The degree to which this orientation retrieval, i.e. the deprojection, is possible, depends on the type of skewed similarity. Depending on the sign of the resulting transformation determinant, each skewed similarity yields constraints on the possible orientations of the contour plane. Odd skewed similarities yield one-parameter families of solutions, whereas even skewed similarities typically yield a pair of mirror orientations as is explained now.

3.1 Even skewed similarities

First suppose that the skewed similarity is even. Deprojection will be performed when the values of the rotation angle ψ corresponding to R_I and the compression factor k in C are found. To this end, consider the relation between these parameters and the skewed similarity matrix A^*:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = R_I C \begin{pmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{pmatrix} C^{-1} R_I^{-1} \tag{8}$$

Equating the corresponding entries, one gets

$$\begin{aligned} a_{11} &= s_1 + s_2(\cos\psi \sin\psi(k - 1/k)) \ , && (9)\\ a_{12} &= s_2(\cos^2\psi/k + \sin^2\psi k) \ , && (10)\\ a_{21} &= -s_2(\cos^2\psi k + \sin^2\psi/k) \text{ and} && (11)\\ a_{22} &= s_1 - s_2(\cos\psi \sin\psi(k - 1/k)) \ . && (12) \end{aligned}$$

We now proceed along the following steps:
From (9) + (12) we obtain

$$s_1 = \frac{a_{11} + a_{22}}{2} \ . \tag{13}$$

Equating s_2 in (9) and (10) yields

$$k^2 = \frac{(a_{11} - s_1)\cos^2\psi + a_{12}\cos\psi\sin\psi}{-(a_{11} - s_1)\sin^2\psi + a_{12}\cos\psi\sin\psi} \ . \tag{14}$$

On the other hand, eliminating s_2 via (10)/(11) yields

$$k^2 = -\frac{a_{21}\cos^2\psi + a_{12}\sin^2\psi}{a_{21}\sin^2\psi + a_{12}\cos^2\psi} \ . \tag{15}$$

Equating k^2 obtained from the two previous steps gives

$$\tan 2\psi = \frac{a_{22} - a_{11}}{a_{21} + a_{12}} \qquad \text{or} \qquad \psi = \frac{1}{2}\arctan\left(\frac{a_{22} - a_{11}}{a_{21} + a_{12}}\right) + n\frac{\pi}{2} \tag{16}$$

with n an arbitrary integer.
As we are only interested in ψ-values in $[0, \pi[$, we find two possible solutions, let's call them ψ_1 and $\psi_2 = \psi_1 + \frac{\pi}{2}$. Using the expression (15) for k^2, we find k_1^2 and k_2^2. One can see that $k_1^2 = 1/k_2^2$. As the compression factor has to be a positive number

smaller than or equal to 1, only the one k value smaller than 1 is acceptable (see also remark 1). The ψ-value to be selected corresponds to the accepted k value. Finally, now that we have values for s_1, ψ, and k, we can use e.g. (10) to calculate s_2:

$$s_2 = \frac{a_{12}}{\cos^2\psi/k + \sin^2\psi k} \ . \tag{17}$$

Remarks

1 When the image and the contour plane happen to be parallel, the equations above become degenerate in the sense that the value of ψ (and possibly also that of k) is undetermined. This was to be expected, since in that case no intersection line can be found between the two planes, and the skewed similarity is already a Euclidean similarity.

2 Numerical unstability may occur when the rotation angle between the different objects is almost 0 or 180 degrees. In that case, $s_2 = 0$, thus yielding a diagonal A^*-matrix (a similarity!), and consequently undetermined values of ψ and k.

3 When searching for coplanar or parallel copies, affine invariants can be used [12, 13, 14]. These techniques yield the correspondence between all points on the contour. The affine transformation (4) can then be obtained by a least-squares fit once the corresponding contour segments have been matched. One should be careful, however, to add additional tests for the parallelism, since affine invariants allow the copies to have arbitrary relative positions.

3.2 Odd skewed similarities

In the case of an odd skewed similarity, we look for a decomposition of the form

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = R_I C \begin{pmatrix} s_1 & s_2 \\ s_2 & -s_1 \end{pmatrix} C^{-1} R_I^{-1} \ , \tag{18}$$

yielding the following relations between the entries:

$$a_{11} = s_1(\cos^2\psi - \sin^2\psi) - s_2\cos\psi\sin\psi(k + 1/k) \ , \tag{19}$$

$$a_{12} = 2s_1\cos\psi\sin\psi + s_2(\cos^2\psi/k - \sin^2\psi k) \ , \tag{20}$$

$$a_{21} = 2s_1\cos\psi\sin\psi - s_2(\sin^2\psi/k - \cos^2\psi k) \ \text{and} \tag{21}$$

$$a_{22} = -a_{11} \ . \tag{22}$$

For odd skewed similarities we thus have 3 equations in 4 unknowns. Therefore, unique deprojection will not be possible on the basis of one such similarity. It is a well-known result that this case leads to a 1-parameter family of solutions. We now proceed with the following steps:
From (20) - (21), we see that

$$s_2 = \frac{a_{12} - a_{21}}{\frac{1}{k} - k} \ . \tag{23}$$

Subsequently, using (20) + (21) we find that

$$s_1 = \frac{(a_{12} + a_{21})(\frac{1}{k} - k) - (a_{12} - a_{21})(\frac{1}{k} + k)\cos 2\psi}{2(\frac{1}{k} - k)\sin 2\psi} \ . \tag{24}$$

Using both (23) and (24) and substituting s_1 and s_2 in (19) gives

$$(2a_{11}\sin 2\psi - (a_{12}+a_{21})\cos 2\psi)(\tfrac{1}{k} - k) + (a_{12}-a_{21})(\tfrac{1}{k}+k) = 0 \ . \qquad (25)$$

Interpreting k as a parameter to choose freely in the [0,1] interval, this equation expresses a 1-parameter family of solutions indeed. Once k has been chosen, one finds the solution for ψ from this equation. Then s_1 and s_2 follow.

Combining two skewed odd similarities

When several skewed odd similarities are present, each gives rise to a 1-parameter family of possible deprojections. In general, the intersection of any two of these families then fixes the right k-value, and consequently permits unique deprojection. We now describe how to extract the appropriate k-value in this case.

Suppose that two skewed odd symmetries are detected with linear parts given by the matrices $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ respectively. Each matrix gives rise to an equation of the form (25). First rewrite these equations as functions of $t = \tan\psi$ by using $\sin 2\psi = \dfrac{2t}{1+t^2}$ and $\cos 2\psi = \dfrac{1-t^2}{1+t^2}$.

For the A-matrix this yields the following formula (after multiplication with the nonzero factor $k(1+t^2)$):

$$[4a_{11}t - (a_{12}+a_{21})(1-t^2)](1-k^2) + (a_{12}-a_{21})(1+t^2)(1+k^2) = 0 \ . \qquad (26)$$

Solving for k yields

$$k^2 = \frac{a_{12}t^2 + 2a_{11}t - a_{21}}{a_{21}t^2 + 2a_{11}t - a_{12}} \ , \qquad (27)$$

and a similar formula for the matrix B. Equating both these expressions, yields, after factoring out the non-zero factor (t^2+1), a quadratic equation for t:

$$(a_{12}b_{21} - a_{21}b_{12})[t^2 - 1] + 2[a_{11}(b_{21}-b_{12}) + b_{11}(a_{12}-a_{21})]t = 0 \ . \qquad (28)$$

Note that this equation has non-negative determinant, and thus we get two real solutions for $t = \tan\psi$, except when $a_{12}b_{21} - a_{21}b_{12} = 0$ and $a_{11}(b_{21}-b_{12}) + b_{11}(a_{12}-a_{21}) = 0$, since in that case all t satisfy the constraint. It is easy to check that this case only occurs when the matrices corresponding to the two odd skewed similarities are equal up to a scaling, i.e. when both include a reflection over axes that are parallel. In that case, the two 1-parameter families coincide. Otherwise, as only ψ-values in $[0,\pi]$ occur, equation (28) gives two possible values for ψ. Let us call them ψ_1 and ψ_2, respectively. Substituting these values into equation (27), we find two values for k (k_1 and k_2, say), only one of which is smaller than 1. This value of k together with its corresponding value of ψ permits a unique deprojection of the planar shapes.

Remark

A simple test to distinguish the skewed odd similarities from more general affine transformations is $a_{11} = -a_{22}$.

4 Implementation

Using affine invariants [14] we find the skewed even similarity between the contour points of b and c in image kampos11 and consequently the deprojection of the contours in the image is obtained (fig. 3).

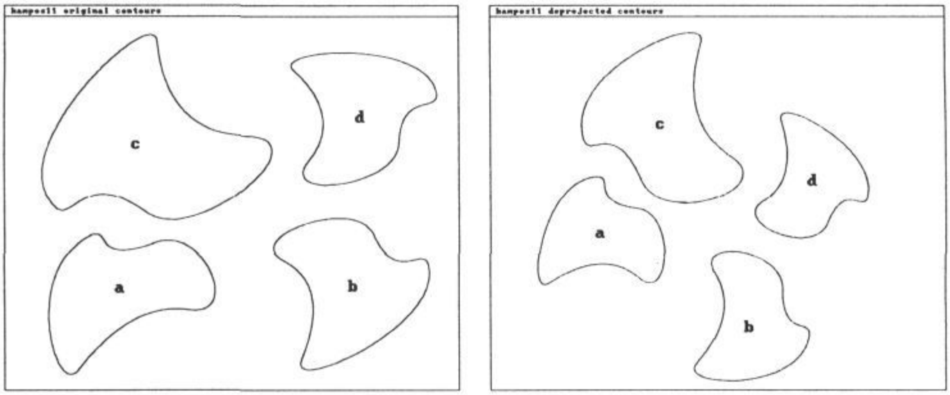

Figure 3: Contours for kampos11 in the image plane and the deprojected contours obtained using copies b and c.

image	symmetry	k	s	ψ
kampos10	$a.b$	0.3316	0.9788	7.2
	$c.d$	0.7535	1.3782	18.6
	$a.d$	0.7642	1.0193	22.3
	$a.c$	0.8314	0.7393	6.1
	$b.c$	0.7481	0.7545	3.9
	$b.d$	0.7879	1.0401	20.6
kampos11	$b.c$	0.7828	0.7549	25.5
	$a.d$	0.7741	1.0367	23.0
	$a\|b+c\|d$	0.8003	0.9647 1.3930	17.6
	$a\|c+b\|d$	0.7688	0.7173 1.0295	19.3
kampos12	$a.c$	0.7926	0.7414	21.3
	$c.d$	0.7635	1.3937	17.7
	$a.d$	0.7712	1.0358	22.7
	$a\|b+d\|b$	0.8048	0.9616 0.9766	18.6
	$b\|c+d\|b$	0.7890	0.7470 0.9766	18.3

Table 1: Camera scaling factor k, similarity scaling factor s and image rotation angle ψ for the images kampos10, kampos11 and kampos12.

In table 1 we show some experimental results derived from three images (see also fig. 4) of coplanar objects taken with the same camera positioning. A dot indicates a rotational symmetry, the vertical bar a mirror symmetry. The combination of a pair of mirror symmetries gives us two estimates for the scaling factor (one for each pair). Object c is twice as large as the others, i.e. a scaling factor of $\sqrt{2}$ or it's inverse is expected. The results obtained by this method are in general quite good, although some examples of (nearly) degenerate solutions can be found in the table. Kampos10 rotations $a.b$, $a.c$ and $b.c$ are nearly multiples of 180 degrees, resulting in worse results for the angle ψ and the compression factor k.

In table 2 we show some distances measured on the original image kampos11 and on the deprojected one. The deprojection was done using the rotation from

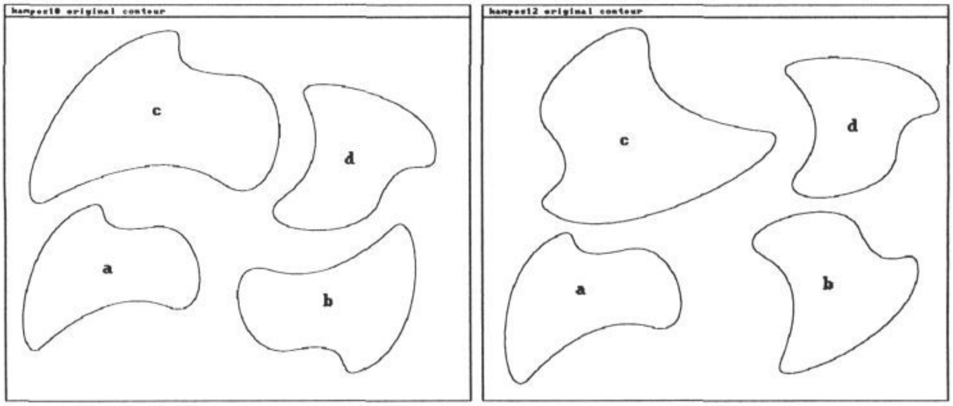

Figure 4: Contours in the images kampos10 and kampos12.

kampos11	original		deprojected	
contour	inside	outside	inside	outside
a	5.0	5.25	4.3	3.85
b	5.85	4.2	4.3	3.8
$c/\sqrt{2}$	4.95	3.85	4.0	3.45
d	5.3	3.75	4.25	3.9

Table 2: Length of longest bitangent line (outside) and distance between the two sharpest tips (inside) for the contours on image kampos11.

object b to object c. The normalized values for the darker object c are somewhat smaller than the others because of border and shadow effects in the edge detection step. The deprojected distances are more or less equal, the distances from the original image are of course distorted by the projection.

5 Conclusions

The problem of similarity-based deprojection was discussed. Skewed odd similarities give rise to a 1-parameter family of solutions, generalizing the results obtained earlier for skewed reflections. Skewed even similarities were shown to often yield unique deprojection (up to the usual mirror ambiguity). It seems justified to preferentially use even skewed similarities for deprojection, since in general a single such match suffices for complete deprojection, whereas at least two odd similarities would be needed. Two main exceptions have to be mentioned though:

1 When there are no even similarities one of course has to resort to odd similarities. Complete deprojection can still be realized when there are two odd similarities between pairs of different copies (otherwise an even similarity is implied).

2 When the even similarity between two copies doesn't allow deprojection (i.e. one has a scaling + translation or halfturn + scaling + translation), but there is a third, reflected copy, then the use of the two odd similarities allows deprojection if the axes of reflection intersect.

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