Attribute-Value Formalization in the Framework of the Logic of Determination of Objects (LDO) and Categorization

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Abstract

There are three major formalisms that are developed around concepts. The first one is Formal Concept Analysis (FCA) by B. Ganter and R. Wille (Ganter & Wille 1999). The second formalism is Description Logic (DL) developped during the 1980s for knowledge representation (Baader *et al.* 2003). The third is Logic of Determination of Objects (LDO) by J.-P. Desclés originating in the 1980s in order to define and articulate notions as concepts and objects, to define and formalize a theory of typicality and an extended theory of quantification (Desclés & Pascu 2006). LDO is a logic applied in natural language processing (NLP) and to the study of natural inferences in common reasoning. In all these formalisms, the notion of property is central. This article constitutes a contribution to an analysis of the notion of property. We present a formal theory of attribute-value in LDO in order to apply it in categorization and semantic annotations.

Keywords : Object, Concept, More or Less Determined Object, Typical, Atypical, Logic of Determination of Objects, Ontologies, Knowledge Representation.

Introduction

The Logic of Determination of Objects (LDO) was introduced in order to analyse the relationship between extension and intension of a concept. It defines, on the one hand, the notion of "determination" and on the other hand the notion of "more or less determined object". In this way, LDO can treat typical and atypical objects. In this paper we analyse the notion of "attribute-value" leading to the notions of "categorising concepts" and "non-categorising concepts". These two classes of concepts are necessary for ontologies which use typical and atypical objects (Rosch 1975) (Rosch & Mervis 1975) (Rosch 1987) and their attributes. In Formal Concept Analysis (FCA) and Description Logic (DL) there are concepts and objects. However, in these formalisms, the notion of concept is rather ambiguous : a concept is sometimes seen as an "attribute" (to be red – having red color for anything which can be red), sometimes as a "capacity to do an action" (to fly for a bird – to be able to fly), sometimes as a "criterion" (more elaborate : to be prime for a number – to be divisible by 1 and itself). Moreover, in FCA, objects are "completely determined", but in the Logic of Determination of Objects (LDO) there are "more or less determined objects" and "completely determined objects" (Desclés 2002), (Desclés & Pascu 2006), (Desclés & Cheong 2006), (Desclés & Pascu 2007a). According to FCA, a "formal context" is a set of determined objects related to a set of attributes. Each object may or may not have any attribute. A "concept in a formal context" (Ganter & Wille 1999) is a pair of two sets : extension which is a subset of the set of objects and intension which is a subset of the set of attributes. The extension and the intension verify the following condition : the set of attributes common to the objects in extension (upper derivation of the extension) equals the intension and, conversely, the objects verifying all attributes of intension (the lower derivation of the intension) equals the extension.

According to LDO, a concept is an operator acting on objects giving a truth value. The context is modelized by a network of concepts (concepts related by a relation of "comprehension"– a concept contents by intension another concept), considered as primitives and there is an operator (τ operator) which associates to each concept f, its objectal representative, τf. A determination operator δ constructs determinations, δg. A determination can be applied to any more or less determined object giving an object more determined than that object.

Let us consider the following example : let Con be the following formal context, in the sense of Wille (Ganter & Wille 1999) : O = {Aristote, Jean-Pierre, Medor, Tutu}, A = {to-be-an-animal, to-be-a-man, to-be-a-dog, from-mythology}. The relation B is given by :

	be-a	be-m	be-dog	in-m
Aristote	1	1		
Charon	1	1		1
Jean-Pierre	1	1		
Medor	1		1	
Tutu	1		1	
Cerber	1		1	1

Let us consider the following pairs : C_1 = (Ext_1, Int_1) = (O, {be-a}) ; C_2 = (Ext_2, Int_2) = ({Aristote, Jean-Pierre}, {be-a, be-m}) ; C_3 = (Ext_3, Int_3) = ({Medor, Tutu }), {be-a, be-dog}) ; C_4 = (Ext_4, Int_4) = ({Aristote, Medor}), {be-a}) ; C_5 = (Ext_5, Int_5) = ({Charon, Cerber}), {be-a, in-m})

According to FCA, C_1, C_2, C_3, C_5 are concepts in the context Con. According to LDO, C_1, C_2, C_3 are associated with the concepts *to-be-an-animal, to-be-a-man, to-be-a-dog,* while C_5 cannot be classified as a concept because it includes a man and a dog. *A-man* is the objectal representative of the concept *to-be-a-man* as for *a-dog,* it is the objectal representative of the concept *to-be-a-dog.*

A first remark is that the notion of concept is not modelized in the same way in FCA and LDO. A second one is that also the notion of "property" is different in LDO in comparison with FCA. Some general distinctive features between the two formalisms are :

Remark 1 *1(a) In FCA the starting point is represented by a set of objects (completely determined), a set of attributes and a relation between them telling us that an object o has the attribute f.*

(b) In LDO there is a set of concepts which are operators, a set of objects which are operands and an operation of application (Curry & Feys 1958). To apply a concept f to an object o corresponds to the predicate $(f\ o) = \top$.

2(a) In FCA the relation between objects and attributes captures only the binary state of an object : possesses – does not posses this attribute.

(b) In LDO, a concept is associated with its "objectal representative". For each object, there is a "privileged" concept f which gives the objectal representative of the concept f, τf (Desclés & Pascu 2006). Each object o, which is in the extension of f, is firstly a τf and then, it is generated by determinations δg_1,...,δg_n. In LDO g_1,...,g_n act as "properties" of FCA. On the other hand, f has a special status : it defines the object o in some ways (Desclés & Pascu 2007a).

(c) Among concepts, there are some which are able to generate categories (to-be-a-man generates the category of *men*) and some which do not generate categories (*to-be-red* does not generate a category ; the *red* is not considered as a category because it has no a material basis – the color).

One of the "innovations" of LDO which has a very impact in object categorization is based on this last remark. It allows us to distinguish from the logical point of view between "to be an x" and "to have the property y" (*to- be-a-man, to-be-red*).

A Mathematical Modelization of the Notion of "Property"

We give a mathematical formalization of the notion of property. We use the formalism of combinatory logic (Curry 1958) with functional types. The two basic types are : the type of individual entity **J**, the type of truth value **H** and the functional type of the form **F** $\alpha\beta$ (the type which applied to an entity of type α gives a result of type β). LDO is an applicative typed system in Curry's sense (Curry & Feys 1958), (Curry, Hindley, & Seldin 1972). Types provide us with a more precise categorization of concepts : a categorising concept and a non-categorising concept, even they are of the same type, they act differently.

If we are starting from a set of objects, $\underline{\mathbf{O}}$, each object x can be characterized by a set of attributes $\underline{A} = \{A_1, A_2, \ldots A_i \ldots, A_n\}$. Attributes depend on the point of view from which we apprehend this object. From this we obtain a first function A :

$$A : \underline{\mathbf{O}} \longrightarrow 2^{\underline{A}} \qquad (1)$$

such that

$$x \vdash \underline{A}_x = \{A_1(x), A_2(x), \ldots A_i(x) \ldots, A_n(x)\} \qquad (2)$$

An **attribute** $A_i(x)$ associated to an object x is a function whose values are intrinsically dependent on this object. These values are named **attribute-values**. Let us designate by v, an attribute-value, by $Y_x(A_i)$ the values space of the attribute A_i for the object x and by $\underline{Y}_x$, the set of all values for different attributes associated to the object x :

$$\underline{Y}_x = \bigcup_i Y_x(A_i) \qquad (3)$$

We can see an attribute as a function associating a space of attribute-values to each object x :

$$x \quad \vdash \quad Y_x(A_i) \qquad (4)$$

So, the attribute diagram is represented in figure 1 :

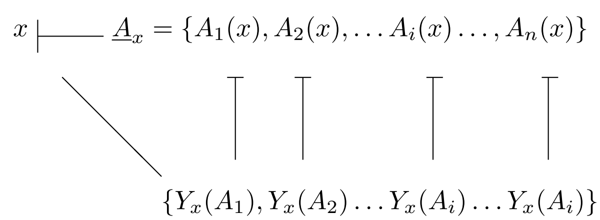

Figure 1: Object – Attribute-Values Diagram

The set of all attributes of all objects is :

$$\mathcal{A} = \bigcup_{i;x \in \mathcal{O}} \mathcal{A}_i\ (x) \qquad (5)$$

The value v(x) is the attribute-value A_i associated to the object x. For example, for "the red color of a car" : the object x is *a-car*, the attribute A_i is *a-color* and the attribute-value $v(x)$ is *red*. It is obvious that the set of attribute-values of an attribute A_i, $Y_x(A_i)$ depends on x ; $Y_x = \{$red, white, blue,....$\}$. In standard functional modelling (using only properties of function theory, without types), the status of an "attribute" and of its "value" are expressed by the commutativity of the diagram in the figure 2, in other words, by relations (6), (7), (8) :

$$f(x) = \underline{A}_x \qquad (6)$$

$$g(\underline{A}_x) = \underline{Y}_x \tag{7}$$

or

$$g(\underline{A}_x) = (g_1, \ldots, g_n) \text{ such that } g_i(A_i) = Y_x(A_i) \tag{8}$$

$$h(x) = v_j \in \text{ such that } \exists A_i \in \underline{A}_x \text{ and } v_j \in g_i(A_i) \tag{9}$$

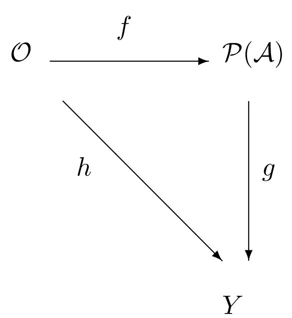

Figure 2: Attribute-Value Diagram

In LDO the "attribute-value" idea is modelled using functional types.

"Attribute-Value" in LDO

The idea is to split the class $\mathcal{F}$ of concepts into two subclasses :

1. categorising concepts – those which can generate a category (to-be-a-man) ;
2. non-categorising concepts (value of attribute concept) – those which cannot generate a category (to-be-red).

This idea leads us to introduce the notion of **attribute-value** in LDO. This notion is captured in the following way : attributes $\mathcal{A}$ are added as a subclass of concepts $\mathcal{F}$: $\mathcal{A} \subset \mathcal{F}$. In LDO, each attribute is a concept, but a "special concept".

Analysis of the Pair "Attribute-Value". Let us take as an example the attribute A = *to-have-a-color*. It can be applied to such objects as *a-car, a-house, a-table....* giving as result the truth value "true" and to such other objects as *a-grammar, a-theory, a-paper, a-job* with the truth value "false" :

(to-have-a-color, a-car) = $\top$

(to-have-a-color, a-theory) = $\bot$

The attribute A has the type : **FJH.**

It determines an operator A' of type **FJ(F(FJH)H).** This operator can be applied to an object x resulting in a function from concepts with values in $\{\top, \bot\}$:

A' **: (J** : x) $\quad |\longrightarrow \quad$ (**F(FJH)H** : ($A'x$))

(to-have-as-color a-car) (to-be-red) = $\top$

(to-have-as-color a-car) (to-be-red) = $\bot$

The attribute is represented in LDO by the two following operators :

Symbol	Type	Language expression
A :	**FJH**	*to-have-a color*
A' :	**FJ(F(FJH)H)**	*to-have-as-color*

Relations between A and A' are :

For each attribute A and each object x such that :

$(A\ x) = \top$, there is a non-categorising concept f such that,

$$(A'x)(f) = \top \tag{10}$$

For each non-categorising concept f such that $(f\ x) = \top$ there is an attribute A such that, $(A\ x) = \top$ and :

$$((A'x))(f) = \top \tag{11}$$

Relations (10) and (11) mean that an attribute A is an attribute of an object x if and only if there is a non-categorising concept (concept value of attribute) f such that the function $(A'x)$ can be applied to f with the value "true". Conversely, for each non-categorising concept (concept as value of attribute) f and for each object x to which it can be applied, there is an attribute of this object, A, such that the function $(A'x)$ can be applied to the concept f with the value "true".

In LDO (which is an applicative system with combinators) the connection between operators A, A' and f is given by[1] the following functional equation with combinators :

$$\mathbf{B}\,(\mathbf{C}^{\star} f)A' \equiv \Phi \wedge f A \tag{12}$$

In order to obtain it we used the β-reduction rules of combinators **B**, **C*** and Φ :

$$\frac{\text{X (Y Z)}}{\mathbf{B}\text{ X Y Z}} \qquad \text{i - }\mathbf{B}$$

$$\frac{\text{Y X}}{\mathbf{C}^{\star}\text{X Y}} \qquad \text{i - }\mathbf{C}^{\star}$$

$$\frac{\text{X (Y U) (Z U)}}{\mathbf{\Phi}\text{X Y Z U}} \qquad \text{i - }\mathbf{\Phi}$$

[1]The symbol $\wedge$ is used for logical conjunction.

The equation 12 is obtained using introduction rules of these combinators as follows :

$(A^{'}\ x)\ f$	$\wedge(f\ x)(A\ x)$
$\Downarrow$ i-$\mathbf{C}^{\star}$	$\Downarrow$ i-Φ
$\mathbf{C}^{\star}\ \ f(A^{'}x)$	$\Phi \wedge f\ A\ x$
$\Downarrow$ i-$\mathbf{B}$	
$\mathbf{B}(\mathbf{C}^{\star}\ f)A^{'}x$	

Equation 12 shows that the conjonction between an object which is f having the attribute A can be expressed by means of a complex operator applied to $A^{'}$.

The corresponding calculus of types is :

$$\frac{\dfrac{\mathbf{FJ(F(FJH)H)}:\ A^{'} \qquad \mathbf{J}:\ x}{\mathbf{F(FJH)H}:\ (A^{'}\ x)} \qquad \mathbf{FJ\ H}:\ f}{\mathbf{H}:\ ((A^{'}\ x)f)}$$

Remark 2 *Attribute-values v such as red, big, young* from the mathematical model correspond, in LDO, to concept values of attributes such as *to-be-red, to-be-big, to-be-young.* They are of type **FJH** and they do not construct categories. The categories of "reds" or of "bigs" are difficult for the human mind to grasp without a material basis (the color or the dimension) An attribute is a kind of basis of its value.

Examples

The following examples give the applicative expressions[2] in LDO associated with phrases of language containing attributes and values.

Example 1 *This example gives the applicative expression associated to the phrase a red car,* constructed with the concept *to-be-red,* the attribute *to-have-a-color,* the operator *to-have-as-color* and the object *a-car.*

Where :

A = *to-have-a-color* ; x = *a-car* ; f = *to-be-red* ; $A^{'}$ = *to-have-as-color.*

If the object *a-car* has the attribute *to-have-a-color,* then the function (*to-have-as-color a-car*) applied to the concept *to-be-red* gives as result the truth value $\top$:

(to-have-as-color a-car) (to-be-red) = $\top$

Example 2 *This example presents the use of LDO's operators and types to construct the applicative expression of a phrase containing attributes and values.*

The car belonging to Jean-Pierre's son is red.

f = *to-be-a-car*; h = *to-be-J.P.'s-son*; g =*to-be-red*; A = *to-have-a-color*; $A^{'}$ = *to-have-as-color*.

The applicative expression associated to this expression is :

$((A^{'}((\delta\ h)(\tau\ f)))g)$

The indetermined object τf is determined by δh and, by means of the operator $A^{'}$, it can receive the attribute value g.

Corresponding types are :

$$\frac{\dfrac{\mathbf{FJ(F(FJH)H)}:A^{'} \qquad \dfrac{\mathbf{FJJ}:(\delta\ h) \quad \mathbf{J}:\ \tau\ f}{\mathbf{J}:\ ((\delta\ h)(\tau\ f))}}{\mathbf{F(FJH)H}:\ (A^{'}((\delta\ h)(\tau\ f))) \qquad \mathbf{FJH}:\ g}}{\mathbf{H}:\ ((A^{'}((\delta\ h)(\tau\ f)))g)}$$

It is important to take into account the idea of attributes as a basis of value because of the fact that there are languages where the position of the adjective changes the meaning of the nominal phrase. The following example is in French and it illustrates the comparison between the analysis of : *grand homme (important man) vs homme grand (tall man).* In the second case (tall man) the attribute is *to-have-height),* in the first (important man), it is *to-have-a-career.* The applicative expressions associated to these forms are illustrated below :

Example 3 *This example shows how we can move the ambiguity by applicative expressions in LDO.*

g = *être-homme*; f = *être grand*; A_1 = *avoir-une-taille* ; A_2 = *avoir-une-carrière* ; $A_1^{'}$ = *avoir-pour-taille* ; $A_2^{'}$ = *avoir-pour-carrière.*

$((\delta f)(\tau g))$ = $((A_1^{'}(\tau g))f)$ = *un-homme-grand (de grande taille)*

$((\delta f)(\tau g))$ = $((A_2^{'}(\tau g))f)$ = *un-grand-homme (important dans la société)*

The determination of τg by non-categorising concept f is equivalent to the application to f of the operator obtained by applying $A^{'}$ to τg.

The Theory Attribute-Value in LDO and Categorization.

LDO has :

1. Concepts $f \in \mathcal{F}$ of type **FJH** ;
   (a) Categorising concepts f (*to-be-a-man, to-be-an-animal*) for which $\tau\ f$ exists (*a-man, an-animal*).
   (b) Non-categorising concepts f:(*to-be-red, to-be-big*) for which $\tau\ f$ does not exist (*a-red, a-big* do not exist). However, we can have *a-red-color, a-great-length* constructed indirectly, with the attribute A : *to-have-a-color, to-have-a-length.* These concepts are a subclass,

[2] An applicative expression is an expression constructed with operators, operands and combinators in combinatory logic.

$\mathcal{F}'$ of $\mathcal{F}$, ($\mathcal{F}' \subset \mathcal{F}$) for which there is a class of attributes $\mathcal{A}$, such that the determination by a concepts of $\mathcal{F}'$ is constructed by the pair (A, A').

2. Objects $x \in \mathcal{O}$ of type **J**;
3. Attributes (A, A') :

A	**FJH**	*to-have-a color*
A'	**FJ(F(FJH)H)**	*to-have-as-color*

The attribute is a pair of operators. This split is proposed in order to clarify some confusions and ambiguties between the notions of concept, object and attribute, sometimes arising in the treatment of semantic networks and ontologies.

The determination operator δ acts differently when applied to a categorising concept and to a non-categorising one, i.e. :

- For a categorising concept f, the operator δ is applied directly to f (i.e. the determination information is built up only from f and δ). This is conveyed by the applicative expression $(\delta f)(\tau g)$.
- For a non-categorising concept f, the operator δ is applied by the means of an attribute. In this case it is the pair $(A' f)$ which builds up the determination with δ. This is expressed by the applicative expression $((A'(\tau g))f)$.

In some problems of categorisation and above all when we meet atypical occurences of an object, we need this distinction ((Desclés 2002), (Desclés & Pascu 2006), (Pascu 2006)). It is also needed in formal ontologies and we can consider itself as a basic concept ontology from which a formal ontology of properties can be constructed. (Guarino 1988), (Guarino 2000).

Attribute-Value and Typicality

The above approach of attribute-value is used in the reasoning with typical and atypical objects. In semantic networks, we often meet the need to express typicality. The typicality was captured in a logical formalism by nonmonotonic logics (M. Minsky), default logics (R. Reiter) and paraconsistent logics (N. da Costa). LDO has its own theory of typicality based on the type of determination. An object x of f is constructed starting from τf by a chain of determinations ((Pascu 2006), (Desclés & Pascu 2007b)) :

$$x = ((\delta g_n \circ \ldots \circ \delta g_1)(\tau f)) \quad (13)$$

Concepts g in this chain can be categorising or non-categorising. If every categorising concept g is in the intension of f, Int f (Pascu 2006) and each non-categorising concept is associated with an attribute with a "typical value", then the object x is a typical object of f.

If there is a categorising concept g such that it is a negation of concept from Int f but it appears in Int-caract(x) ((Pascu 2006), (Desclés & Pascu 2007b)), or there is a non-categorising concept g such that it it corresponds to an attribute with an "atypical value", then the object x is an atypical object of f.

Let us take the well known example from AI literature, the Nixon's Diamond problem. We consider the following sentences :
Republicans are not pacifists.
Quakers are pacifists.
Nixon is a republican.
Nixon is a quaker.
Nixon is a pacifist.

So, Nixon must be obtained as beeing a typical representative of quakers and an atypical representative of republicans versus the property "opinion against the war".

One considers more or less determined objects : a-man, a-republican, a-quaker and the completely determined object Nixon. They are each obtained by chains of determinations, as follows (see fig.3) :
a-quaker = (Δ_1 *a-man)*
a-republican = (Δ_2 *a-man)*
Nixon = (Δ_4 *a-quaker)*
Nixon = (Δ_5 *a-republican)*
Nixon = (Δ *a-man)*
The following condition is fullfiled:
(Δ a-man) = (Δ_4 a-quaker) = (Δ_5 a-republican)

In the figure 3, these determinations are represented by arrows.

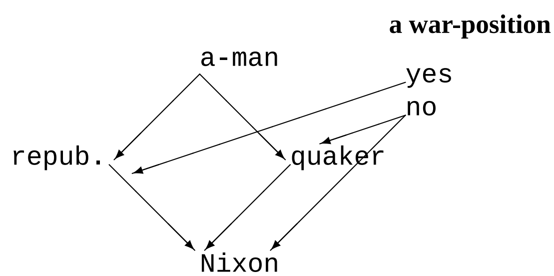

Figure 3: Nixon's Diamond

The attribute *to-have-a-war-position* is an attribute of objects Nixon, a-republican, a-quaker i.e. :
(to-have-a-war-position Nixon) = $\top$
(to-have-a-war-position republican) = $\top$
(to-have-a-war-position quaker) = $\top$
The operator *to-have-as-war-position* gives :
(to-have-as-war-position a-republican)(yes)= $\top$
(to-have-as-war-position a- republican)(no)= $\bot$
(to-have-as-war-position a- quaker)(yes)= $\bot$
(to-have-as-war-position a-quaker)(no)= $\top$
(to-have-as-war-position Nixon)(yes)= $\bot$
(to-have-as-war-position Nixon)(no)= $\top$

In the above applicative expressions, *yes*, *no* are abreviations of non-categorising concepts : *to-be-of-war-position-yes* and respectively *to-be-of-war-position-no.*

The object *Nixon* is a typical object of the concept *to-be-a-quaker.* In the chain of determinations obtaining it starting to the object *a-quaker* there is the determination by the concept *war-position-no.* The same object *Nixon* is an atypical object of the concept *to-be-a-republican.* In the chain of determinations obtaining it starting to the object *a-republican* the determination *war-position-yes,* inherited from *a-republican* is replaced by the determination *war-position-no.*

Conclusions

In classical logics, sometimes, the notions of attribute and property are confused. Description logics, which are supposed to represent knowledge, do not offer a sound enough theoretical framework of these notions.

The model presented in this paper is a part of LDO. LDO propose a logical notion of property, by defining the "attribute" and the "attribute-value" in connection with objects and concepts. LDO tries to explain the distinction between "attribute" and "attribute-value" by a mathematical formalization.

Moreover the distinction between "categorising concepts" and "non-categorising concepts" is emphasized. This distinction is essential at least in natural language processing (NLP) and in computer-assisted translation (CAT) to remove the ambiguity. It is also essential in the study of natural deduction as a cognitive process, in the study of connections between mind, natural languages and artificial languages.

The theory of typicality build up within LDO is an alternative way to work with "contradictions", to avoid "contradictions".

For knowledge representation, in Artificial Intelligence, where reasoning about objects and their properties, it can help to solve local contradictions.

The choice of combinatory logic as the logical framework of LDO is due, on one hand, to the fundamental idea of operator-operand belonging to the combinatory logic, and, on the other hand, to the very easy machine-implemention of combinators by a functional language.

LDO is a system for managing semantic networks. More generally, LDO gives a logical foundation to categorization and inference processes with semantic networks and a framework to formal ontologies.

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