

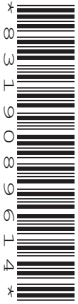


Oxford Cambridge and RSA

**Thursday 08 October 2020 – Afternoon**

**A Level Further Mathematics A Y541/01 Pure Core 2**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

Solve the equation  $4z^2 - 20z + 169 = 0$ . Give your answers in modulus-argument form. [5]

**2 In this question you must show detailed reasoning.**

The roots of the equation  $3x^3 - 2x^2 - 5x - 4 = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ .

(a) Find a cubic equation with integer coefficients whose roots are  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$ . [4]

(b) Find the exact value of  $\frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha\beta\gamma}$ . [2]

**3 In this question you must show detailed reasoning.**

(a) Use partial fractions to show that  $\sum_{r=5}^n \frac{3}{r^2 + r - 2} = \frac{37}{60} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}$ . [5]

(b) Write down the value of  $\lim_{n \rightarrow \infty} \left( \sum_{r=5}^n \frac{3}{r^2 + r - 2} \right)$ . [1]

**4 The equations of two intersecting lines  $l_1$  and  $l_2$  are**

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 7 \\ 9 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

where  $a$  is a constant.

The equation of the plane  $\Pi$  is

$$\mathbf{r} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = -14.$$

$l_1$  and  $\Pi$  intersect at  $Q$ .

$l_2$  and  $\Pi$  intersect at  $R$ .

(a) Verify that the coordinates of  $R$  are  $(13, 3, -14)$ . [2]

(b) Determine the exact value of the length of  $QR$ . [7]

- 5 A capacitor is an electrical component which stores charge. The value of the charge stored by the capacitor, in suitable units, is denoted by  $Q$ . The capacitor is placed in an electrical circuit.

At any time  $t$  seconds, where  $t \geq 0$ ,  $Q$  can be modelled by the differential equation

$$\frac{d^2Q}{dt^2} - 2\frac{dQ}{dt} - 15Q = 0.$$

Initially the charge is 100 units and it is given that  $Q$  tends to a finite limit as  $t$  tends to infinity.

- (a) Determine the charge on the capacitor when  $t = 0.5$ . [6]
- (b) Determine the finite limit of  $Q$  as  $t$  tends to infinity. [1]
- 6 The equation of a curve in polar coordinates is  $r = \ln(1 + \sin \theta)$  for  $\alpha \leq \theta \leq \beta$  where  $\alpha$  and  $\beta$  are non-negative angles. The curve consists of a single closed loop through the pole.
- (a) By solving the equation  $r = 0$ , determine the smallest possible values of  $\alpha$  and  $\beta$ . [2]
- (b) Find the area enclosed by the curve, giving your answer to 4 significant figures. [2]
- (c) Hence, by considering the value of  $r$  at  $\theta = \frac{\alpha + \beta}{2}$ , show that the loop is **not** circular. [2]

- 7 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 0.6 & 2.4 \\ -0.8 & 1.8 \end{pmatrix}$ .

- (a) Find  $\det \mathbf{A}$ . [1]

The matrix  $\mathbf{A}$  represents a stretch parallel to one of the coordinate axes followed by a rotation about the origin.

- (b) By considering the determinants of these transformations, determine the scale factor of the stretch. [2]
- (c) Explain whether the stretch is parallel to the  $x$ -axis or the  $y$ -axis, justifying your answer. [1]
- (d) Find the angle of rotation. [2]

**8 In this question you must show detailed reasoning.**

The complex number  $-4 + i\sqrt{48}$  is denoted by  $z$ .

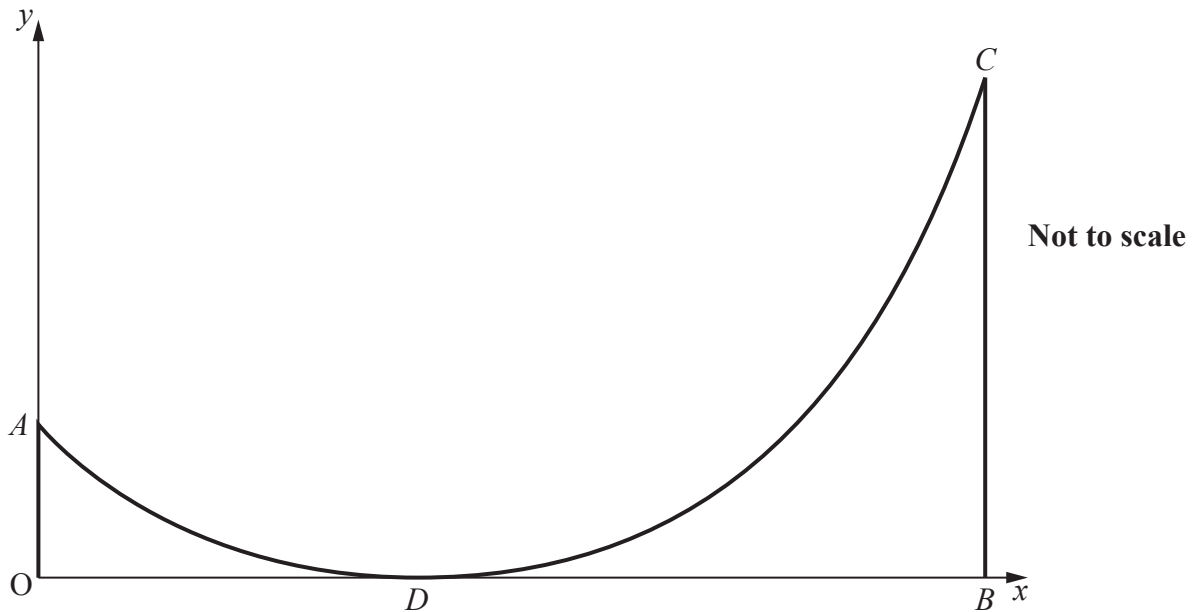
- (a) Determine the cube roots of  $z$ , giving the roots in exponential form. [6]

The points which represent the cube roots of  $z$  are denoted by  $A$ ,  $B$  and  $C$  and these form a triangle in an Argand diagram.

- (b) Write down the angles that any lines of symmetry of triangle  $ABC$  make with the positive real axis, justifying your answer. [3]

- 9 Two thin poles,  $OA$  and  $BC$ , are fixed vertically on horizontal ground. A chain is fixed at  $A$  and  $C$  such that it touches the ground at point  $D$  as shown in the diagram.

On a coordinate system the coordinates of  $A$ ,  $B$  and  $D$  are  $(0, 3)$ ,  $(5, 0)$  and  $(2, 0)$ .



It is required to find the height of pole  $BC$  by modelling the shape of the curve that the chain forms.

Jofra models the curve using the equation  $y = k \cosh(ax - b) - 1$  where  $k$ ,  $a$  and  $b$  are positive constants.

- (a) Determine the value of  $k$ . [2]
- (b) Find the exact value of  $a$  and the exact value of  $b$ , giving your answers in logarithmic form. [5]

Holly models the curve using the equation  $y = \frac{3}{4}x^2 - 3x + 3$ .

- (c) Write down the coordinates of the point,  $(u, v)$  where  $u$  and  $v$  are both non-zero, at which the two models will agree. [1]
- (d) Show that Jofra's model and Holly's model disagree in their predictions of the height of pole  $BC$  by 3.32 m to 3 significant figures. [3]

10 Let  $f(x) = \sin^{-1}(x)$ .

- (a) (i) Determine  $f''(x)$ . [2]
- (ii) Determine the first two non-zero terms of the Maclaurin expansion for  $f(x)$ . [3]
- (iii) By considering the first two non-zero terms of the Maclaurin expansion for  $f(x)$ , find an approximation to  $\int_0^{\frac{1}{2}} f(x) dx$ . Give your answer correct to 6 decimal places. [2]
- (b) By writing  $f(x)$  as  $\sin^{-1}(x) \times 1$ , determine the value of  $\int_0^{\frac{1}{2}} f(x) dx$ . Give your answer in exact form. [3]

**END OF QUESTION PAPER**

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