OCR
Oxford Cambridge and RSA

Monday 4 October 2021 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

7820437810

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 **(a)** Sketch on a single Argand diagram the loci given by

(i) $|z-1+2\mathrm{i}|=3$, **[2]**

(ii) $|z+1|=|z-2|$. **[2]**

(b) Indicate, by shading, the region of the Argand diagram for which $|z-1+2\mathrm{i}| \leqslant 3$ and $|z+1| \leqslant |z-2|$. **[2]**

2 You are given that $\mathrm{f}(x) = \tan^{-1}(1+x)$.

(a) **(i)** Find the value of $\mathrm{f}(0)$. **[1]**

(ii) Determine the value of $\mathrm{f}'(0)$. **[2]**

(iii) Show that $\mathrm{f}''(0) = -\frac{1}{2}$. **[3]**

(b) Hence find the Maclaurin series for $\mathrm{f}(x)$ up to and including the term in x^2. **[2]**

3 A function $\mathrm{f}(z)$ is defined on all complex numbers z by $\mathrm{f}(z) = z^3 - 3z^2 + kz - 5$ where k is a real constant. The roots of the equation $\mathrm{f}(z) = 0$ are α, β and γ. You are given that $\alpha^2 + \beta^2 + \gamma^2 = -5$.

(a) Explain why $\mathrm{f}(z) = 0$ has only one real root. **[3]**

(b) Find the value of k. **[3]**

(c) Find a cubic equation with integer coefficients that has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. **[2]**

4 Points A, B and C have coordinates (4, 2, 0), (1, 5, 3) and (1, 4, −2) respectively. The line l passes through A and B.

(a) Find a cartesian equation for l. **[3]**

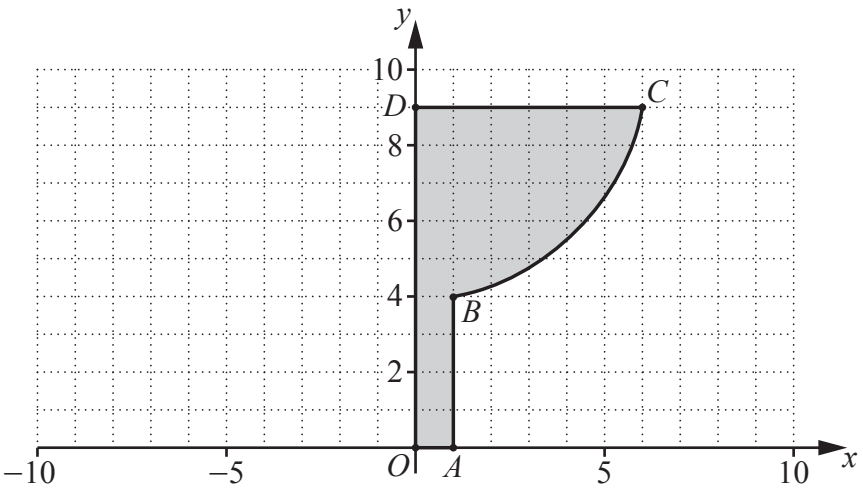
M is the point on l that is closest to C.

(b) Find the coordinates of M. **[4]**

(c) Find the exact area of the triangle ABC. **[4]**

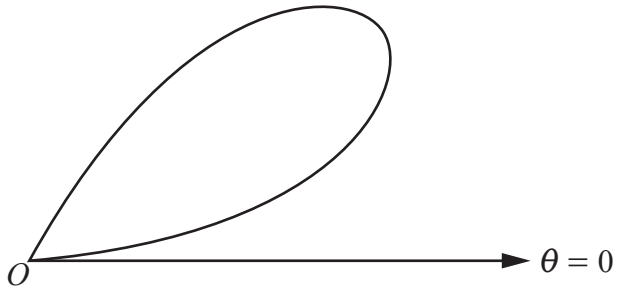
5 Use de Moivre's theorem to find the constants A, B and C in the identity $\sin^5\theta \equiv A\sin\theta + B\sin 3\theta + C\sin 5\theta$. **[4]**

6 O is the origin of a coordinate system whose units are cm.
The points A, B, C and D have coordinates (1, 0), (1, 4), (6, 9) and (0, 9) respectively.
The arc BC is part of the curve with equation $x^2 + (y-10)^2 = 37$.
The closed shape $OABCD$ is formed, in turn, from the line segments OA and AB, the arc BC and the line segments CD and DO (see diagram).
A funnel can be modelled by rotating $OABCD$ by 2π radians about the y-axis.



Find the volume of the funnel according to the model. **[3]**

7 The diagram below shows the curve with polar equation $r = \sin 3\theta$ for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.



(a) Find the values of θ at the pole. **[1]**

(b) Find the polar coordinates of the point on the curve where r takes its maximum value. **[2]**

(c) **In this question you must show detailed reasoning.**

Find the exact area enclosed by the curve. **[4]**

(d) Given that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, find a cartesian equation for the curve. **[2]**

8 You are given that $\mathrm{f}(x) = 4\sinh x + 3\cosh x$.

(a) Show that the curve $y = \mathrm{f}(x)$ has no turning points. **[3]**

(b) Determine the exact solution of the equation $\mathrm{f}(x) = 5$. **[5]**

9 You are given that the matrix $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation T.

(a) You are given that the line with equation $y = kx$ is invariant under T.

Determine the value of k. **[4]**

(b) Determine whether the line with equation $y = kx$ in part **(a)** is a line of invariant points under T. **[1]**

10 Using an algebraic method, determine the least value of n for which $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} \geqslant 0.49$. **[8]**

11 The displacement of a door from its equilibrium (closed) position is measured by the angle, θ radians, which the door makes with its closed position. The door can swing either side of the equilibrium position so that θ can take positive and negative values. The door is released from rest from an open position at time $t = 0$.

A proposed differential equation to model the motion of the door for $t \geqslant 0$ is

$\frac{d^2\theta}{dt^2} + \lambda\frac{d\theta}{dt} + 3\theta = 0$ where λ is a constant and $\lambda \geqslant 0$.

(a) (i) According to the model, for what value of λ will the motion of the door be simple harmonic? **[1]**

(ii) Explain briefly why modelling the motion of the door as simple harmonic is unlikely to be realistic. **[1]**

(b) Find the range of values of λ for which the model predicts that the door will never pass through the equilibrium position. **[2]**

(c) Sketch a possible graph of θ against t when λ lies **outside** the range found in part **(b)** but the motion is not simple harmonic. **[1]**

END OF QUESTION PAPER

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