Weight of a shadow

Supplementary information

Radiation pressure [N/m^2]

The general equation is:

$$p = \frac{\alpha I}{c}, \tag{1}$$

where I is the intensity of the light, c is the speed of light and α is a parameter that takes into account the absorption capacity of the body, $1 \leq \alpha \leq 2$. For a black body (perfect absorber), $\alpha = 1$ and for a white body (perfect reflector), $\alpha = 2$

$$p = \frac{2I}{c}, \quad \text{perfect reflector} \tag{2}$$

$$p = \frac{I}{c}, \quad \text{perfect absorber} \tag{3}$$

In the case of a human body, we will assume that it absorbs half of the radiation (0.5) so $p_{body} = \frac{1.5I}{c}$

For the calculations, let us consider the following points:

- Area of a person (172 cm tall, i.e., me):
  → **5640.3 cm^2** (*exact area*)
  → 6880 cm^2 (estimating a width of 40 cm, size L corresponds to 35.5cm back width)
- Area of Madrid: 605 km^2
- Sunlight wavelength: white light (visible) comprises wavelengths in the range of 400 nm to 800 nm. The whole radiation spectra goes from 200 nm to 3000 nm (ultraviolet, visible and infrared). As the intensity of the light depends not only on the wavelength but also on the distance to the emitter and the angle, it is easier to look for the values of the sunlight intensity in certain places (measured):

  Intensity of sunlight (*includes visible light and infrared*)

  In Madrid, on average: 1466 W/m^2

  Worldwide average: 1360 W/m^2

  *As for the calculations, we need the intensity of the light, we will not use the wavelength of the light

CALCULATIONS:

Radiation pressure from sunlight on a person:

$$p_{body} = \frac{1.5I}{c} = \frac{1.5 \cdot 1466\ W/m^2}{3 \cdot 10^8\ m/s} = 733 \cdot 10^{-8}\ \text{N/m}^2$$

$$F_{light} = p_{body} \cdot A = 733 \cdot 10^{-8}\ \text{N/m}^2 \cdot 0.56403\ \text{m}^2 = \boxed{4.1344 \cdot 10^{-6}\ \text{N}}$$

Weight of a person (55 kg):

$$F_{weight} = m \cdot g = 55\ kg \cdot 9.8\ \text{m/s}^2 = \boxed{539\ \text{ N}}$$

Translating the force of the photons into mass:

$$m^* = \frac{F_{light}}{g} = \frac{4.1344 \cdot 10^{-6}\ \text{N}}{9.8\ \text{m/s}^2} = \boxed{0.42\ \cdot 10^{-6}\ \text{kg}}$$

In the case of Chicago... (trying to check what can be found online)

Apparently, it weights 140kg more on a sunny day (its area is 605 km^2, same area as Madrid). Let us see:

$$F_{light} = m^* \cdot g = 140\ \text{kg} \cdot 9.8\ \text{m/s}^2 = 1372\ \text{N}$$

The radiation pressure is then:

$$p_{body} = \frac{F_{light}}{A} = \frac{1372\ \text{N}}{605 \cdot 10^6\ \text{m}^2} = 2.26 \cdot 10^{-6}\ \text{N/m}^2$$

Finally, the intensity of the light and the absorption coefficient are:

$$\alpha \cdot I = p_{body} \cdot c = 2.26 \cdot 10^{-6}\ \text{N/m}^2 \cdot 3 \cdot 10^8\ \text{m/s} = 680.3\ \text{W/m}^2$$

If $I \approx 1000$ W/m^2, then $\boxed{\alpha = 0.68}$, which is smaller than the coefficient for a perfect absorber body (black body) (remember that $1 \leq \alpha \leq 2$). If $I = 1466$ W/m^2, α is even smaller.

In the case of a person, according to the article...

Apparently, it weights $3 \cdot 10^{-6}$ kg more on a sunny day (its area is ≈ 0.925 m^2, height 185 cm, back width 50 cm as upper bounds). Let us see:

$$F_{light} = m^* \cdot g = 3 \cdot 10^{-6}\ \text{kg} \cdot 9.8\ \text{m/s}^2 = 2.94 \cdot 10^{-5}\ \text{N}$$

The radiation pressure is then:

$$p_{body} = \frac{F_{light}}{A} = \frac{2.94 \cdot 10^{-5}\ \text{N}}{0.925\ \text{m}^2} = 3.17 \cdot 10^{-5}\ \text{N/m}^2$$

Finally, the intensity of the light and the absorption coefficient are:

$$\alpha \cdot I = p_{body} \cdot c = 3.17 \cdot 10^{-5}\ \text{N/m}^2 \cdot 3 \cdot 10^8\ \text{m/s} = 9535\ \text{W/m}^2$$

If $I \approx 1000$ W/m^2, then $\boxed{\alpha = 9.53}$, which is MUCH larger than the coefficient for a perfect reflective body (white body). If $I = 1466$ W/m^2 and the area is 2 m^2 α (average area of the skin of our body), $\alpha \approx 3$ which is still larger than the coefficient for a white body.