MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Given set = {1, 2, 3, …, 50}

One number is selected randomly from set. Find probability that number is multiple of 4 or 6 or 7.

(1) $\frac{21}{50}$ (2) $\frac{18}{50}$

(3) $\frac{8}{25}$ (4) $\frac{21}{25}$

Answer (1)

Sol. Take $P(A)$ = Probability that number is multiple of 4

$P(B)$ = Probability that number is multiple of 6

$P(C)$ = Probability that number is multiple of 7

$P(A) = \frac{12}{50}$, $P(B) = \frac{8}{50}$, $P(C) = \frac{7}{50}$

$P(A \cap B) = \frac{4}{50}$ (Multiple of 12)

$P(B \cap C) = \frac{1}{50}$ (Multiple of 42)

$P(A \cap C) = \frac{1}{50}$ (Multiple of 28)

$P(A \cap B \cap C) = 0$ (Multiple of 84)

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} + 0$

$= \frac{21}{50}$

2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\, dx$ is

(1) $\sqrt{2} - \sqrt{3} + 1$ (2) $2\sqrt{2} - \sqrt{3} - 1$

(3) $2\sqrt{2} + \sqrt{3} - 1$ (4) $\sqrt{2} + \sqrt{3} - 1$

Answer (2)

Sol. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x| dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x) dx$

$= (\sin x + \cos x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} + (-\sin x - \cos x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(\sin\frac{\pi}{6} + \cos\frac{\pi}{6}\right)\right] + \left[\left(-\sin\frac{\pi}{3} - \cos\frac{\pi}{3}\right) - \left(-\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)\right]$

$= \left[\sqrt{2} - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)\right] + \left[-\frac{\sqrt{3}}{2} - \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

$= 2\sqrt{2} - \sqrt{3} - 1$

3. A = {1, 2, 3, 4} minimum number of elements added to make it equivalence relation on set A containing (1, 3) and (1, 2) in it.

(1) 8 (2) 9

(3) 12 (4) 16

Answer (1)

Sol. Set A = {1, 2, 3, 4}

For reflexive relation

We need to have (1, 1), (2, 2), (3, 3), (4, 4).

For symmetric,

(1, 3) $\in$A

So (3, 1) should be added

And (1, 2) $\in$A

So (2, 1) should be added

set has become {(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1)}

Now (3, 1) $\in$A

(1, 2) $\in$A

So (3, 2) should be added (for transitive)

Then (2, 3) should be added (for symmetric)

So set becomes

{(1, 1), (2, 2), (3, 3), (4, 4), (1, 3), (3, 1), (1, 2), (2, 1), (3, 2), (2, 3)}

So minimum 8 elements are added

4. If ln a, ln b, ln c are in AP and ln a – ln $2b$, ln $2b$ – ln $3c$, ln $3c$ – ln a are in AP then $a : b : c$ is

(1) 1 : 2 : 3 (2) 7 : 7 : 4

(3) 9 : 9 : 4 (4) 4 : 4 : 9

Answer (3)

Sol. ln a, ln b, ln $c \to$ AP

$\Rightarrow b^2 = ac$ ...(i)

$\ln\frac{a}{2b}, \ln\frac{2b}{3c}, \ln\frac{3c}{a} \to$ AP

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b}\times\frac{3c}{a}$$

$$\frac{4b^2}{9c^2} = \frac{3c}{2b}$$

$8b^3 = 27c^3$

$\boxed{2b = 3c}$ ...(ii)$\Rightarrow \boxed{4b = 9c}$

$4b^2 = 9c^2$

$4ac = 9c^2$

$\Rightarrow \boxed{4a = 9c}$ ...(iii)

From (ii) & (iii)

$4a = 9c = 4b = k$

$$a = \frac{k}{4}\; b = \frac{k}{4}\; c = \frac{k}{9}$$

$$a : b : c = \frac{1}{4} : \frac{1}{4} : \frac{1}{9}$$

$a : b : c = 9 : 9 : 4$

5. If $r = |z|$, $\theta = \arg(z)$ and $z = 2 - 2i\tan\left(\frac{5\pi}{8}\right)$ then find (r, θ)

(1) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ (2) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(3) $\left(2\tan\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (4) $\left(2\tan\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Answer (2)

Sol.
$$z = 2 - 2i\frac{\sin\frac{5\pi}{8}}{\cos\frac{5\pi}{8}}$$

$$= \frac{2}{\cos\frac{5\pi}{8}}\left(\cos\frac{5\pi}{8} - i\sin\frac{5\pi}{8}\right)$$

$$= \frac{2}{\cos\frac{5\pi}{8}} e^{i\frac{(-5\pi)}{8}}$$

$$= 2\sec\left(\frac{5\pi}{8}\right) e^{i\frac{(-5\pi)}{8}}$$

$$= 2\sec\left(\frac{3\pi}{8}\right) e^{ir}\, e^{i\frac{(-5\pi)}{8}}$$

$$= 2\sec\frac{3\pi}{8} e^{i\frac{(3\pi)}{8}}$$

$$\theta = \frac{3\pi}{8},\; r = 2\sec\frac{3\pi}{8}$$

6. In which interval the function $f(x) = \frac{x}{x^2 - 6x - 16}$ is increasing?

(1) ϕ (2) $\left[1, \frac{3}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$

(3) $\left(\frac{5}{4}, \infty\right)$ (4) $\left[\frac{3}{4}, \frac{5}{4}\right]$

Answer (1)

Sol. $f(x) = \frac{x}{x^2 - 6x - 16}$

$$f'(x) = \frac{(x^2 - 6x - 16) - (x)(2x - 6)}{(x^2 - 6x - 16)^2}$$

$$\Rightarrow \frac{-x^2 - 16}{(x^2 - 6x - 16)^2} < 0\; \forall\; x \in D_f$$

$\therefore\; x \in \phi$

7. (α, β) lie on the parabola $y^2 = 4x$ and (α, β) also lie on chord with mid-point $\left(1, \frac{5}{4}\right)$ of another parabola $x^2 = 8y$, then value of $|(8 - \beta)(\alpha - 28)|$ is

(1) 192 (2) 92

(3) 64 (4) 128

Answer (1)

Sol. Chord with point, $T = S_1$

$\Rightarrow\; xx_1 - 4(y + y_1) = x_1^2 - 8y_1$

$(x_1, y_1) \equiv \left(1, \frac{5}{4}\right) \Rightarrow x - 4\left(y + \frac{5}{4}\right) = \frac{1 - 8\times 5}{4}$

$x - 4y - 5 = -9$

$\Rightarrow x - 4y + 4 = 0$ $(L1)$

(α, β) lie on $(L1)$ and also $y^2 = 4x$

$\Rightarrow \alpha - 4\beta + 4 = 0$

$\beta^2 = 4\alpha$

$\beta^2 = 4(4\beta - 4)$

$\beta^2 - 16\beta + 16 = 0$

$\Rightarrow (\beta-8)^2 = 64-16 = 48$

$\Rightarrow \beta = 8 \pm 4\sqrt{3}$

$\alpha = 4\beta - 4$

$= 28 \pm 16\sqrt{3}$

$(28+16\sqrt{3},\ 8+4\sqrt{3})$ and $(28-16\sqrt{3},\ 8-4\sqrt{3})$

$(8-\beta)(\alpha-28)$

$\Rightarrow (-4\sqrt{3})(16\sqrt{3})$

$= -192$

8. Unit vector $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ makes angles $\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ with $\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}\right), \left(\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right)$, $\left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$ respectively and $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ find $|\vec{u} - \vec{v}|$.

(1) $\sqrt{\frac{5}{2}}$ (2) $\sqrt{\frac{7}{2}}$

(3) $\sqrt{\frac{2}{5}}$ (4) $\sqrt{\frac{2}{7}}$

Answer (1)

Sol. $\frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$ ...(1)

$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2}$ ...(2)

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{-1}{2}$ ...(3)

$\Rightarrow y = 0,\ z = \frac{1}{\sqrt{2}},\ x = \frac{-1}{\sqrt{2}}$

$\vec{v} - \vec{u} = \sqrt{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$|\vec{v} - \vec{u}| = \sqrt{2 + \frac{1}{2}}$

$= \sqrt{\frac{5}{2}}$

9. If first term of non-constant GP be $\frac{1}{8}$ and every term is AM of next two, then $\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$ is

(1) 2^{15} (2) -2^{15}

(3) -2^{18} (4) 2^{18}

Answer (2)

Sol. $a_1 = \frac{1}{8}$

a, ar, ar^2, ar^3 ......

$2ar = ar^2 + ar^3$

$2 = r + r^2$

$r^2 + r - 2 = 0$

$(r + 2)(r - 1) = 0$

$r \neq 1$

$\Rightarrow r = -2$

$\sum_{r=1}^{20} T_r - \sum_{r=1}^{18} T_r$

$= \frac{a(1-r^{20})}{1-r} - \frac{a(1-r^{18})}{1-r}$

$= \frac{1}{8}\left[\frac{1}{3}\left[1-r^{20}-1+r^{18}\right]\right]$

$= \frac{1}{24}2^{18}[1-4]$

$= -\frac{2^{18}}{8} \Rightarrow -2^{15}$

10. The mean of 5 observations is $\frac{24}{5}$ and variance is $\frac{194}{25}$. If the mean of first four observations is $\frac{7}{2}$, then the variance of first four observations is

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$

(3) $\frac{5}{4}$ (4) $\frac{2}{3}$

Answer (3)

Sol. $\sum_{i=1}^{5} x_i = 24$

$\frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$

$\Rightarrow \sum x_i^2 = \frac{770}{25} \times 5 = 154$

5^{th} observation $= 24 - \frac{7}{2} \times 4 = 10$

New variance $= \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{7}{2}\right)^2$

$= \frac{154-100}{4} - \frac{49}{4}$

$= \frac{5}{4}$

11.
12.
13.
14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The remainder when $64^{32^{32}}$ is divided by 9 is

Answer (1)

Sol. $64 \equiv 1 (\text{mod } 9)$

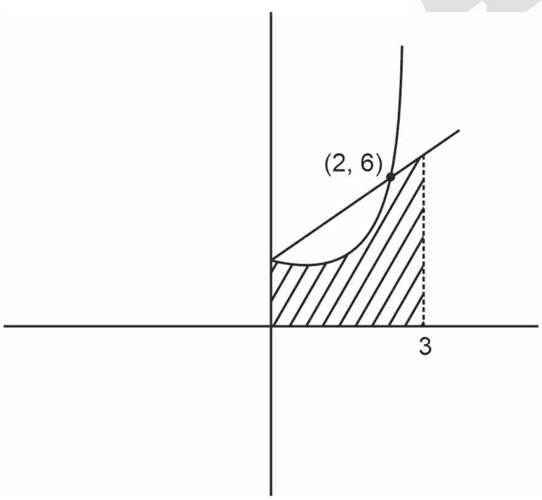
$64^{32^{32}} \equiv 1^{32^{32}}$ (mod 9)

$\Rightarrow$ Remainder = 1

22. Area bounded by $0 \le y \le \min\{x^2 + 2, 2x + 2\}$, $x \in [0, 3]$ is A, then 12 A is

Answer (164)

Sol. $\min\{x^2+2,\ 2x+2\}\begin{cases} x^2+2 & 0 \le x \le 2 \\ 2x+2 & 2 \le x \le 3 \end{cases}$



Area = $A = \int_0^2 (x^2+2)dx + \frac{1}{2}[6+8]\times 1$

$= \left.\frac{x^3}{3} + 2x\right]_0^2 + 7$

$\frac{8}{3} + 4 + 7 = \left(\frac{8}{3} + 11\right)$ unit

$12A = 12\left(\frac{8}{3} + 11\right) = 164$

23. The number of ways to distribute 8 identical books into 4 distinct bookshelf is (where any bookshelf can be empty)

Answer (165)

Sol. $x_1 + x_2 + x_3 + x_4 = 8$

Number of ways = $\binom{8+4-1}{4-1}$

= $\binom{11}{3}$

= 165

24. If $f(x) = \ln\left(\frac{1-x^2}{1+x^2}\right)$ then value of $225(f'(x) - f''(x))$ at $x = \frac{1}{2}$

Answer (736)

Sol. $f(x) = \ln(1 - x^2) - \ln(1 + x^2)$

$f'(x) = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2}$

$= -2x\left[\frac{2}{1-x^4}\right]$

$f'(x) = \frac{4x}{x^4-1}$

$f''(x) = 4\left[\frac{(x^4-1) - 4x^4}{(x^4-1)^2}\right]$

$= 4\left[\frac{-3x^4-1}{(x^4-1)^2}\right]$

$f'(x) - f''(x) = 4\left[\frac{x}{x^4-1} + \frac{3x^4+1}{(x^4-1)^2}\right]$

At $x = \frac{1}{2}$

$225\left[f'(x) - f''(x)\right] = 736$

25. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$, then find sum of roots,

Answer (1)

Sol. $\therefore$

$$\frac{\cos 2x(3+\cos^2 2x)}{(\cos^2 x - \sin^2 x)[\sin^4 x + \cos^4 x + \sin^2 x\cos^2 x]},$$

$\cos^2 x - \sin^2 x = \cos 2x$

$$= \frac{3+\cos^2 2x}{1-\sin^2 x\cos^2 x} = 4\left(\frac{3+\cos^2 2x}{4-\sin^2 2x}\right) = 4$$

$\Rightarrow \; x^3 - x^2 + 6 = 4$

$\Rightarrow \; x^3 - x^2 + 2 = 0$

$\therefore$ therefore sum of roots = 1

26. $x\left(\cos\left(\dfrac{y}{x}\right)\right)\dfrac{dy}{dx} = y\cos\left(\dfrac{y}{x}\right) + x$

where $\sin\left(\dfrac{y}{x}\right) = \ln|x| + \dfrac{\alpha}{2}$ and $f(1) = \dfrac{\pi}{3}$

Find α^2.

Answer (3)

Sol. $\because \left(\cos\dfrac{y}{x}\right)\dfrac{dy}{dx} = \dfrac{y}{x}\cos\dfrac{y}{x} + 1$

Putting $y = vx$

$$\Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$\Rightarrow \cos v\left(x\frac{dv}{dx} + v\right) = v\cos v + 1$$

$$\Rightarrow \int \cos v\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \sin\frac{y}{x} = \ln|x| + c$$

where $c = \dfrac{\alpha}{2}$

putting initial condition,

$$2\sin\frac{\pi}{3} = \alpha$$

$$\Rightarrow \alpha = \sqrt{3}$$

$$\Rightarrow \alpha^2 = 3$$

27. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and area of ΔOAC is S and a parallelogram with sides parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ and diagonal $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$, has area equal to B, then $\dfrac{B}{S}$ is equal to

Answer (96)

Sol. $S = \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

$B = \left|12\vec{a}\times 4\vec{b}\right|$

$$\Rightarrow \frac{B}{S} = \frac{48\left|\vec{a}\times\vec{b}\right|}{\frac{1}{2}\left|\vec{a}\times\vec{b}\right|} = 96$$

28.

29.

30.

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