Temporal OBDA with *LTL* and *DL-Lite*

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Abstract. We investigate various types of query rewriting over ontologies given in the standard temporal logic *LTL* as well as combinations of *LTL* with *DL-Lite* logics. In particular, we consider FO($<$)-rewritings that can use the temporal precedence relation, FO($<, +$)-rewritings that can also employ the arithmetic predicate PLUS, and rewritings to finite automata with data given on the automaton tape.

1 Introduction

Ontology-based data access (OBDA), one of the most promising applications of description logics, has recently been extended to temporal ontologies and data. Motivated by applications in temporal data management, data stream management and monitoring in context-aware systems [11, 4, 7, 3, 15], a few approaches to temporal OBDA have been suggested. In all of them, data is stored in an ABox as timestamped assertions. It is assumed to be incomplete and supplemented by an ontology representing information about the domain or system under consideration. A user can access the data by means of queries in the vocabulary of the ontology that refer to time via either the temporal precedence relation directly or standard temporal operators from logics such as *LTL*.

Within this framework, a fundamental challenge is to minimally restrict the use of temporal constructs in ontologies and queries so that query answering is tractable (in data complexity) and can ideally be delegated to existing temporal or stream data management tools after query rewriting into an appropriate query language. For example, [4, 7] have explored the data complexity of query answering and the possibility of first-order rewritability for standard ontologies, which do *not* employ any temporal constructs, and conjunctive queries extended with the temporal operators of *LTL*. The only temporal aspect the ontology can represent in this approach is that some roles and concepts can be declared rigid (time-invariant), which has a strong impact on the complexity of query answering. An extension of SPARQL with temporal operators was designed in [15].

In contrast, the aim of [3] was to identify most expressive combinations of the ontology language *OWL 2 QL* (and other logics of the *DL-Lite* family) with *LTL* that would ensure rewritability of temporal ontologies and conjunctive queries with timestamped atoms into first-order queries with the temporal precedence relation $<$. One such combination, *TQL*, allows the temporal operators $\Diamond_P$ (sometime in the past) and $\Diamond_F$ (sometime in the future) in axioms of the form

$$\Diamond_P \mathit{signsForMU} \sqcap \Diamond_F \mathit{endOfContract} \ \sqsubseteq \ \mathit{onMUcontract}, \tag{1}$$

saying that since the signing of a contract with Manchester United and until its end, one is contracted to Manchester United. We can then use the query

$$\boldsymbol{q}(x) \ = \ \exists y, t \left((n_1 \leq t \leq n_2) \wedge \mathit{Striker}(x,t) \wedge \mathit{onMUcontract}(x,y,t) \right)$$

to find MU strikers in the time interval $[n_1, n_2]$. It was also observed that axioms with the next- and previous-time operators $\bigcirc_F$ and $\bigcirc_P$ such as

$$\mathit{HamstringPulled} \ \sqsubseteq \ \bigcirc_F^{\leq 21} \mathit{Injured},$$

saying that if a player pulls his hamstring then he is injured for at least three weeks, can ruin FO($<$)-rewritability. It was conjectured, however, that the addition of the operations $+$ and $\times$ could be enough to guarantee rewritability in this case.

In this paper, we launch a more systematic investigation of various types of query rewritability over temporal extensions of *DL-Lite* logics. To begin with, we consider 'ontologies' formulated in pure *LTL*, which is interpreted over the timeline $(\mathbb{Z}, <)$, and 'data instances' with atoms of the form $p(n)$, where p is a propositional variable and n a moment of time in $\mathbb{Z}$. Ontology axioms in this case are of the form $\boxast \varphi$, where $\boxast$ is the temporal operator 'always' and φ is any *LTL*-formula. Our initial observation is that any FO-query $\boldsymbol{q}$ with atoms $p(\tau)$, $\tau < \tau'$ and $\tau = \tau'$ (where τ, τ' are variables or constants from $\mathbb{Z}$) and any *LTL* ontology $\mathcal{T}$ can be 'rewritten' to a nondeterministic finite automaton (NFA) that computes answers to $(\mathcal{T}, \boldsymbol{q})$ over any data instance given on the automaton tape. In complexity-theoretic terms, this means that the entailment problem for such ontology-mediated queries is in the class NC^1 for data complexity.

We then investigate rewritability of two types of FO-queries over ontologies given in the fragments of *LTL* that have been identified in [2]. More specifically, we consider atomic queries (AQs) as well as positive existential queries that can also use a built-in successor relation over $\mathbb{Z}$ (UCQ^+, for short). The fragments from [2] operate with *LTL*-formulas in *clausal normal form*

$$\boxast(\neg\lambda_1 \vee \cdots \vee \neg\lambda_n \vee \lambda_{n+1} \vee \cdots \vee \lambda_{n+m}), \tag{2}$$

where the temporal literals λ_i are defined by the grammar

$$\lambda \ ::= \ \bot \ \mid \ p \ \mid \ \bigcirc_F \lambda \ \mid \ \bigcirc_P \lambda \ \mid \ \Box_F \lambda \ \mid \ \Box_P \lambda. \tag{3}$$

(Here $\Box_F$ and $\Box_P$ are the operators 'always in the future' and 'always in the past'.) Similarly to [10], one can show that any *LTL*-ontology can be transformed to clausal normal form. For example, axiom (1) can be replaced by three axioms

$$\mathit{signsForMU} \sqsubseteq \Box_F B, \quad \mathit{endOfContract} \sqsubseteq \Box_P E, \quad B \sqcap E \sqsubseteq \mathit{onMUContract}$$

with fresh concepts B and E. Borrowing the terminology from [1], we distinguish four types of clauses $\neg\lambda_1 \lor \dots \lor \neg\lambda_n \lor \lambda_{n+1} \lor \dots \lor \lambda_{n+m}$ in (2): *bool* clauses with arbitrary n and m; *horn* clauses with $m \leq 1$; *krom* clauses with $n+m \leq 2$; and *core* clauses with $n+m \leq 2$ and $m \leq 1$. We consider 12 fragments of *LTL* denoted by $\mathsf{LTL}_\alpha^\Box$, $\mathsf{LTL}_\alpha^\bigcirc$ and $\mathsf{LTL}_\alpha^{\Box\bigcirc}$, where $\alpha \in \{\mathit{bool}, \mathit{horn}, \mathit{krom}, \mathit{core}\}$ indicates the type of clauses and the superscript indicates the temporal operators that can be used in the literals λ in the clauses of the fragments. (For instance, $\mathsf{LTL}_{core}^\bigcirc$ contains formulas in clausal normal form with core clauses whose literals can only contain $\bigcirc_F$ and $\bigcirc_P$.) The obtained results are summarised in the table below:

	$\mathsf{LTL}_\alpha^\Box$		$\mathsf{LTL}_\alpha^\bigcirc$		$\mathsf{LTL}_\alpha^{\Box\bigcirc}$	
	AQ	UCQ$^+$	AQ	UCQ$^+$	AQ	UCQ$^+$
bool	FO(<)	NFA	NFA	NFA	NFA	NFA
horn	FO(<)	FO(<)	NFA	NFA	NFA	NFA
krom	FO(<)	NFA	FO(<, +)	NFA	≤NFA	NFA
core	FO(<)	FO(<)	FO(<, +)	FO(<, +)	≤NFA	≤NFA

Thus, if only the operators $\Box_F$ and $\Box_P$ can be used in ontology axioms, then ontology-mediated UCQ$^+$s are rewritable to FO(<)-queries for all Horn ontologies. If the operators $\bigcirc_F$ and $\bigcirc_P$ are allowed then in the best case we can only have FO(<, +)-rewritings, which use $<$ and $+$. (By '≤NFA' we indicate that the exact complexity is still unknown.)

We then 'lift' these rewritability results to the temporal description logics $T^\Box\mathit{DL\text{-}Lite}_{horn}^{flat}$ and $T^\bigcirc\mathit{DL\text{-}Lite}_{core}^{flat}$ whose concept and role inclusions are $\mathsf{LTL}_{horn}^\Box$ and $\mathsf{LTL}_{core}^\bigcirc$ clauses with concepts or roles in place of propositional variables and without $\exists R$ on the right-hand side (we impose this restriction to focus on the temporal aspects of query rewriting). We prove that temporal UCQ$^+$s over $T^\Box\mathit{DL\text{-}Lite}_{horn}^{flat}$-ontologies are FO(<)-rewritable, while over $T^\bigcirc\mathit{DL\text{-}Lite}_{core}^{flat}$-ontologies they are FO(<, +)-rewritable. We also show that the UCQ$^+$-entailment problem over $T^{\Box\bigcirc}\mathit{DL\text{-}Lite}_{horn}^{flat}$-ontologies, which do not contain role inclusions, is NC^1-complete for data complexity.

2 OBDA with *LTL*

We begin by considering the *propositional temporal logic LTL* over the flow of time $(\mathbb{Z}, <)$. *LTL-formulas* are built from propositional variables $\mathcal{P} = \{p_0, p_1, \dots\}$, propositional constants $\top$ and $\bot$, the Booleans $\land$, $\lor$, $\to$ and $\neg$, and the temporal operators: $\boxast$ (always), $\Diamond\!\!\!\!\ast$ (sometime), $\mathcal{S}$ (since) and $\mathcal{U}$ (until), $\Diamond_P$ (sometime in the past) and $\Diamond_F$ (sometime in the future), $\Box_P$ (always in the past) and $\Box_F$ (always in the future), $\bigcirc_P$ (in the previous moment) and $\bigcirc_F$ (in the next moment).

A *temporal interpretation*, $\mathcal{I}$, defines a truth-relation, $\models$, between moments of time $n \in \mathbb{Z}$ and propositional variables p_i. We write $\mathcal{I}, n \models p_i$ to indicate that p_i is true at n in $\mathcal{I}$. This truth-relation is extended to all *LTL*-formulas in the following way (the Booleans are interpreted as expected):

– $\mathcal{I}, n \models \boxast\varphi$ iff $\mathcal{I}, k \models \varphi$, for all $k \in \mathbb{Z}$,

- $\mathcal{I}, n \models \diamondsuit \varphi$ iff $\mathcal{I}, k \models \varphi$, for some $k \in \mathbb{Z}$,
- $\mathcal{I}, n \models \varphi \,\mathcal{U}\, \psi$ iff there is $k > n$ with $\mathcal{I}, k \models \psi$ and $\mathcal{I}, m \models \varphi$, for $n < m < k$,
- $\mathcal{I}, n \models \Box_F \varphi$ iff $\mathcal{I}, k \models \varphi$, for all $k > n$,
- $\mathcal{I}, n \models \Diamond_F \varphi$ iff $\mathcal{I}, k \models \varphi$, for some $k > n$,
- $\mathcal{I}, n \models \bigcirc_F \varphi$ iff $\mathcal{I}, n+1 \models \varphi$,

and symmetrically for the past-time operators. (Note that the interpretation of $\mathcal{U}$ and $\mathcal{S}$ only involves moments of time that are 'strictly' in the future and, respectively, past; all other temporal operators are expressible via such $\mathcal{S}$ and $\mathcal{U}$.)

An *LTL-ontology*, $\mathcal{O}$, is a finite set of *LTL*-formulas of the form $\boxast \varphi$. With each $p \in \mathcal{P}$ we associate a unary predicate over $\mathbb{Z}$, also denoted p. A *data instance* is a finite set, $\mathcal{D}$, of atoms of the form $p(\ell)$ with $p \in \mathcal{P}$ and $\ell \in \mathbb{Z}$. We denote by $\min \mathcal{D}$ ($\max \mathcal{D}$) the minimal (maximal) number occurring in $\mathcal{D}$ and set $\Delta_\mathcal{D} = \{\ell \mid \min \mathcal{D} \leq \ell \leq \max \mathcal{D}\}$. To simplify presentation, we assume in this paper that $\min \mathcal{D} = 0$ and $\max \mathcal{D} \geq 1$. A *temporal knowledge base* (KB) is a pair $\mathcal{K} = (\mathcal{O}, \mathcal{D})$. We call an interpretation $\mathcal{I}$ a *model* of $\mathcal{K}$ and write $\mathcal{I} \models \mathcal{K}$ if $\mathcal{I}, n \models \varphi$, for all $\boxast \varphi$ in $\mathcal{O}$ and $n \in \mathbb{Z}$, and $\mathcal{I}, \ell \models p$, for all $p(\ell) \in \mathcal{D}$.

A *temporal first-order query* (*FOQ*) is any FO($<$)-formula, $\boldsymbol{q}(\boldsymbol{t})$, built from atoms $p_i(\tau)$, $\tau = \tau'$ or $\tau < \tau'$, where τ and τ' are *temporal terms* (individual variables or constants from $\mathbb{Z}$). Although the *successor relation* $S(\tau, \tau')$ (saying that $\tau' = \tau + 1$) is expressible in this language, we assume that it can be used as a basic atom. The free variables $\boldsymbol{t}$ of $\boldsymbol{q}(\boldsymbol{t})$ are the *answer variables* of $\boldsymbol{q}$; if there are no answer variables, $\boldsymbol{q}$ is called *Boolean*. A *union of conjunctive queries* (or *UCQ*$^+$, with $+$ indicating that we can use the successor relation) is a disjunction of FOQs of the form $\exists \boldsymbol{s}\, \varphi(\boldsymbol{s}, \boldsymbol{t})$, where φ is a conjunction of atoms. Finally, an *atomic query* (or *AQ*) takes the form $p_i(\tau)$, for a temporal term τ. A *certain answer* to a FOQ $\boldsymbol{q}(\boldsymbol{t})$ with $\boldsymbol{t} = t_1, \dots, t_m$ *over a KB* $(\mathcal{O}, \mathcal{D})$ is any tuple $\boldsymbol{\ell} = \ell_1, \dots, \ell_m$ of elements in $\Delta_\mathcal{D}$ such that, for every model $\mathcal{I}$ of $(\mathcal{O}, \mathcal{D})$, we have $\mathcal{I} \models \boldsymbol{q}(\boldsymbol{\ell})$ regarding $\mathcal{I}$ as a first-order structure; in this case we write $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}(\boldsymbol{\ell})$. We refer to the problem '$\mathcal{O}, \mathcal{D} \models \boldsymbol{q}$?', where $\boldsymbol{q}$ is a Boolean FOQ, as *FOQ-entailment over temporal ontologies*.

Let $\boldsymbol{q}(\boldsymbol{t})$ be a FOQ and $\mathcal{O}$ an *LTL*-ontology. Denote by $\mathcal{I}_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$ the finite first-order structure defined as follows. The domain $\Delta_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$ of $\mathcal{I}_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$ is the minimal closed interval in $\mathbb{Z}$ that contains $\Delta_\mathcal{D}$ as well as some *fixed* integers that only depend on $\boldsymbol{q}$ and $\mathcal{O}$ (for example, all $\ell \notin \Delta_\mathcal{D}$ occurring in $\boldsymbol{q}$ should be included into $\Delta_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$). For any $\ell \in \Delta_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$, we set $\mathcal{I}_\mathcal{D}^{\boldsymbol{q},\mathcal{O}} \models p(\ell)$ iff $p(\ell) \in \mathcal{D}$. We also assume that the binary relations $<$, $=$ and S and ternary relation PLUS are interpreted over $\Delta_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$ in a natural way (e.g., PLUS(n, m, k) is true iff $n = m + k$). If we extend the domain of $\mathcal{I}_\mathcal{D}^{\boldsymbol{q},\mathcal{O}}$ to $\mathbb{Z}$, then the resulting structure is denoted by $\mathcal{I}_\mathcal{D}^\mathbb{Z}$.

A FOQ $\boldsymbol{q}'(\boldsymbol{t})$ is an FO($<$)-*rewriting of* $\boldsymbol{q}(\boldsymbol{t})$ *and* $\mathcal{O}$ if, for any data instance $\mathcal{D}$ and any tuple $\boldsymbol{\ell}$ from $\Delta_\mathcal{D}$, we have $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}(\boldsymbol{\ell})$ iff $\mathcal{I}_\mathcal{D}^{\boldsymbol{q},\mathcal{O}} \models \boldsymbol{q}'(\boldsymbol{\ell})$. If we allow the use of PLUS in $\boldsymbol{q}'(\boldsymbol{t})$, then it is called an FO($<, +$)-*rewriting of* $\boldsymbol{q}(\boldsymbol{t})$ and $\mathcal{O}$.

Example 1. Consider the ontology $\mathcal{O} = \{\boxast(\bigcirc_P p \to q), \boxast(\bigcirc_P q \to p)\}$. Then $\exists s, n\, [(p(s) \land (t - s = 2n \geq 0)) \lor (q(s) \land (t - s = 2n + 1 \geq 0))]$ is an FO($<, +$)-rewriting of the AQ $p(t)$ and $\mathcal{O}$, where $t - s = 2n \geq 0$ is a shortcut for

$\exists n_2\,(\mathrm{PLUS}(n_2,n,n)\wedge\mathrm{PLUS}(t,s,n_2)\wedge(n_2\geq 0))$ and $t-s=2n+1\geq 0$ is defined similarly. However, $p(t)$ and $\mathcal{O}$ do not have an FO($<$)-rewriting because properties of numbers such as 't is even' are not expressible by FO($<$)-formulas [14]. On the other hand, for any $\ell\in\mathbb{Z}$, the Boolean AQ $p(\ell)$ and $\mathcal{O}$ are FO($<$)-rewritable: for instance, $p(3)\vee q(2)\vee p(1)\vee q(0)$ is an FO($<$)-rewriting of $p(3)$ and $\mathcal{O}$. Consider next the ontology $\mathcal{O}'$ with the axioms:

$$\boxast(\star\to p_0),\qquad \boxast(\bigcirc_F p_k\wedge\bar{a}\to p_k)\text{ and }\boxast(\bigcirc_F p_k\wedge a\to p_{1-k}),\qquad k=0,1.$$

For $\boldsymbol{w}=(w_1,\dots,w_n)\in\{0,1\}^n$, let $\mathcal{D}_{\boldsymbol{w}}$ contain $a(i)$ if $w_i=1$, $\bar{a}(i)$ if $w_i=0$, as well as $\star(n+1)$. Then $\mathcal{O}',\mathcal{D}_{\boldsymbol{w}}\models p_0(0)$ iff the number of 1s in $\boldsymbol{w}$ is even. It follows that the AQ $p_0(0)$ and the ontology $\mathcal{O}'$ are not FO-rewritable even if we are allowed to use arbitrary arithmetic predicates (not only $<$, S and PLUS) [12]. This observation motivates the following definitions.

An NFA $\mathfrak{A}$ is an *NFA-rewriting* of a *Boolean* FOQ $\boldsymbol{q}$ and an *LTL*-ontology $\mathcal{O}$ if, for any data instance $\mathcal{D}$, we have $\mathcal{O},\mathcal{D}\models\boldsymbol{q}$ iff $\mathfrak{A}$ accepts $\mathcal{D}$ written on the tape as the word $\mathcal{D}_0,\dots,\mathcal{D}_k,\mathcal{D}_{k+1}$, where $\mathcal{D}_i=\{p\mid p(i)\in\mathcal{D}\}$, $k=\max\mathcal{D}$ and $\mathcal{D}_{k+1}=\star$ is a special 'end of data' marker. Thus, the alphabet of $\mathfrak{A}$ is $\Sigma_{\boldsymbol{q},\mathcal{O}}=2^{\mathsf{sig}(\boldsymbol{q},\mathcal{O})}\cup\{\star\}$, where $\mathsf{sig}(\boldsymbol{q},\mathcal{O})$ is the set of variables from $\mathcal{P}$ used in $\boldsymbol{q}$ and $\mathcal{O}$. Now suppose we are given a FOQ $\boldsymbol{q}(\boldsymbol{t})$ with non-empty $\boldsymbol{t}=t_1,\dots,t_m$. We say that an NFA $\mathfrak{A}$ with the tape alphabet $\Sigma_{\boldsymbol{q},\mathcal{O}}\times 2^{\{t_1,\dots,t_m\}}$ is an *NFA-rewriting* of $\boldsymbol{q}(\boldsymbol{t})$ and $\mathcal{O}$ when, for any $\mathcal{D}$ and $\boldsymbol{\ell}=\ell_1,\dots,\ell_m$ in $\Delta_{\mathcal{D}}$, we have $\mathcal{O},\mathcal{D}\models\boldsymbol{q}(\boldsymbol{\ell})$ iff $\mathfrak{A}$ accepts the input word $(\mathcal{D}_0,V_0),\dots,(\mathcal{D}_k,V_k),(\mathcal{D}_{k+1},\emptyset)$ with the $\mathcal{D}_i$ as above and $V_i=\{t_j\mid\ell_j=i\}$, for $0\leq i\leq k$. Note that instead of NFA-rewritability we could define an equivalent notion of *MSO*($<$)*-rewritability* (cf. [17, Theorem 1.1]).

It is known [6] that if a Boolean FOQ $\boldsymbol{q}$ and an *LTL*-ontology $\mathcal{O}$ are FO($<,+$)-rewritable then the problem '$\mathcal{O},\mathcal{D}\models\boldsymbol{q}$?' is in LogTime-uniform AC^0 for data complexity. It is also known [13] that if $\boldsymbol{q}$ and $\mathcal{O}$ are NFA-rewritable then this problem is in the class NC^1 for data complexity.

Theorem 2. *Any FOQ $\boldsymbol{q}(\boldsymbol{t})$ and any LTL-ontology $\mathcal{O}$ are NFA-rewritable.*

Proof. Suppose $\boldsymbol{t}=t_1,\dots,t_m$. For a data instance $\mathcal{D}$, let $\mathcal{I}^{\mathbb{N}}_{\mathcal{D}}$ be the restriction of $\mathcal{I}^{\mathbb{Z}}_{\mathcal{D}}$ to $\mathbb{N}$. It is easy to construct an MSO-formula $\mu(\boldsymbol{t})$ such that $\mathcal{I}^{\mathbb{N}}_{\mathcal{D}}\models\mu(\boldsymbol{\ell})$ iff $\mathcal{O},\mathcal{D}\models\boldsymbol{q}(\boldsymbol{\ell})$, for all tuples $\boldsymbol{\ell}$ in $\mathbb{N}$. By Büchi's theorem [8], one can now construct a Büchi automaton $\mathfrak{B}$ which accepts an ω-word $(G_0,V_0),(G_1,V_1),\dots$ over the alphabet $\Sigma_{\boldsymbol{q},\mathcal{O}}\times 2^{\{t_1,\dots,t_m\}}$ iff (i) there is exactly one k with $G_k=\star$; (ii) $V_{k+1}=\emptyset$ and $(G_{k'},V_{k'})=(\emptyset,\emptyset)$, for all $k'>k$; (iii) for any $\mathcal{D}$ with $\mathcal{D}_i=G_i$ for $i<k$, we have $\mathcal{I}^{\mathbb{N}}_{\mathcal{D}}\models\mu(\boldsymbol{\ell})$ for $\ell_j=i$ if $t_j\in V_i$. Finally, one can transform $\mathfrak{B}$ to an NFA $\mathfrak{A}$ that accepts a finite word $(G_0,V_0),\dots,(G_k,V_k)$ iff $\mathfrak{B}$ accepts its ω-extension with $(\emptyset,\emptyset),(\emptyset,\emptyset),\dots$. ❑

Let us now assume that the axioms of *LTL*-ontologies are represented in clausal normal form (2) and consider the fragments $\mathsf{LTL}^{\Box}_{\alpha}$, $\mathsf{LTL}^{\bigcirc}_{\alpha}$ and $\mathsf{LTL}^{\Box\bigcirc}_{\alpha}$, for $\alpha\in\{\mathit{bool},\mathit{horn},\mathit{krom},\mathit{core}\}$, defined in Section 1. The next theorem establishes NC^1-hardness of query entailment for various types of queries and ontologies.

Theorem 3. (i) *AQ-entailment over* $LTL^{\bigcirc}_{horn}$*-ontologies is* NC^1*-hard.*

(ii) *FOQ-entailment over the empty ontology is* NC^1*-hard.*

(iii) *UCQ*$^+$*-entailment over* $LTL^{\Box}_{krom}$*- and* $LTL^{\bigcirc}_{krom}$*-ontologies is* NC^1*-hard.*

Proof. We use the fact [5] that there exist NC^1-complete regular languages. Suppose $\mathfrak{A}$ is an NFA, q_0 its initial state and q_1 the only accepting state. Given an input word $\boldsymbol{w} = w_0 \dots w_n$, we set $\mathcal{D}_{\boldsymbol{w}} = \{\, w_i(i) \mid i \leq n \,\} \cup \{\, q_1(n+1) \,\}$.

(i) Let $\mathcal{O} = \{\boxast(\bigcirc_{\!F} q' \land a \to q) \mid q \to_a q'\}$. Then $\mathfrak{A}$ accepts $\boldsymbol{w}$ iff $\mathcal{O}, \mathcal{D}_{\boldsymbol{w}} \models q_0(0)$. (ii) Let $\boldsymbol{q} = q_0(0) \lor \bigvee_{q \to_a q'} \exists t, t' \big(S(t,t') \land q'(t') \land a(t) \land \neg q(t)\big)$. Then $\mathfrak{A}$ accepts $\boldsymbol{w}$ iff $\emptyset, \mathcal{D}_{\boldsymbol{w}} \models \boldsymbol{q}$. (iii) Let $\boldsymbol{q}'$ be the result of replacing each occurrence of $\neg q(t)$ in $\boldsymbol{q}$ with $\bar{q}(t)$, for a fresh $\bar{q}$, and let $\mathcal{O}'$ be an ontology containing $\boxast(\bar{q} \leftrightarrow \neg q)$ for every such $\bar{q}$. Then $\mathfrak{A}$ accepts $\boldsymbol{w}$ iff $\mathcal{O}', \mathcal{D}_{\boldsymbol{w}} \models \boldsymbol{q}'$. ❑

We now establish the FO($<,+$)- and FO($<$)-rewritability results advertised in the introduction.

Theorem 4. *Any AQ and* $LTL^{\bigcirc}_{krom}$*-ontology are FO*$(<,+)$*-rewritable, but not necessarily FO*$(<)$*-rewritable.*

Proof. Suppose we are given an AQ $q(\tau)$ and an $LTL^{\bigcirc}_{krom}$-ontology $\mathcal{O}$. (That they are not necessarily FO($<$)-rewritable was shown in Example 1.) Without loss of generality we assume that $\mathcal{O}$ does not contain nested $\bigcirc_{\!F}$ and $\bigcirc_{\!P}$ and write $\bigcirc^n \varphi$ in place of $\bigcirc_{\!F}^n \varphi$ if $n > 0$, φ if $n = 0$, and $\bigcirc_{\!P}^{-n} \varphi$ if $n < 0$. By a *literal*, L, we mean a propositional variable from $\mathsf{sig}(q, \mathcal{O})$ or its negation. Given literals L and L', we write $\mathcal{O} \models \boxast(L \to \bigcirc^k L')$ to say that $\boxast(L \to \bigcirc^k L')$ is true in all models of $\mathcal{O}$. Let $\mathfrak{A}^{\mathcal{O}}_{L,L'}$ be an NFA whose tape alphabet is $\{0\}$, the states are all the literals, with L being the initial and L' the accepting states, and whose transitions are of the form $L_1 \to_0 L_2$, for $\mathcal{O} \models \boxast(L_1 \to \bigcirc L_2)$.

Lemma 5. *For any* $k > 0$*, the NFA* $\mathfrak{A}^{\mathcal{O}}_{L,L'}$ *accepts* 0^k *iff* $\mathcal{O} \models \boxast(L \to \bigcirc^k L')$*.*

Every unary automaton $\mathfrak{A}^{\mathcal{O}}_{L,L'}$ with N states gives rise (via Chrobak normal form [9, 18]) to $M = O(N^2)$ arithmetic progressions $a_i + b_i\mathbb{N} = \{a_i + b_i \cdot m \mid m \geq 0\}$, for $i \leq M$, such that $1 \leq a_i, b_i \leq N$ and, for any $k > 0$, $\mathfrak{A}^{\mathcal{O}}_{L,L'}$ accepts 0^k iff $\mathsf{div}_{b_i}(k - a_i)$, for some $i \leq M$, where $\mathsf{div}_b(v) = \exists n\, ((0 \leq n \leq v) \land (v = bn))$. Let

$$P^{\mathcal{O}}_{L,L'}(t) = \bigvee_{i=1}^{M} \mathsf{div}_{b_i}(t - a_i) \quad \text{and} \quad E^{\mathcal{O}}_{L,L'}(t,t') = \begin{cases} (t = t'), & \text{if } \mathcal{O} \models \boxast(L \to L'), \\ \bot, & \text{otherwise.} \end{cases}$$

Lemma 6. (i) *If* $(\mathcal{O}, \mathcal{D})$ *is consistent then, for any* $\ell \in \mathbb{Z}$*, we have* $\mathcal{O}, \mathcal{D} \models q(\ell)$ *iff either* $\mathcal{O} \models \boxast\, q$ *or there exists* $p(m) \in \mathcal{D}$ *such that* $\mathcal{O} \models \boxast(p \to \bigcirc^{\ell - m} q)$*.*

(ii) *If* $(\mathcal{O}, \mathcal{D})$ *is inconsistent then either* $\mathcal{O} \models \boxast\, \bot$ *or there are* $p(m) \in \mathcal{D}$ *and* $r(n) \in \mathcal{D}$ *such that* $\mathcal{O} \models \boxast(p \to \bigcirc^{n-m} \neg r)$*.*

The conditions of this lemma can be encoded as an FO($<,+$)-formula $\varphi_q(t)$, which is $\top$ if $\mathcal{O} \models \boxast\, \bot$ or $\mathcal{O} \models \boxast\, q$, and otherwise a disjunction of the following formulas, for $p, r \in \mathsf{sig}(q, \mathcal{O})$,

$$\exists s, s' \big(p(s) \land r(s') \land (P^{\mathcal{O}}_{p,\neg r}(s' - s) \lor E^{\mathcal{O}}_{p,\neg r}(s', s)))\big), \tag{4}$$

$$\exists s \,\big(p(s) \land \big(P^{\mathcal{O}}_{p,q}(t - s) \lor E^{\mathcal{O}}_{p,q}(t, s) \lor P^{\mathcal{O}}_{\neg q,\neg p}(s - t)\big)\big). \tag{5}$$

Thus, $\mathcal{O}, \mathcal{D} \models q(\ell)$ iff $\mathcal{I}_{\mathcal{D}}^{\mathbb{Z}} \models \varphi_q(\ell)$, for any $\mathcal{D}$ and $\ell \in \mathbb{Z}$. In fact, for a slight modification $\varphi'_q(t)$ of $\varphi_q(t)$, we can show that $\mathcal{O}, \mathcal{D} \models q(\ell)$ iff $\mathcal{I}_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}} \models \varphi'_q(\ell)$ where $\Delta_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}} = [\min\{0,\ell\}, \max\{\max \mathcal{D}, |\ell| + O(|\mathcal{O}|)\}]$. It follows that $\varphi'_q(\tau)$ is a FO($<,+$)-rewriting of $q(\tau)$ and $\mathcal{O}$ over $\mathcal{I}_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}}$. ❑

By Theorem 3, UCQ$^+$s and $LTL_{krom}^{\bigcirc}$-ontologies are not always FO($<,+$)-rewritable. In the next theorem, we restrict attention to $LTL_{core}^{\bigcirc}$-ontologies.

Theorem 7. *Any UCQ$^+$ $\boldsymbol{q}(\boldsymbol{t})$ and $LTL_{core}^{\bigcirc}$-ontology $\mathcal{O}$ are FO($<,+$)-rewritable.*

Proof. Let $\boldsymbol{q}'(\boldsymbol{t})$ result from replacing any $q(\tau)$ in $\boldsymbol{q}(\boldsymbol{t})$ with the above mentioned $\varphi'_q(\tau)$. Assuming that $\mathcal{O}$ and $\mathcal{D}$ are consistent, denote by $\mathcal{C}_{\mathcal{O},\mathcal{D}}$ the *canonical model of* $\mathcal{O}$ and $\mathcal{D}$. We then have $\mathcal{C}_{\mathcal{O},\mathcal{D}} \models \boldsymbol{q}(\boldsymbol{\ell})$ iff $\mathcal{I}_{\mathcal{D}}^{\mathbb{Z}} \models \boldsymbol{q}'(\boldsymbol{\ell})$. Let $\mathcal{I}_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}}$ be the interpretation defined in the proof of Theorem 4, where ℓ is the constant in $\boldsymbol{q}$ with maximal $|\ell|$, if any. Let $\boldsymbol{max}$ and $\boldsymbol{min}$ be the maximum and minimum of $\Delta_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}}$. By analysing the structure of (4) and (5), we obtain:

Lemma 8. *Suppose $\boldsymbol{q}'(\boldsymbol{t}) = \exists \boldsymbol{s}\, \psi(\boldsymbol{t}, \boldsymbol{s})$ with quantifier-free ψ and $\boldsymbol{s} = s_1, \dots, s_m$. If $\mathcal{I}_{\mathcal{D}}^{\mathbb{Z}} \models \boldsymbol{q}'(\boldsymbol{\ell})$, for some $\boldsymbol{\ell}$ in $\Delta_{\mathcal{D}}$, then there is a tuple $\boldsymbol{n} = n_1, \dots, n_m$ in the interval $[\boldsymbol{min} - |\boldsymbol{q}| \cdot O(|\mathcal{O}|^{|\boldsymbol{q}|}), \boldsymbol{max} + |\boldsymbol{q}| \cdot O(|\mathcal{O}|^{|\boldsymbol{q}|})]$ such that $\mathcal{I}_{\mathcal{D}}^{\mathbb{Z}} \models \psi(\boldsymbol{\ell}, \boldsymbol{n})$.*

We can now transform $\boldsymbol{q}'$ to an FO($<,+$)-rewriting $\boldsymbol{q}''(\boldsymbol{t})$ of $\boldsymbol{q}$ and $\mathcal{O}$ over $\mathcal{I}_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}}$ using FO($<,+$)-sentences φ_q^k such that, for $k < 0$ and $\ell = \boldsymbol{min} + k$ or for $k > 0$ and $\ell = \boldsymbol{max} + k$, we have $\mathcal{I}_{\mathcal{D}}^{\mathbb{Z}} \models q(\ell)$ iff $\mathcal{I}_{\mathcal{D}}^{\boldsymbol{q},\mathcal{O}} \models \varphi_q^k(\ell)$. To construct such sentences φ_q^k, we require the $(\boldsymbol{max}+1)$-ary representation of numbers and a technique from [12, Sec. 1.4]. ❑

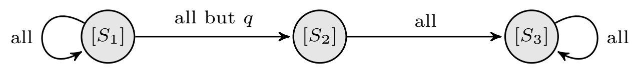
Consider next the *LTL*-ontologies whose axioms (in clausal normal form) can only contain the operators $\Box_P$ and $\Box_F$. All AQs over arbitrary $LTL_{bool}^{\Box}$-ontologies turn out to be FO($<$)-rewritable.

Theorem 9. *Any AQ and $LTL_{bool}^{\Box}$-ontology are FO($<$)-rewritable.*

Proof. Let $\mathcal{O}$ be an $LTL_{bool}^{\Box}$-ontology. First we construct an NFA $\mathfrak{A}$ that describes the structure of models of $\mathcal{O}$ and a given data instance $\mathcal{D}$ written on the tape as defined above (cf. [19]). Denote by Φ the set of all subformulas of $\mathcal{O}$ and their negations. Each state of $\mathfrak{A}$ is a maximal consistent subset $S \subseteq \Phi$; let $\boldsymbol{S}$ be the set of all such states. For any $S, S' \in \boldsymbol{S}$ and any tape symbol $X \neq \star$, we set $S \to_X S'$ just in case (*i*) $X \subseteq S'$, (*ii*) $\Box_F \psi \in S$ iff $\psi, \Box_F \psi \in S'$, and (*iii*) $\Box_P \psi \in S'$ iff $\psi, \Box_P \psi \in S$. We also set $S \to_\star S'$ iff $S \to_\emptyset S'$. A state $S \in \boldsymbol{S}$ is *accepting* if $\mathfrak{A}$ contains an infinite 'ascending' chain of transitions of the form $S \to_\emptyset S_1 \to_\emptyset S_2 \to_\emptyset \dots$; S is *initial* if $\mathfrak{A}$ contains an infinite 'descending' chain $\dots \to_\emptyset S_2 \to_\emptyset S_1 \to_\emptyset S$. It is not hard to check that $\mathfrak{A}$ accepts $\mathcal{D}$ iff $\mathcal{O}$ and $\mathcal{D}$ are consistent.

An NFA is called *partially ordered* [16] if it contains no cycles other than trivial loops. We now convert $\mathfrak{A}$ to an equivalent partially-ordered NFA $\mathfrak{A}'$. Define an equivalence relation, $\sim$, on $\boldsymbol{S}$ by taking $S \sim S'$ iff either $S = S'$ or $\mathfrak{A}$ contains a cycle with both S and S'. Denote by $[S]$ the $\sim$-equivalence class of S. It is readily seen that if $S \to_X S'$ then $S_1 \to_X S'$, for any $S_1 \in [S]$. The states of $\mathfrak{A}'$ are of the form $[S]$, for $S \in \boldsymbol{S}$, and it contains a transition $[S] \to_X [S']$ iff $S_1 \to_X S_1'$, for some $S_1 \in [S]$ and $S_1' \in [S']$. The initial (accepting) states of $\mathfrak{A}'$ are all $[S]$ with initial (accepting) S. Clearly, $\mathfrak{A}'$ is partially ordered, and it follows from the construction that $\mathfrak{A}'$ and $\mathfrak{A}$ accept the same language.

Example 10. Let $\mathcal{O} = \{\boxast(p \to \Box_P q),\ \boxast(\Box_P q \to r)\}$. The NFA $\mathfrak{A}'$ for $\mathcal{O}$ is shown below:



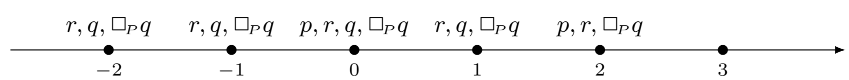
where all states are initial and accepting, $[S_1] = \{\{\Box_P q, q, p, r\}, \{\Box_P q, q, r\}\}$, $[S_2] = \{\{\Box_P q, p, r\}, \{\Box_P q, r\}\}$, $[S_3] = 2^{\{r,q\}}$ (negative literals are omitted).

Consider a *maximal* path *pi* in $\mathfrak{A}'$ of the form $[S_0] \to_{X_1} \dots \to_{X_n} [S_n]$, where all the $[S_i]$ are pairwise distinct, $[S_0]$ is initial and $[S_n]$ accepting. We call π a *prime path* in $\mathfrak{A}'$. Suppose $[S_i] \to_X [S_i]$ in $\mathfrak{A}'$. The operation of *duplicating* $[S_i]$ (required for technical reasons that will be clear from Lemma 11) in a prime path π replaces $[S_i]$ in π with $[S_i] \to_X [S_i]$. Denote by $\Psi_{\mathcal{O}}$ the set of all paths obtained by at most *two* applications of duplication to some prime path. The length of each path in $\Psi_{\mathcal{O}}$ is polynomial in $|\mathcal{O}|$ and $|\Psi_{\mathcal{O}}| = 2^{O(|\mathcal{O}|^2)}$. Given an interpretation $\mathcal{I}$ and $\ell \in \mathbb{Z}$, let $\boldsymbol{t}_{\mathcal{I}}(\ell) = \{p \in \Phi \cap \mathcal{P} \mid \mathcal{I}, \ell \models p\}$ (the $\mathcal{I}$-type of ℓ).

Lemma 11. *For any AQ $q(t)$, interpretation $\mathcal{I}$ and $\ell \in \mathbb{Z}$, we have $\mathcal{I} \models \mathcal{O} \cup \mathcal{D}$ and $\mathcal{I} \not\models q(\ell)$ iff there exist a path $\pi = [S_0] \to_{X_1} \dots \to_{X_n} [S_n]$ in $\Psi_{\mathcal{O}}$, an injective function $f\colon \{0, \dots, n\} \to \mathbb{Z}$ and numbers k, b, e in $\{0, \dots, n\}$ such that*

- *$f(k) = \ell$, $f(b) = 0$, $f(e) = \max \mathcal{D}$ and $b < e$;*
- *for any $m \le f(0)$, there is $S_0' \in [S_0]$ such that $\boldsymbol{t}_{\mathcal{I}}(m) = S_0'$;*
- *for any m with $f(i) \le m < f(i+1)$, there is $S_i' \in [S_i]$ with $\boldsymbol{t}_{\mathcal{I}}(m) = S_i'$;*
- *for any $m \ge f(n)$, there is $S_n' \in [S_n]$ such that $\boldsymbol{t}_{\mathcal{I}}(m) = S_n'$.*

Example 12. Consider again the ontology $\mathcal{O}$ from Example 10, $\mathcal{D} = \{p(0), p(2)\}$ and the model $\mathcal{I}$ of $\mathcal{O}$ and $\mathcal{D}$ shown below:



We have $\mathcal{I}, 3 \not\models r$ and, for the path $[S_1] = [S_0] \to_{\{p\}} [S_1] \to_{\{p\}} [S_2] \to_{\emptyset} [S_3]$ in $\Psi_{\mathcal{O}}$, we can take $f(0) = -1$, $f(1) = 0$, $f(2) = 2$, $f(3) = 3$, $k = 3$, $b = 1$ and $e = 2$. The conditions of Lemma 11 for π can be expressed by an FO($<$)-formula such as

$$\begin{aligned}\varphi_{\pi,r}(t) \quad = \quad & \exists t_b, t_e [(t_b = 0) \land \star(t_e + 1) \land (t > t_e) \land \\ & \phi_{\pi,[S_1]}(t_b) \land \phi_{\pi,[S_2]}(t_e) \land \forall t''((t_b < t'' < t_e) \to \phi_{[S_1]}(t''))],\end{aligned}$$

where $\phi_{\pi,[S_1]}(t) = \phi_{\pi,[S_2]}(t) = p(t) \land \neg q(t) \land \neg r(t)$ characterises the transitions $[S_0] \to_{\{p\}} [S_1] \to_{\{p\}} [S_2]$, and $\phi_{[S_1]}(t) = \top(t)$ the loops $[S_1] \to_{all} [S_1]$.

Lemma 13. *For any AQ $q(t)$ and $LTL^{\Box}_{bool}$-ontology $\mathcal{O}$, there is an FO($<$)-formula $\chi_q(t)$ such that $\mathcal{O}, \mathcal{D} \models q(\ell)$ iff $\mathcal{I}^{\mathbb{Z}}_{\mathcal{D}} \models \chi_q(\ell)$, for all $\mathcal{D}$ and $\ell \in \mathbb{Z}$.*

The formula $\chi_q(t)$ is defined as a negation of a disjunction of formulas such as $\varphi_{\pi,r}(t)$ in Example 12, for all possible choices of π, k, b and e. To complete the proof of Theorem 14, we proceed in the same way as in Theorem 4. ❑

By using the canonical models and the technique of the proof of Theorem 7, we also obtain:

Theorem 14. *Any UCQ$^+$ and $LTL^{\Box}_{horn}$-ontology are FO(<)-rewritable.*

Our next aim is to 'lift' these results to temporal extensions of *DL-Lite* logics.

3 OBDA with Temporal *DL-Lite*

Let α be one of *bool*, *horn*, *krom* or *core*, and let β be one of $\Box$, $\bigcirc$ or $\Box\bigcirc$. We denote by T^β*DL-Lite*$_\alpha$ the *temporal description logic* with inclusions of the form

$$\lambda_1 \sqcap \cdots \sqcap \lambda_n \ \sqsubseteq\ \lambda_{n+1} \sqcup \cdots \sqcup \lambda_{n+m} \tag{6}$$

such that $\neg\lambda_1 \vee \cdots \vee \neg\lambda_n \vee \lambda_{n+1} \vee \cdots \vee \lambda_{n+m}$ is an α-clause (according to the definition in Section 1) and the λ_i are all either *DL-Lite* basic concepts or *DL-Lite* roles, possibly prefixed with temporal operators indicated in β. Recall [1] that *DL-Lite basic concepts*, B, are of the form $B \ ::= \bot \ \mid \ A \ \mid \exists R$, where A is a *concept name* and R a role. *DL-Lite roles* are defined by $R \ ::= \ \bot \ \mid \ P \ \mid \ P^-$, where P is a *role name*. A T^β*DL-Lite*$_\alpha$ *TBox* $\mathcal{T}$ (*RBox* $\mathcal{R}$) is a finite set of T^β*DL-Lite*$_\alpha$ concept (respectively, role) inclusions. A TBox is said to be *flat* if its concept inclusions do not contain $\exists R$ on the right-hand side. We denote by T^β*DL-Lite*$_\alpha^{flat}$ the fragment of T^β*DL-Lite*$_\alpha$ in which only flat TBoxes are allowed. (Note that *DL-Lite*$_{core}^{flat}$ is basically RDFS.) An *ABox* $\mathcal{A}$ is a set of atoms of the form $A(a,n)$ and $P(a,b,n)$, where a, b are object names and $n \in \mathbb{Z}$.

A *temporal interpretation* is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}(n)})$, in which $\Delta^{\mathcal{I}} \neq \emptyset$ and $\mathcal{I}(n) = (\Delta^{\mathcal{I}}, a_0^{\mathcal{I}}, \dots, A_0^{\mathcal{I}(n)}, \dots, P_0^{\mathcal{I}(n)}, \dots)$ is a standard DL interpretation for each time instant $n \in \mathbb{Z}$. Thus, we assume that the domain $\Delta^{\mathcal{I}}$ and the interpretations $a_i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ of the object names are the same for all $n \in \mathbb{Z}$. The DL and temporal constructs are interpreted in $\mathcal{I}(n)$ as expected, for example,

$$(\bigcirc_F B)^{\mathcal{I}(n)} \ = \ B^{\mathcal{I}(n+1)} \quad \text{and} \quad (\Box_F B)^{\mathcal{I}(n)} \ = \ \bigcap\nolimits_{k>n} B^{\mathcal{I}(k)}.$$

Concept and role inclusions are interpreted in $\mathcal{I}$ *globally*, that is, $\mathcal{I} \models \bigsqcap \lambda_i \ \sqsubseteq \ \bigsqcup \lambda_j$ just in case $\bigcap \lambda_i^{\mathcal{I}(n)} \ \subseteq \ \bigcup \lambda_j^{\mathcal{I}(n)}$, for *all* $n \in \mathbb{Z}$. Given an ABox $\mathcal{A}$, we denote by $\mathcal{I}_{\mathcal{A}}^{\mathbb{Z}}$ the interpretation with domain $ind(\mathcal{A})$, which consists of the object names in $\mathcal{A}$, such that $\mathcal{I}_{\mathcal{A}}^{\mathbb{Z}} \models A(a,n)$ iff $A(a,n) \in \mathcal{A}$ and $\mathcal{I}_{\mathcal{A}}^{\mathbb{Z}} \models P(a,b,n)$ iff $P(a,b,n) \in \mathcal{A}$, for $a, b \in ind(\mathcal{A})$ and $n \in \mathbb{Z}$.

Temporal FOQs are built in the same way as in the *LTL* case, but from atoms of the form $A(\xi,\tau)$, $P(\xi,\zeta,\tau)$ using $=$, $<$, S and PLUS. As before, we are interested in various types of FO-rewriting of a FOQ $\boldsymbol{q}$ and a T^β*DL-Lite*$_\alpha$ ontology $\mathcal{T}$, $\mathcal{R}$ over finite first-order structures $\mathcal{I}_{\mathcal{A}}^{\boldsymbol{q},\mathcal{T},\mathcal{R}}$ which are restrictions of $\mathcal{I}_{\mathcal{A}}^{\mathbb{Z}}$ to the minimal closed intervals in $\mathbb{Z}$ containing all time instants from $\mathcal{A}$ as well as some fixed integers that only depend on $\boldsymbol{q}$ and $\mathcal{T}$, $\mathcal{R}$.

Theorem 15. *Any UCQ⁺ and $T^{\bigcirc}$DL-Lite$^{flat}_{core}$ ontology are FO$(<,+)$-rewritable.*

Proof. Without loss of generality, we assume that the TBox, $\mathcal{T}$, and RBox, $\mathcal{R}$, do not contain nested temporal operators; together with any role inclusion, $\mathcal{R}$ contains the respective inclusion for the inverses; and for every role inclusion in $\mathcal{R}$, say $\bigcirc_P P^- \sqsubseteq R$, there is an inclusion for their domains, $\bigcirc_P \exists P^- \sqsubseteq \exists R$, in $\mathcal{T}$. It can be seen that, for any ABox $\mathcal{A}$ consistent with $\mathcal{T}$, $\mathcal{R}$ and any $a, b \in ind(\mathcal{A})$ and $\ell \in \mathbb{Z}$, we have: $\mathcal{R}, \mathcal{A} \models R(a, b, \ell)$ iff $\mathcal{T}, \mathcal{R}, \mathcal{A} \models R(a, b, \ell)$, and $\mathcal{T}, \mathcal{A} \models B(a, \ell)$ iff $\mathcal{T}, \mathcal{R}, \mathcal{A} \models B(a, \ell)$.

Denote by $\mathcal{T}^p$ ($\mathcal{R}^p$) the $LTL^{\bigcirc}_{core}$-ontology that contains all inclusions from $\mathcal{T}$ ($\mathcal{R}$) in which every basic concept (role) is regarded as a propositional variable. For a concept name A (role name P), let $\varphi_A(t)$ (respectively, $\varphi_P(t)$) be the FO$(<,+)$-rewriting of $A(t)$ and $\mathcal{T}^p$ (of $P(t)$ and $\mathcal{R}^p$) constructed in the proof of Theorem 4. For any ABox $\mathcal{A}$ and $a, b \in ind(\mathcal{A})$, denote by $\mathcal{A}_{a,b}$ the LTL data instance containing $P(n)$ if $P(a, b, n) \in \mathcal{A}$ and $P^-(n)$ if $P(b, a, n) \in \mathcal{A}$. Similarly, for any $a \in ind(\mathcal{A})$, let $\mathcal{A}_a$ contain $A(n)$ if $A(a, n) \in \mathcal{A}$ and $\exists R(n)$ if $R(n) \in \mathcal{A}_{a,b}$, for some b. Then $\mathcal{R}^p, \mathcal{A}_{a,b} \models R(n)$ iff $\mathcal{R}, \mathcal{A} \models R(a, b, n)$, and $\mathcal{T}^p, \mathcal{A}_a \models B(n)$ iff $\mathcal{T}, \mathcal{A} \models B(a, n)$.

Now, given an atom $P(\xi, \zeta, \tau)$, we construct an FO$(<,+)$-formula $P^*(\xi, \zeta, \tau)$ by replacing every $R(\tau')$ in $\varphi_P(\tau)$ with $R(\xi, \zeta, \tau')$ if R is a role name, and with $Q(\zeta, \xi, \tau')$ if $R = Q^-$. Similarly, given an atom $A(\xi, \tau)$, we construct an FO$(<,+)$-formula $A^*(\xi, \tau)$ by replacing every $B(\tau')$ in $\varphi_A(\tau)$ with $B(\xi, \tau')$ if B is a concept name, with $\exists y\, P^*(\xi, y, \tau')$ if $B = \exists P$, and with $\exists y\, P^*(y, \xi, \tau')$ if $B = \exists P^-$. As a consequence of Theorem 4 and the observations above, we obtain:

Lemma 16. *For any ABox $\mathcal{A}$, any $a, b \in ind(\mathcal{A})$ and any $\ell \in \mathbb{Z}$, we have: $\mathcal{T}, \mathcal{R}, \mathcal{A} \models P(a, b, \ell)$ iff $\mathcal{I}^{\mathbb{Z}}_{\mathcal{A}} \models P^*(a, b, \ell)$, and $\mathcal{T}, \mathcal{R}, \mathcal{A} \models A(a, \ell)$ iff $\mathcal{I}^{\mathbb{Z}}_{\mathcal{A}} \models A^*(a, \ell)$.*

The remainder of the proof is a straightforward modification of the corresponding part in the proof of Theorem 7. ❑

Example 17. Suppose $\boldsymbol{q} = \exists x, y, t\, (P(x, y, t) \wedge A(x, t))$ and let $\mathcal{T}$ contain $\exists P^- \sqsubseteq A$ and $\mathcal{R}$ contain $\bigcirc_P P^- \sqsubseteq P$. We will also assume that $\bigcirc_P P \sqsubseteq P^-$ is in $\mathcal{R}$ and so, both $\bigcirc_P \exists P^- \sqsubseteq \exists P$ and $\bigcirc_P \exists P \sqsubseteq \exists P^-$ are in $\mathcal{T}$. We obtain the following FO$(<,+)$-rewriting of $\boldsymbol{q}$ and $\mathcal{T}, \mathcal{R}$, where all the variables are existentially quantified:

$$\underbrace{[(P(x,y,s) \wedge (t - s = 2n \geq 0)) \vee (P(y,x,s) \wedge (t - s = 2n+1 \geq 0))]}_{P^*(x,y,t)} \wedge$$
$$\underbrace{[A(x,t) \vee \exists z\, ((P(z,x,s) \wedge (t - s = 2n \geq 0)) \vee (P(x,z,s) \wedge (t - s = 2n+1 \geq 0)))]}_{A^*(x,t)}.$$

Theorem 18. *Any UCQ⁺ and any $T^{\Box}$DL-Lite$^{flat}_{horn}$ ontology are FO$(<)$-rewritable.*

Proof. We proceed similarly to the proof of Theorem 15. One important difference is that, given an RBox $\mathcal{R}$, we extend it with $\Box_F R \equiv F_R$, for every $\Box_F R$ in $\mathcal{R}$, and $\Box_P R \equiv P_R$, for every $\Box_P R$ in $\mathcal{R}$. (These new roles are auxiliary and do not occur in any ABox.) Treating (the extended) $\mathcal{R}$ as an $\mathit{LTL}^{\Box}_{horn}$-ontology, we take the rewriting $\chi_R(t)$ of $R(t)$ and $\mathcal{R}$ constructed in the proof of Theorem 14. Let $R^*(\xi,\zeta,\tau)$ be the result of replacing every $P(\tau')$ in $\chi_R(t)$ with $P(\xi,\zeta,\tau')$ and every $P^-(\tau')$ with $P(\zeta,\xi,\tau')$. To construct a rewriting for atoms $A(\xi,\tau)$, we extend $\mathcal{T}$ with $\exists F_R \sqsubseteq \Box_F \exists R$, for every F_R in $\mathcal{R}$, and $\exists P_R \sqsubseteq \Box_P \exists R$, for every P_R in $\mathcal{R}$. (The need for such axioms, connecting $\mathcal{T}$ and $\mathcal{R}$, is explained by Example 19.) Treating (the extended) $\mathcal{T}$ as an $\mathit{LTL}^{\Box}_{horn}$-ontology, we take the rewriting $\chi_A(t)$ of $A(t)$ and $\mathcal{T}$ from Theorem 14. Denote by $A^*(\xi,\tau)$ the result of replacing every $A'(\tau')$ in $\chi_A(t)$ with $A'(\xi,\tau')$, every $\exists P(\tau')$ with $\exists y\, P^*(\xi,y,\tau')$, and every $\exists P^-(\tau')$ with $\exists y\, P^*(y,\xi,\tau')$. The remainder of the proof is now routine.

Example 19. Consider $\mathcal{R} = \{R \sqsubseteq \Box_P T\}$, $\mathcal{T} = \{\Box_P \exists T \sqsubseteq A\}$ and the AQ $A(x,t)$. Then $\chi_T(t) = T(t) \lor \exists s\,[(s > t) \land R(s)]$. If we did not extend $\mathcal{T}$ with $\exists P_T \sqsubseteq \Box_P \exists T$, we would have $\chi_A(t) = A(t)$. But due to this inclusion, we obtain

$$\chi_A(t) = A(t) \lor \exists t'\,[(t' \geq t) \land \exists P_T(t')] \lor \exists t'[(t' < t) \land \exists P_T(t') \land \forall s\,((t' \leq s < t) \to \exists T(s))],$$
$$\chi_{P_T}(t) = \exists t'\,[(t' \geq t) \land R(t')] \lor \exists t'\,[(t' < t) \land R(t') \land \forall s\,((t' \leq s < t) \to T(s))].$$

Theorem 20. *The UCQ⁺-entailment problem over $T^{\Box\bigcirc}$DL-Lite$^{flat}_{horn}$-ontologies without role inclusions is* NC^1*-complete for data complexity.*

Proof. To sketch the idea, let us assume that the given TBox $\mathcal{T}$ does not contain roles (roles are mostly harmless without role inclusions). Let $\boldsymbol{q} = \exists \boldsymbol{x} \exists \boldsymbol{t}\, \varphi(\boldsymbol{x},\boldsymbol{t})$ be the given UCQ⁺. We treat the atoms $A(x_i,t)$ in φ as unary predicates $Ax_i(t)$ and, using the construction from [19], define an NFA $\mathfrak{A}$ such that, for any ABox $\mathcal{A}$ and any tuple $\boldsymbol{a}$ from $\mathcal{A}$, we have $\mathcal{T},\mathcal{A} \not\models \exists \boldsymbol{t}\,\varphi(\boldsymbol{a},\boldsymbol{t})$ iff $\mathfrak{A}$ accepts a word $\mathcal{D}_{\boldsymbol{a}}$, constructed by a constant-size circuit and encoding the data in $\mathcal{A}$ relevant to $\boldsymbol{a}$. We convert $\mathfrak{A}$ to an NC^1 circuit and then use multiple copies of the constructed circuits to assemble an NC^1 circuit answering $\boldsymbol{q}$ over any given ABox. ❑

4 Conclusions

This paper launches a systematic investigation of the rewritability problem for temporal queries and ontologies. We first consider this problem for AQs and UCQ⁺s over *LTL*-ontologies in clausal normal form and classify them in terms of FO(<)-, FO(<, +)- or NFA-rewritability. However, it remains unclear whether NFA-rewritability of AQs over $\mathsf{LTL}^{\Box\bigcirc}_{krom}$-ontologies, and of AQs and UCQ⁺s over $\mathsf{LTL}^{\Box\bigcirc}_{core}$-ontologies can be improved to FO(<, +)-rewritability. Another interesting problem is to analyse rewritability of general FO-queries in more detail.

We also show that some of our rewritability results over *LTL*-ontologies can be lifted to the corresponding flat *DL-Lite*-ontologies. Extending these results to the non-flat case looks more of less straightforward but is technically challenging. We are working on the extension of Theorem 20 to the case with role inclusions and on a complete classification of temporalised *DL-Lite* logics according to query rewritability.

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