Tableau-based revision in $\mathcal{SHIQ}$

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Introduction The problem of revising a description logic-based ontology (called DL ontology) is closely related to the problem of belief revision which has been widely discussed in the literature. Among early works on belief revision, the AGM theory (Alchourrón *et al.*, 1985) introduced intuitive and plausible constraints (namely AGM postulates) which should be satisfied by any rational belief revision operator. However, it is not trivial to adapt belief revision operators to DLs because DLs have their own features (Flouris *et al.*, 2005) (Qi and Yang, 2008). One main difficult for such revision is that DL ontologies often incur infinitely many models. To address this issue, we propose a finite set of finite structures, namely a set $\mathsf{MT}(\mathcal{O})$ of completion trees, for characterizing a possibly infinite set of models of an ontology $\mathcal{O}$. Then, we define a distance over a set of completion trees. This distance allows one to determine how far an ontology is from another one. Another problem our approach has to address is that there may not exist a revision ontology such that (i) it is expressible in the logic used for expressing initial ontologies $\mathcal{O}, \mathcal{O}'$, and (ii) it admits *exactly* a set of models $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ computed from $\mathsf{MT}(\mathcal{O})$ and $\mathsf{MT}(\mathcal{O}')$. For this reason, we borrow the notion of *maximal approximation* (De Giacomo *et al.*, 2007) which allows us to build a *minimal* revision ontology admitting $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$.

Construction of the revision ontology First, we define a novel tableau algorithm, namely $\mathbb{TA}$, for a $\mathcal{SHIQ}$ ontology without individuals by replacing expansion $\sqsubseteq$-, $\sqcap$-, $\sqcup$, ch-rules by a new rule, namely sat-rule which chooses a subset $\mathcal{S}$ from a set $\mathsf{sub}(\mathcal{O})$ including all sub-concepts of a $\mathcal{SHIQ}$ ontology $\mathcal{O}$. Note that all concepts in the form of conjunctions or of disjunctions are removed from $\mathsf{sub}(\mathcal{O})$ and replaced with their conjuncts and disjuncts. This can be performed by a function $\mathsf{Flat}(C)$ that flattens conjunctions and disjunctions of a concept C into subsets of sub-concepts occurring in C. For example, $\mathsf{Flat}(A \sqcap (\exists R.B \sqcup C)) = \{\{A, \exists R.B\}, \{A, C\}\}$.

sat**-rule.** If sat-rule has never been applied to a node x then we choose a subset $S \subseteq \mathsf{sub}(\mathcal{O})$ such that $L(x) \cup \bigcup_{C \sqsubseteq D \in \mathcal{T}} f(C \sqsubseteq D) \subseteq S$ where $f(C \sqsubseteq D) \in \mathsf{Flat}(\neg C \sqcup D)$ for each $C \sqsubseteq D \in \mathcal{T}$, and set $L(x) := S \cup \bar{S}$ where $\bar{S} = \{\neg C \mid C \in \mathsf{sub}(\mathcal{O}) \setminus S\}$

In this paper, a *completion tree* for $\mathcal{O}$ is a tree $T = (V, E, L, \widehat{x})$ where V is a set of nodes with the root node $\widehat{x} \in V$. Each node $x \in V$ is labeled with a function $L(x) \subseteq \mathsf{sub}(\mathcal{O})$. E is a set of edges and each edge $\langle x, y\rangle \in E$ is labeled with a function $L(\langle x, y\rangle)$ containing a set of $\mathcal{SHIQ}$ roles.

The sat-rule that is applied to each node of a completion tree introduces a lot of non-determinisms. We need this "bad" behavior of the new tableau algorithm to control the generation process of completion trees in such a way that allows one to infer the ontology when knowing completion trees and its signature. We use $\mathsf{MT}(\mathcal{O})$ to denote the set of all completion trees which are generated by running the novel tableau algorithm $\mathbb{TA}$ for an ontology $\mathcal{O}$. Note that $\mathbb{TA}$ does not necessarily terminate when a complete and clash-free completion tree is built. It should terminate when all non-determinisms are considered. We can extend straightforwardly $\mathsf{MT}(\mathcal{O})$ to $\mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ as follows. The set $\mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ is built by the tableau algorithm $\mathbb{TA}$ for $\mathcal{O}'$ with an extra set of concepts $\mathsf{sub}(\mathcal{O})$ that is taken into account when applying the sat-rule. In this case one can import additional concepts into node labels of a completion tree for $\mathcal{O}'$ while respecting the axioms of $\mathcal{O}'$. Importing $\mathsf{sub}(\mathcal{O})$ to $\mathsf{MT}(\mathcal{O}')$ ensures that $\mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ captures semantic constraints from $\mathcal{O}$ which are compatible with $\mathcal{O}'$.

Next, we introduce a distance between two completion trees T and T' which allows one to talk about the similarity between two ontologies. This distance is defined for two completion trees which are isomorphic, i.e., there is an isomorphism π that maintains the successor relationship from two nodes of a completion tree to the two corresponding nodes of the other one via π. Note that we can always obtain such an isomorphism between two completion trees by adding empty nodes and edges to completion trees since node and edge labels are ignored in the definition of isomorphisms.

Definition 1 (Distance). *Let $T = \langle V, L, E, \widehat{x}\rangle$ and $T' = \langle V', L', E', \widehat{x}'\rangle$ two completion trees. Let $\Pi(T, T')$ be the set of all isomorphisms between T and T'. The distance between T and T', denoted $T \vartriangle T'$, is defined as follows:* $T \vartriangle T' = \min\limits_{\pi\in\Pi(T,T')}\{\max\limits_{x\in V}(|L(x) \vartriangle L'(\pi(x))|) + \max\limits_{\langle x,y\rangle\in E}(|L(\langle x,y\rangle) \vartriangle L'(\langle \pi(x),\pi(y)\rangle)|)\}$

We can check that $\vartriangle$ is a distance over a set of isomorphic trees with the operator $\vartriangle$ defined over two node or edge labels α, α' as follows: $L(\alpha) \vartriangle L'(\alpha') = (L(\alpha) \cup L'(\alpha')) \setminus (L(\alpha) \cap L'(\alpha'))$. Based on this distance, we now define a set of completion trees a revision ontology of an ontology $\mathcal{O}$ by another $\mathcal{O}'$ should admit.

Definition 2 (Revision operation). *Let $\mathcal{O}$ and $\mathcal{O}'$ be two consistent $\mathcal{SHIQ}$ ontologies. A set of tree models $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ of the revision of $\mathcal{O}$ by $\mathcal{O}'$ is defined as follows:*

$$\mathsf{MT}(\mathcal{O}, \mathcal{O}') = \{T \in \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O})) \mid \exists T_0 \in \mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')), \forall T' \in \mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')), T'' \in \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O})) : T \vartriangle T_0 \leq T' \vartriangle T''\}$$

Intuitively, $\mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ includes completion trees from $\mathsf{MT}(\mathcal{O}')$ each node of which is consistently filled by an arbitrary set of concepts imported from $\mathsf{sub}(\mathcal{O})$ such that each axiom of $\mathcal{O}'$ remains satisfied. Among these completion trees, $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ retains only those which are closest to completion trees from $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}'))$ thanks to the operator $T \vartriangle T'$ that characterizes the difference between T and T'. We consider the following example. Let $\mathcal{O} = \{\top \sqsubseteq A \sqcap \exists R.(\neg B) \sqcap \neg B\}$ and $\mathcal{O}' = \{\neg A \sqsubseteq \forall R.B, \neg B \sqsubseteq A \sqcap \forall R.B\}$. By running the algorithm $\mathbb{TA}$ for $\mathcal{O}$, we build the set $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}'))$ which contains a unique tree model T_1 with nodes $\{a, b\}$ and labels $L(a) = \{A, \exists R.(\neg B), \neg B\}, L(b) = \{A, \exists R.(\neg B), \neg B\}, E = \{R(a, b)\}$. In the same way, $\mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ has 4 tree models one of which is T_1' with nodes $\{a', b'\}$ and labels $L(a') = \{A, \exists R.(\neg B), B\}$, $L(b') = \{\neg B, A, \forall R.B\}, \{R(a', b')\}$. According to Definition 2, we have $T_1' \vartriangle T_1 = 2$ that is minimal. Thus, $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ contains a unique tree model T_1'.

We obtain a strong result which states that the all AGM postulates rephrased (Qi *et al.*, 2006) for DL ontologies in our setting hold. This result relies on a total pre-order over a set of all completion trees that can be devised from the distance according to

Definition 1. The main difference between the postulates presented by Qi et al. and those reformulated in our setting is that the set of models $\mathsf{Mod}(\mathcal{O})$ of an ontology $\mathcal{O}$ is replaced with $\mathsf{MT}(\mathcal{O})$. To illustrate this point, we consider a postulate by Qi et al. **(G2)**: *If* $\mathsf{Mod}(\mathcal{O}) \cap \mathsf{Mod}(\mathcal{O}') \neq \emptyset$ *then* $\mathsf{Mod}(\mathcal{O}, \mathcal{O}') = \mathsf{Mod}(\mathcal{O}) \cap \mathsf{Mod}(\mathcal{O}')$; and our corresponding postulate: **(P2)** *If* $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')) \cap \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O})) \neq \emptyset$ *then* $\mathsf{MT}(\mathcal{O}, \mathcal{O}') = \mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')) \cap \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$. A proof of **(P2)** can be obtained straightforwardly from the definition of $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}'))$ and $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$.

By soundness and completeness of the tableau algorithm, we can show that $\mathsf{Mod}(\mathcal{O})$ is semantically equivalent to $\mathsf{MT}(\mathcal{O})$, i.e., $\mathsf{MT}(\mathcal{O}) \models \alpha$ iff $\mathsf{Mod}(\mathcal{O}) \models \alpha$ for some axiom α. Moreover, it holds that $\mathsf{Mod}(\mathcal{O}) \cap \mathsf{Mod}(\mathcal{O}') \neq \emptyset$ iff $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')) \cap \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O})) \neq \emptyset$. Therefore, as **(G2)** our postulate **(P2)** captures the fact that if $\mathcal{O} \cup \mathcal{O}'$ is consistent, then the revision ontology of $\mathcal{O}$ by $\mathcal{O}'$ should admit exactly shared models of $\mathcal{O}$ and $\mathcal{O}'$. Such models are encapsulated in $\mathsf{MT}(\mathcal{O}, \mathsf{sub}(\mathcal{O}')) \cap \mathsf{MT}(\mathcal{O}', \mathsf{sub}(\mathcal{O}))$ by our setting.

Finally, our goal is to build from $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ a revision ontology $\widehat{\mathcal{O}}$ that admits exactly $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ as tree models. However, we can show that there may not exist such an ontology $\widehat{\mathcal{O}}$ by reconsidering the example above with $\mathsf{MT}(\mathcal{O}, \mathcal{O}') = \{T_1'\}$. Assume that there exists an ontology $\widehat{\mathcal{O}}$ with $\mathsf{sub}(\widehat{\mathcal{O}}) = \{A, \neg A, B, \neg B, \exists R.(\neg B), \forall R.B\}$ which admits the unique T_1' as tree model. Due to the specific behavior of the sat-rule with $\mathsf{sub}(\widehat{\mathcal{O}})$, if we apply $\mathbb{TA}$ to $\widehat{\mathcal{O}}$ for building $\mathsf{MT}(\widehat{\mathcal{O}})$, we must obtain T_1 and another tree model T_2' with one node $\{x\}$, $L(x) = \{A, \forall R.B, B\}$, which is a contradiction.

For this reason, we use the notion of maximal approximation (De Giacomo *et al.*, 2007) to define an ontology $\mathcal{O}^*$ which satisfies the following conditions: (i) $\mathcal{O}^*$ is expressible in $\mathcal{SHIQ}$, (ii) it admits tree models in $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$, and (iii) it is a "smallest" ontology admitting $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$. Such an ontology $\mathcal{O}^*$, namely *maximal approximation*, can be built from the node labels of all tree models in $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$.

Definition 3 (Revision ontology). *Let $\mathcal{O}$ and $\mathcal{O}'$ be two consistent $\mathcal{SHIQ}$ ontologies with revision operation $\mathsf{MT}(\mathcal{O}, \mathcal{O}') = \{T_1, \cdots, T_n\}$ where $T_i = \langle V_i, L_i, E_i, \widehat{x_i}\rangle$ for $1 \leq i \leq n$. A revision ontology $\mathcal{O}^* = (\mathcal{T}, \mathcal{R})$ of $\mathcal{O}$ by $\mathcal{O}'$ can be built from completion trees in $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$ as follows: $\mathcal{R}$ includes the role hierarchy of $\mathcal{O}'$ and the one of $\mathcal{O}$; $\mathcal{T}$ contains all axioms of $\mathcal{O}'$ and the following axiom :* $\top \sqsubseteq \bigsqcup\limits_{1\leq i\leq n} (\bigsqcup\limits_{x\in V_i} (\bigsqcap\limits_{C\in L_i(x)} C))$.

Theorem 1. *Let $\mathcal{O}$ and $\mathcal{O}'$ be two consistent $\mathcal{SHIQ}$ ontologies. The revision ontology $\mathcal{O}^*$ of $\mathcal{O}$ by $\mathcal{O}'$ is a maximal approximation from $\mathsf{MT}(\mathcal{O}, \mathcal{O}')$. Additionally, the size of $\mathcal{O}^*$ is bounded by a doubly exponential function in the size of $\mathcal{O}$ and $\mathcal{O}'$.*

Conclusion The main limitation of our approach is to omit individuals in ontologies. However, our approach can be extended in order to deal with individuals by extending the distance defined for completion trees to graphs. Another limitation is that the obtained revision ontology is very large. This exponential blow-up in size arises from doubly exponential size of completion trees. We believe that our procedure can be improved by using a method for compressing completion trees generated from tableau algorithms. Such a method has been proposed by Le Duc et al. (Le Duc *et al.*, 2013).
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