Every Concept Lattice With Hedges Is Isomorphic To Some Generalized Concept Lattice *

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Abstract. We show the relationship between two different types of common platforms of till known fuzzifications of a concept lattice, namely that the notion of a concept lattice with hedges is a special case of our generalized concept lattice.

1 Introduction

There are some approaches to fuzzify (i. e. generalize for fuzzy case too) the classical Ganter-Wille construction of concept lattice. If we omit the (maybe slightly naive) attempt done by Burusco & Fuentes-Gonzalez ([6]), the first approach, which was theoretically and practically well developed, was given by Bělohlávek ([1]) and Pollandt ([11]). It use (L-)fuzzy subsets of objects and (L-)fuzzy subsets of attributes. The another approach, so-called one-sided fuzzy concept lattice was invented independently by Ben Yahia & Jaoua ([5]), by Bělohlávek et al. ([4]) and by the author ([8]). It considers fuzzy subsets of attributes but ordinary/classical/crisp subsets of objects (or vice versa). Because there is no inclusion between these two fuzzy approaches the natural asking for some common platform of both had arose.

We know about two such generalizations. One of them was shown by Bělohlávek et al. ([3] and partially in [2]) and it uses so-called hedges (or truth-stressers) (details below). The second one was given by author ([9]) and its idea is to separate between the ranges of fuzzy sets of objects and fuzzy sets of attributes (again details below). Till now it seemed that these two generalizing approaches are not compatible, but in this paper we try to show that the first is contained in the second.

2 A generalized concept lattice

Let us shortly recall a notion of a generalized concept lattice given by the author. All these results are proven in [9] (and/or in [10]).

Let P be a poset, C and D be complete lattices. Let $\bullet : C \times D \to P$ be monotone and left-continuous in both their arguments, i.e.

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Radim Bělohlávek, Václav Snášel (Eds.): CLA 2005, pp. 1–9, ISBN 80–248–0863–3.

1a) $c_1 \leq c_2$ implies $c_1 \bullet d \leq c_2 \bullet d$ for all $c_1, c_2 \in C$ and $d \in D$.
1b) $d_1 \leq d_2$ implies $c \bullet d_1 \leq c \bullet d_2$ for all $c \in C$ and $d_1, d_2 \in D$.
2a) If $c \bullet d \leq p$ holds for $d \in D$, $p \in P$ and for all $c \in X \subseteq C$, then

$$\sup X \bullet d \leq p.$$

2b) If $c \bullet d \leq p$ holds for $c \in C$, $p \in P$ and for all $d \in Y \subseteq D$, then

$$c \bullet \sup Y \leq p.$$

Let A and B be non-empty sets and let R be P-relation on their Cartesian product, i.e. $R : A \times B \to P$.

Define the following mapping $\nearrow : {}^B D \to {}^A C$ (by ${}^S T$ we understand the set of all mappings from the set S to the set T):

If $g : B \to D$ then $\nearrow(g) : A \to C$ is defined as follows:

$$\nearrow(g)(a) = \sup\{c \in C : (\forall b \in B) c \bullet g(b) \leq R(a, b)\}.$$

Symmetrically we define the mapping $\swarrow : {}^A C \to {}^B D$:

If $f : A \to C$ then $\swarrow(f) : B \to D$ is defined as follows:

$$\swarrow(f)(b) = \sup\{d \in D : (\forall a \in A) f(a) \bullet d \leq R(a, b)\}.$$

Because these two mappings $\swarrow$ and $\nearrow$ form a Galois connection, it can be repeated a classical construction of concept lattice. The result of this construction is called a generalized concept lattice and the following basic theorem on generalized concept lattice holds (the proofs are in [9] and [10]):

Theorem 1. *1) The generalized concept lattice $\mathfrak{L}$ is a complete lattice in which*

$$\bigwedge_{i \in I} \langle g_i, f_i \rangle = \left\langle \bigwedge_{i \in I} g_i, \nearrow\left(\swarrow\left(\bigvee_{i \in I} f_i\right)\right)\right\rangle$$

and

$$\bigvee_{i \in I} \langle g_i, f_i \rangle = \left\langle \swarrow\left(\nearrow\left(\bigvee_{i \in I} g_i\right)\right), \bigwedge_{i \in I} f_i\right\rangle.$$

2) Let moreover P have the least element 0_P and $0_C \bullet d = 0_P$ and $c \bullet 0_D = 0_P$ for every $c \in C$ and $d \in D$. Then a complete lattice V is isomorphic to $\mathfrak{L}$ if and only if there are mappings $\alpha : A \times C \to V$ and $\beta : B \times D \to V$ s.t.
1a) α is non-increasing in the second argument.
1b) β is non-decreasing in the second argument.
2a) $\alpha[A \times C]$ is infimum-dense.
2b) $\beta[B \times D]$ is supremum-dense.
3) For every $a \in A$, $b \in B$, $c \in C$, $d \in D$

$$\alpha(a, c) \geq \beta(b, d) \qquad \textit{if and only if} \qquad c \bullet d \leq R(a, b).$$

This approach is really a generalization of Bělohlávek's fuzzy concept lattice and of one-sided fuzzy concept lattice (and, of course, of a classical crisp case).

3 A concept lattice with hedges

Bělohlávek consider a (complete) residuated lattice $\mathbf{L} = \langle L, \vee, \wedge, \otimes, \rightarrow, 0, 1\rangle$, where $\otimes$ and $\rightarrow$ are connectives on L which form an adjoint pair, i.e. $x \otimes y \leq z$ iff $x \leq y \rightarrow z$. $\otimes$ is isotone in both their arguments, $\rightarrow$ is antitone in the first argument and isotone in the second one, $\otimes$ is commutative and $x \otimes 1 = 1 \otimes x = x$.

Moreover he has sets X and Y and an incidence relation $I : X \times Y \rightarrow L$. Then he defines the mappings $' : L^X \rightarrow L^Y$ and $'' : L^Y \rightarrow L^X$ as follows: If $A \in L^X$ and $B \in L^Y$ then

$$A'(y) = \bigwedge_{x \in X} (A(x) \rightarrow I(x, y))$$

and

$$B''(x) = \bigwedge_{y \in Y} (B(y) \rightarrow I(x, y)).$$

The new idea of Bělohlávek (et al.)'s is to modify these definitions in this way:

$$A^{\uparrow}(y) = \bigwedge_{x \in X} (A(x)^{*_X} \rightarrow I(x, y))$$

and

$$B^{\downarrow}(x) = \bigwedge_{y \in Y} (B(y)^{*_Y} \rightarrow I(x, y)),$$

where $*_X$ and $*_Y$ are so-called hedges on L. A hedge is a function $*$ on L which fulfills these properties:

$$1^* = 1,$$

$$a^* \leq a,$$

$$(a \rightarrow b)^* \leq a^* \rightarrow b^*,$$

$$a^{**} = a^*.$$

(Note that the last one can be rewrite in the form

$$* \circ * = *$$

where $\circ$ is the composition of mappings.) Moreover they work with the following functions:

- For arbitrary $A : U \rightarrow L$ (U is some universe) define:

$$\lfloor A \rfloor = \{\langle u, a\rangle \in U \times L : a \leq A(u)\}.$$

- For arbitrary $B \subseteq U \times L$ take the function $\lceil B \rceil : U \rightarrow L$ defined by:

$$\lceil B \rceil(u) = \bigvee\{a \in L : \langle u, a\rangle \in B\}.$$

– For arbitrary $A : U \to L$ and $* : L \to L$ define the function A^* given pointwise by:
$$A^*(u) = (A(u))^*.$$
– For arbitrary $B \subseteq U \times L$ and $* : L \to L$ define:
$$B^* = \{\langle x, a^*\rangle : \langle x, a\rangle \in B\}.$$

Then they have taken the set
$$\mathcal{B}(X^{*_X}, Y^{*_Y}, I) = \{\langle A, B\rangle : A^{\Uparrow} = B, B^{\Downarrow} = A\}$$
of all fixpoints of the pair $\langle^{\Uparrow}, {}^{\Downarrow}\rangle$ and showed that this structure is isomorphic to the ordinary concept lattice $\mathcal{B}(X \times *_X[L], Y \times *_Y[L], I_{\langle \curlywedge, \curlyvee\rangle})$ where
$$A^{\curlyvee} = \lfloor \lceil A\rceil^{\Uparrow} \rfloor^{*_X}$$
and
$$B^{\curlywedge} = \lfloor \lceil B\rceil^{\Downarrow} \rfloor^{*_Y},$$
and relation $I_{\langle \curlywedge, \curlyvee\rangle}$ is given by
$$\langle\langle x, a\rangle, \langle y, b\rangle\rangle \in I_{\langle \curlywedge, \curlyvee\rangle} \qquad \text{iff} \qquad a \otimes b \leq I(x, y).$$

Finally, they have proven this basic theorem for concept lattice with hedges (we present here only its second part):

Theorem 2. *An arbitrary complete lattice $\langle V, \leq\rangle$ is isomorphic to the complete lattice $\mathcal{B}(X^{*_X}, Y^{*_Y}, I)$ iff there are mappings $\gamma : X \times *_X[L] \to V$ and $\mu : Y \times *_Y[L] \to V$ s. t.*

1a) $\mu[Y \times *_Y[L]]$ is infimum-dense.
1b) $\gamma[X \times *_X[L]]$ is supremum-dense.
2)
$$\gamma(x, a) \leq \mu(y, b) \qquad \textit{if and only if} \qquad a \otimes b \leq I(x, y).$$

4 A few observations

We add some small, but useful assertions about these notions:

Lemma 1. *Every hedge is monotonous, i. e. $a \leq b$ implies $a^* \leq b^*$.*

Proof. By properties of a hedge we obtain: $a \leq b$ iff $1 \otimes a \leq b$ iff $1 \leq a \to b$ iff $1 = a \to b$, which implies $1 = (a \to b)^* \leq a^* \to b^*$, and then $a^* = 1 \otimes a^* \leq b^*$.

Lemma 2. *For every hedge $*$ if $A \subseteq *[L]$ then* $\sup A \in *[L]$.

Proof. For every $a \in A$ we know $a \leq \sup A$, and then (from the previous lemma) $a = a^* = (\sup A)^*$. It means that $(\sup A)^*$ is an upper bound of A, which implies $\sup A \leq (\sup A)^*$. But it follows $\sup A = (\sup A)^*$, i. e. $\sup A \in *[L]$.

Lemma 3. *The function* $\lfloor\cdot\rfloor$ *is monotonous.*

Proof. If $A_1, A_2 : U \to L$ and $A_1 \leq A_2$, then obviously $\lfloor A_1 \rfloor = \{\langle u, a\rangle \in U \times L : a \leq A_1(u)\} \subseteq \{\langle u, a\rangle \in U \times L : a \leq A_2(u)\} = \lfloor A_2 \rfloor$.

Lemma 4. *The function* $\lceil\cdot\rceil$ *is monotonous.*

Proof. If $B_1 \subseteq B_2 \subseteq U \times L$, then for all $u \in U$ we have clearly $\lceil B_1\rceil(u) = \bigvee\{a \in L : \langle u, a\rangle \in B_1\} = \bigvee\{a \in L : \langle u, a\rangle \in B_2\} = \lceil B_2\rceil(u)$.

Lemma 5. *For arbitrary* $A : U \to L$ *it is true that* $\lceil\lfloor A\rfloor\rceil = A$.

Proof. $\lceil\lfloor A\rfloor\rceil(u) = \bigvee\{a \in L : \langle u, a\rangle \in \lfloor A\rfloor\} = \bigvee\{a \in L : a \leq A(u)\} = A(u)$.

5 Relationship between these two approaches

We can see that the basic theorem for concept lattice with hedges and the basic theorem for our generalized concept lattice are very similar. And it is suspicious. So take such special case of our generalized concept lattice given by the following table:

general	special
P	L
A	Y
B	X
C	$*_Y[L]$
D	$*_X[L]$
$\bullet$	$\otimes$
$R : A \times B \to P$	$I : X \times Y \to L$

Then our definitions can be rewritten in this form:
If $g : X \to *_X[L]$ then $\nearrow(g) : Y \to *_Y[L]$ is defined as follows:

$$\nearrow(g)(y) = \sup\{c \in *_Y[L] : (\forall x \in X) c \otimes g(x) \leq I(x, y)\},$$

and if $f : Y \to *_Y[L]$ then $\swarrow(f) : X \to *_X[L]$ is defined by:

$$\swarrow(f)(x) = \sup\{d \in *_X[L] : (\forall y \in Y) f(y) \otimes d \leq I(x, y)\}.$$

And now we will try to prove that such special case of generalized concept lattice is isomorphic to the lattice from [3]:

Theorem 3. *The lattices* $\mathfrak{L} = \mathfrak{L}({}^X(*_X[L]), {}^Y(*_Y[L]), I)$ *and* $\mathcal{B} = \mathcal{B}(X \times *_X[L], Y \times *_Y[L], I_{\langle\curlywedge,\curlyvee\rangle})$ *are isomorphic and the isomorphisms are*

$$\Phi(\langle g, f\rangle) = \langle\lfloor g\rfloor, \lfloor f\rfloor\rangle$$

and

$$\Psi(\langle S, T\rangle) = \langle\lceil S\rceil, \lceil T\rceil\rangle,$$

where $g : X \to *_X[L]$, $f : Y \to *_Y[L]$, $S \subseteq X \times *_X[L]$, $T \subseteq Y \times *_Y[L]$.

It is enough to prove the following six claims:

Claim 1 *If $\langle g, f\rangle \in \mathfrak{L}$ then $\Psi(\langle g, f\rangle) \in \mathcal{B}$.*

Proof. Let $\langle g, f\rangle \in \mathfrak{L}$, i. e. $g = \swarrow(f)$ and $f = \nearrow(g)$, we want to prove $\lfloor g\rfloor = \lfloor f\rfloor^{\curlyvee} = \lfloor\lceil\lfloor f\rfloor\rceil^{\downarrow}\rfloor^{*} = \lfloor f^{\downarrow}\rfloor^{*}$ (because from the lemma 5 we have $\lceil\lfloor f\rfloor\rceil = f$) and $\lfloor f\rfloor = \lfloor g\rfloor^{\curlywedge} = \lfloor\lceil\lfloor g\rfloor\rceil^{\uparrow}\rfloor^{*} = \lfloor g^{\uparrow}\rfloor^{*}$ (because again $\lceil\lfloor g\rfloor\rceil = g$), which will mean $\Phi(\langle g, f\rangle) = \langle\lfloor g\rfloor, \lfloor f\rfloor\rangle \in \mathcal{B}$, what we want to show.

By above definitions we obtain:

$\lfloor f^{\downarrow}\rfloor^{*}$
$= \{\langle x, d^{*_X}\rangle \in X \times *_X[L] : \langle x, d\rangle \in \lfloor f^{\downarrow}\rfloor\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : \langle x, d^{*_X}\rangle \in \lfloor f^{\downarrow}\rfloor\}$, (because $*_X \circ *_X = *_X$, we have $d^{*_X} = d$ for all $d \in *_X[L]$),
$= \{\langle x, d\rangle \in X \times *_X[L] : d \leq f^{\downarrow}(x)\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : d \leq \bigwedge_{y\in Y}(f(y)^{*_Y} \to I(x, y))\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : (\forall y \in Y)(f(y)^{*_Y} \otimes d \leq I(x, y))\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : (\forall y \in Y)(f(y) \otimes d \leq I(x, y))\}$ (because $*_Y \circ *_Y = *_Y$, we have $c^{*_Y} = c$ for all $c \in *_Y[L]$, especially $f(y) \in *_Y[L]$),
$= \{\langle x, d\rangle \in X \times *_X[L] : d \leq \sup\{e \in *_X[L] : (\forall y \in Y) f(y) \otimes e \leq I(x, y)\}\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : d \leq \swarrow(f)(x)\}$,
$= \{\langle x, d\rangle \in X \times *_X[L] : d \leq g(x)\}$,
$= \lfloor g\rfloor$,

So we have $\lfloor f^{\downarrow}\rfloor^{*} = \lfloor g\rfloor$, and the equality $\lfloor g^{\uparrow}\rfloor^{*} = \lfloor f\rfloor$ can be proven symmetrically.

Claim 2 *If $\langle S, T\rangle \in \mathcal{B}$ then $\Phi(\langle S, T\rangle) \in \mathfrak{L}$.*

Proof. Let $\langle S, T\rangle \in \mathcal{B}$, i. e. $S = T^{\curlywedge} = \lfloor\lceil T\rceil^{\downarrow}\rfloor^{*}$ and $T = S^{\curlyvee} = \lfloor\lceil S\rceil^{\uparrow}\rfloor^{*}$, we want to prove $\nearrow(\lceil S\rceil) = \lceil T\rceil$ and $\swarrow(\lceil T\rceil) = \lceil S\rceil$ which will mean $\Psi(\langle S, T\rangle) = \langle\lceil S\rceil, \lceil T\rceil\rangle \in \mathfrak{L}$, what we want to show.

Firstly we show that for all $y \in Y$ we have $\lceil T\rceil(y) \in *^{Y}[L]$:

$\lceil T\rceil(y)$
$= \lceil\lfloor\lceil S\rceil^{\uparrow}\rfloor^{*}\rceil(y)$,
$= \bigvee\{a \in L : \langle y, a\rangle \in \lfloor\lceil S\rceil^{\uparrow}\rfloor^{*}\}$,
$= \bigvee\{a \in *^{Y}[L] : \langle y, a\rangle \in \lfloor\lceil S\rceil^{\uparrow}\rfloor^{*}\}$,
$\in *^{Y}[L]$ (because of lemma 2).

Hence by above definitions we obtain for every $x \in X$:

$\swarrow(\lceil T\rceil)(x)$
$= \sup\{d \in *_X[L] : (\forall y \in Y)\lceil T\rceil(y) \otimes d \leq I(x, y)\}$,
$= \sup\{d \in *_X[L] : (\forall y \in Y)\lceil T\rceil(y)^{*_Y} \otimes d \leq I(x, y)\}$ (because $*_Y \circ *_Y = *_Y$, we have $c^{*_Y} = c$ for all $c \in *_Y[L]$, especially for $\lceil T\rceil(y)$ as we have shown above),
$= \sup\{d \in *_X[L] : d \leq \bigwedge_{y\in Y}(\lceil T\rceil(y)^{*_Y} \to I(x, y))\}$,

$= \sup\{d \in *_X[L] : d \leq \lceil T \rceil^{\downarrow}(x)\}$,
$= \sup\{d \in *_X[L] : \langle x, d\rangle \in \lfloor \lceil T \rceil^{\downarrow} \rfloor\}$,
$= \sup\{d \in *_X[L] : \langle x, d^{*_X}\rangle \in \lfloor \lceil T \rceil^{\downarrow} \rfloor^*\}$,
$= \sup\{d \in *_X[L] : \langle x, d\rangle \in \lfloor \lceil T \rceil^{\downarrow} \rfloor^*\}$ (because $*_X \circ *_X = *_X$, we have $d^{*_X} = d$ for all $d \in *_X[L]$),
$= \sup\{d \in L : \langle x, d\rangle \in \lfloor \lceil T \rceil^{\downarrow} \rfloor^*\}$ (the (stronger) condition $d \in *_X[L]$ is covered by the condition $\langle x, d\rangle \in \lfloor \lceil T \rceil^{\downarrow} \rfloor^*$),
$= \sup\{d \in L : \langle x, d\rangle \in S\}$,
$= \bigvee\{d \in L : \langle x, d\rangle \in S\}$,
$= \lceil S \rceil(x)$.

So we have $\swarrow(\lceil T \rceil) = \lceil S \rceil$, and the equality $\nearrow(\lceil S \rceil) = \lceil T \rceil$ can be proven symmetrically.

Claim 3 *If* $\langle g, f\rangle \in \mathfrak{L}$ *then*

$$\Phi(\Psi(\langle g, f\rangle)) = \langle g, f\rangle.$$

Proof. $\Phi(\Psi(\langle g, f\rangle)) = \langle \lceil \lfloor g \rfloor \rceil, \lceil \lfloor f \rfloor \rceil\rangle = \langle g, f\rangle$ because of lemma 5. (You can see that assumption $\langle g, f\rangle \in \mathfrak{L}$ is only formal, we do not need it.)

Claim 4 *If* $\langle S, T\rangle \in \mathcal{B}$ *then*

$$\Psi(\Phi(\langle S, T\rangle)) = \langle S, T\rangle.$$

Proof. By definition we obtain $\Psi(\Phi(\langle S, T\rangle)) = \langle \lfloor \lceil S \rceil \rfloor, \lfloor \lceil T \rceil \rfloor\rangle$, so we want to prove that $\lfloor \lceil S \rceil \rfloor = S$ and $\lfloor \lceil T \rceil \rfloor = T$.

For one inclusion we will need the following observation: Because $\langle S, T\rangle \in \mathcal{B}$, we have $S = T^{\curlywedge} = \lfloor \lceil T \rceil^{\downarrow} \rfloor^*$, i. e. $S = \lfloor g \rfloor^*$ for some $g : X \to *_X[L]$.

Using definitions we obtain:

$\langle x, d\rangle \in \lfloor \lceil S \rceil \rfloor$
iff $d \leq \lceil S \rceil(x)$,
iff $d \leq \bigvee\{e \in L : \langle x, e\rangle \in S\}$,
iff $d \leq \bigvee\{e \in L : \langle x, e\rangle \in \lfloor g \rfloor^*\}$,
iff $d \leq \bigvee\{e \in *_X[L] : \langle x, e^{*_X}\rangle \in \lfloor g \rfloor^*\}$ (because if $\langle x, e\rangle \in \lfloor g \rfloor^*$ then $e \in *_X[L]$ and $e = e^{*_X}$),
iff $d \leq \bigvee\{e \in *_X[L] : \langle x, e\rangle \in \lfloor g \rfloor\}$,
iff $d \leq \bigvee\{e \in *_X[L] : e \leq g(x)\} = g(x)$,
iff $\langle x, d\rangle \in \lfloor g \rfloor$ and $d \in *_X[L]$,
iff $\langle x, d^{*_X}\rangle \in \lfloor g \rfloor^*$ and $d \in *_X[L]$,
iff $\langle x, d\rangle \in \lfloor g \rfloor^*$ (because $*_X \circ *_X = *_X$, so $d = d^{*_X}$ for all $d \in *_X[L]$),
iff $\langle x, d\rangle \in S$.

So we have $\lfloor \lceil S \rceil \rfloor = S$, and the second equality $\lfloor \lceil T \rceil \rfloor = T$ can be proven symmetrically.

Claim 5 Φ *is order-preserving.*

Proof. Let $\langle g_1, f_1\rangle, \langle g_2, f_2\rangle \in \mathfrak{L}$ and $\langle g_1, f_1\rangle \leq \langle g_2, f_2\rangle$. This inequality means that $g_1 \leq g_2$ and $f_1 \geq f_2$ (pointwise) and by monotonicity of $\lfloor\cdot\rfloor$ we obtain $\lfloor g_1\rfloor \subseteq \lfloor g_2\rfloor$ and $\lfloor f_1\rfloor \supseteq \lfloor f_2\rfloor$, which implies $\Phi(\langle g_1, f_1\rangle) = \langle\lfloor g_1\rfloor, \lfloor f_1\rfloor\rangle \leq \langle\lfloor g_2\rfloor, \lfloor f_2\rfloor\rangle = \Phi(\langle g_2, f_2\rangle)$.

Claim 6 *Ψ is order-preserving.*

Proof. Let $\langle S_1, T_1\rangle, \langle S_2, T_2\rangle \in \mathcal{B}$ and $\langle S_1, T_1\rangle \leq \langle S_2, T_2\rangle$. This inequality means that $S_1 \subseteq S_2$ and $T_1 \supseteq T_2$ and by monotonicity of $\lceil\cdot\rceil$ we obtain $\lceil S_1\rceil \leq \lceil S_2\rceil$ and $\lceil T_1\rceil \geq \lceil T_2\rceil$, which implies $\Psi(\langle S_1, T_1\rangle) = \langle\lceil S_1\rceil, \lceil T_1\rceil\rangle \leq \langle\lceil S_2\rceil, \lceil T_2\rceil\rangle = \Psi(\langle S_2, T_2\rangle)$.

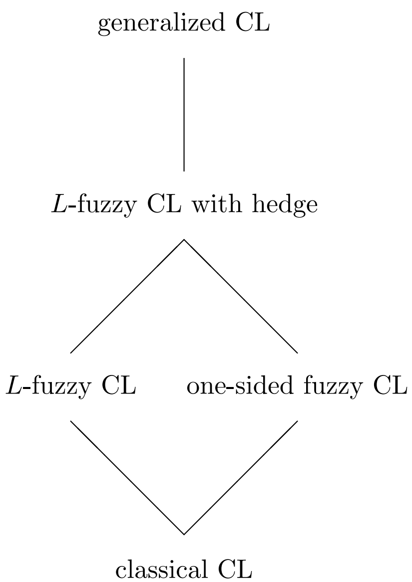
Now we have proven that the lattices $\mathcal{B}$ and $\mathfrak{L}$ are isomorphic. Hence we have this:

Corrolary 1 *The lattices $\mathfrak{L}({}^X(*_X[L]), {}^Y(*_Y[L]), I)$ and $\mathcal{B}(X^{*_X}, Y^{*_Y}, I)$ are isomorphic.*

Or we can say it in another words, that *every concept lattice with hedges is isomorphic to some generalized concept lattice.*

6 Conclusions

In this paper we showed that the notion of a generalized concept lattice defined in [10] contains as a part the notion of a fuzzy concept lattice with hedges. Hence relationships between all classes of concept lattices mentioned in Introduction can be depicted in this diagram:



The inverse question arises how big is distinction between these classes, e. g. what additional conditions are needed for a generalized concept lattices to be isomorphic to some fuzzy concept lattice with hedges. Or are these notions the same?...

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