An Analytic Model of the Impact of Skeptical Agents on the Dynamics of Compromise

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***Abstract*—This paper studies analytically the dynamics of the opinion in multi-agent systems where two classes of agents coexist. The fact that the population of considered multi-agent systems is divided into two classes is meant to account for agents with different propensity to change their opinions. Skeptical agents, which are agents that are not inclined to change their opinions, can be modeled together with moderate agents, which are agents that are moderately open to take into account the opinions of the others. The studied analytic model of opinion dynamics involves only compromise, which describes how agents change their opinions to try to reach consensus. The adopted model of compromise is stochastic in order to give agents some level of autonomy in their decisions. Presented results show, analytically, that after a sufficient number of interactions consensus is reached, regardless of the initial distribution of the opinion. Analytic results are confirmed by simulations shown in the last part of the paper.**

I. Introduction

This paper discusses an analytic framework that can be used to study collective and asymptotic properties of multi-agent systems. The properties of studied multi-agent systems evolve because of binary interactions among agents, where the term (binary) interaction is used here to denote a message exchange among two agents. Each interaction identifies a single step of the evolution of the systems, regardless of how often interactions occur. Note that studied multi-agent systems are completely decentralized and that they involve no form of supervised coordination. In particular, in this paper we assume that each agent is associated with a scalar property, which changes because of interactions with other agents, and we assume that such a property represents the opinion of the agent on a given topic. Notably, we remark that the proposed approach can be used to study other collective and asymptotic properties of multi-agent systems, and that it is not limited to the study of the opinion, even if the development of specific analytic results is needed if different properties are considered.

We start by considering proper rules that describe the effects of interactions among agents on their opinions at a microscopic level. Then, the dynamics of macroscopic properties of the considered multi-agent system are analytically derived, taking inspiration from physical models. In detail, the proposed framework is related to the kinetic theory of gases, a branch of physics which studies the temporal evolution of macroscopic properties of gases starting from the description of microscopic collisions among molecules. The idea of generalizing the framework of kinetic theory of gases to study social phenomena relies on a proper parallelism between *molecules and their collisions* in gases, and *agents and their interactions* in multi-agent systems. Such an idea is not new and it recently gave birth to a discipline called *sociophysics* (see, e.g., [1]). Note that the details of collisions among molecules are different from the details of interactions among agents and, hence, analytic results obtained at the macroscopic level for the dynamics of the properties of multi-agent systems are significantly different from those of kinetic theory of gases. For the specific case of the study of opinion dynamics, the major advantage of the proposed approach consists in the derivation of analytic results, in contrast with simulation results that are typically studied in the literature on the subject. While the validity of simulations depends on the specific tool adopted, and on the choice of the parameters of simulated models, analytic results are valid as long as the hypothesis used to derive them are valid.

We have already applied the proposed approach to the study of various models of opinion dynamics (see, [2]–[9]), and analytic results were always confirmed by independent simulations. In this paper, we enrich previous studies on the subject by considering multi-agent systems where two types of agents, grouped into two disjoint *classes*, coexist. Each class is associated with different parameters, such as a different number of agents, a different initial distribution of the opinion, and a different inclination of contained agents at changing their opinions. Results concerning a similar model, where only deterministic interactions among agents are considered, have already been presented in [10]. Here, we extend those results by adding stochastic parameters in order to describe the behavior of agents that exhibit some level of autonomy. Moreover, we concentrate on the impact of the presence of skeptical agents in the multi-agent system. Note that, in order to account for the existence of two classes of agents, we take inspiration from kinetic theory of gas mixtures, which is used to study gases composed of different types of molecules.

This paper is organized as follows. Section II describes the details of the proposed kinetic framework. Section III derives relevant analytic results concerning the average opinion in the multi-agent system. Section IV shows simulation results that are used to confirm analytic results. Finally, Section V concludes the paper.

II. A Kinetic Framework of Opinion Dynamics

In kinetic theory of gases, each molecule of a gas is associated with specific physical quantities, such as its position and its velocity, and the macroscopic features of gases, concerning, for example, its temperature and pressure, are derived on the basis of a proper balance equation known as *Boltzmann equation*. Similarly, in the context of opinion dynamics, we assume that each agent is associated with a single scalar value which models its opinion, and macroscopic characteristics of the considered multi-agent system, such as the average opinion and the standard deviation of the opinion, are studied analytically using a proper balance equation, whose formulation is inspired by the Boltzmann equation.

Opinion is modeled as a continuous variable v, which is defined in a closed interval I_v. Without loss of generality [11], interval I_v is typically set to

$$I_v = [-1, 1] \tag{1}$$

where values close to 0 represent moderate opinions while values close to -1 or to $+1$ correspond to extremal opinions.

The first step to derive analytic results consists in the definition of the microscopic rules that govern the effects of an interaction among two agents. As detailed in the introduction, we assume that two classes of agents characterize the considered multi-agent system. Let us consider an interaction between an agent of class $s \in \{1, 2\}$ and an agent of class $r \in \{1, 2\}$. Denoting as v the pre-interaction opinion of the agent of class s, and as w the pre-interaction opinion of the agent of class r, we assume that the post-interaction opinions of the two interacting agents can be computed according to the following rules

$$\begin{cases} v^* = v - \Gamma_{sr}(v - w) \\ w^* = w - \Gamma_{rs}(w - v) \end{cases} \tag{2}$$

where v^* and w^* are the opinions of the two agents after the interaction, and $\{\Gamma_{sr}\}_{s,r=1}^{2}$ are four mutually independent random variables.

Assuming that the support of random variables $\{\Gamma_{sr}\}_{s,r=1}^{2}$ is a subset of $(0, 1)$, then it is guaranteed that the post-interaction opinions v^* and w^* still belong to I_v. In addition, this choice of the support of random variables allows using interaction rules (2) to model *compromise*, which is the sociological phenomenon that describes the tendency of agents to change their opinions towards those of the agents they interact with. In fact, considering, for instance, the first rule, it can be observed that if the value of Γ_{sr} is close to 0, then the post-interaction opinion v^* of the first agent is close to its pre-interaction opinion v. At the opposite, if the value of Γ_{sr} is close to 1, then the post-interaction opinion v^* of the first agent is close to the pre-interaction opinion w of the second agent. Hence, it can be concluded that Γ_{sr} measures the propensity of an agent of type s to change its opinion in favor of that of an agent of type r after an interaction. In addition, in order to properly model the sociological characteristics of compromise, we also aim at reproducing the fact that the post-interaction opinion of an agent is closer to its pre-interaction opinion than to the pre-interaction opinion of the other agent. This phenomenon can be reproduced in terms of the following inequalities

$$\begin{aligned} |v^* - v| &< |v^* - w| \\ |w^* - w| &< |w^* - v|. \end{aligned} \tag{3}$$

Simple algebraic manipulations show that a sufficient condition for (3) to hold is that the support of random variables $\{\Gamma_{sr}\}_{s,r=1}^{2}$ is restricted to a subset of

$$I_\Gamma = \left(0, \frac{1}{2}\right). \tag{4}$$

This is the reason why, in the remaining of this paper, we assume that the supports of random variables $\{\Gamma_{sr}\}_{s,r=1}^{2}$ are subsets of I_Γ.

The expected value of the sum of post-interaction opinions of two interacting agents can be computed from (2) as

$$\mathbb{E}[v^* + w^*] = v + w + (\overline{\Gamma}_{rs} - \overline{\Gamma}_{sr})(v - w) \tag{5}$$

where $\overline{\Gamma}_{sr}$ denotes the average value of random variable Γ_{sr}, and $\overline{\Gamma}_{rs}$ is the average value of random variable Γ_{rs}. Observe that (5) shows that the opinion is not conserved through single interactions. Actually, the average value of the sum of the opinions of two interacting agents can increase or decrease depending on the sign of $(\overline{\Gamma}_{rs} - \overline{\Gamma}_{sr})(v - w)$. Note that if the two random variables Γ_{sr} and Γ_{rs} have the same average value, then opinion is conserved, on average, after single interactions. Similarly, the difference between post-interaction opinions of two interacting agents is

$$v^* - w^* = \varepsilon_{rs}(v - w). \tag{6}$$

where $\varepsilon_{rs} = 1 - (\overline{\Gamma}_{rs} + \overline{\Gamma}_{sr})$ and, according to (4), $\varepsilon_{rs} \in (0, 1)$. From (6) it can than be concluded that the difference between post-interaction opinions is smaller than the difference between pre-interaction opinions. According to these considerations, it is reasonable to expect that, after a sufficiently large number of interactions, all agents would eventually end up with the same opinion, regardless of their classes.

We now describe the kinetic framework which allows deriving analytic results starting from microscopic rules (2). As in kinetic theory of gas mixtures, we need a *distribution function* $f_s(v, t)$ for each class $s \in \{1, 2\}$, where $f_s(v, t)\mathrm{d}v$ represents the number of agents of class s with opinion in $(v, v + \mathrm{d}v)$. Observe that, using this notation, the number of agents of class s at time t, denoted as $n_s(t)$, can be computed as

$$n_s(t) = \int_{I_v} f_s(v, t)\mathrm{d}v \qquad s \in \{1, 2\}. \tag{7}$$

We also denote the total number of agents at time t as $n(t)$, which can be computed as

$$n(t) = n_1(t) + n_2(t). \tag{8}$$

In analogy with the kinetic theory of gas mixtures, the temporal evolution of each distribution function is described by

$$\frac{\partial f_s}{\partial t}(v,t) = \mathcal{I}_s \qquad s \in \{1,2\} \tag{9}$$

where $\mathcal{I}_s$ can be computed as

$$\mathcal{I}_s = \sum_{r=1}^{2} \mathcal{Q}_{sr}(f_s, f_r) \qquad s \in \{1,2\}, \tag{10}$$

and $\mathcal{Q}_{sr}(f_s, f_r)$ is a proper operator that depends on distribution functions $f_s(v,t)$ and $f_r(v,t)$. Equation (9) is a balance equation that plays in the described framework the same role that the Boltzmann equation plays in kinetic theory of gas mixtures. For this reason, we use the same nomenclature and we call such an equation *Boltzmann equation*, and its right-hand size *collisional operator* relative to class s. However, note that the explicit expression of the collisional operator in kinetic theory of gas mixtures depends on the details of interactions among molecules, which are different from the details of interactions among agents, expressed in (2). Hence, the explicit expression of the collisional operator (10) is different from that of kinetic theory, leading to different developments of analytic results.

In order to obtain analytic results, the *weak form* of the Boltzmann equation needs to be considered. Just like in kinetic theory of gas mixtures, the weak form of the Boltzmann equation is obtained by multiplying (9) by a test function $\phi(v)$, a smooth function with compact support, and by integrating the result with respect to v (see, e.g., [12]). Hence, the weak form of Boltzmann equation (9) is

$$\int_{I_v} \frac{\partial f_s}{\partial t} \phi(v) \mathrm{d}v = \sum_{r=1}^{2} \int_{I_v} \mathcal{Q}_{sr}(f_s, f_r) \phi(v) \mathrm{d}v. \tag{11}$$

By generalizing the results in [1] and in [10], the right-hand side of (11) can be rewritten as

$$\begin{aligned} \sum_{r=1}^{2} \int_{I_v} \int_{I_v} \int_{B_{sr}} \int_{B_{rs}} & \beta \Theta_{sr}(\Gamma_{sr}) \Theta_{rs}(\Gamma_{rs}) \\ & \cdot f_s(v) f_r(w) (\phi(v^*(v,w)) - \phi(v)) \mathrm{d}v \mathrm{d}w \mathrm{d}\Gamma_{rs} \mathrm{d}\Gamma_{sr} \end{aligned} \tag{12}$$

where

1) $\Theta_{sr}(\cdot)$ and $\Theta_{rs}(\cdot)$ are the distributions of random variables Γ_{sr} and Γ_{rs};
2) B_{sr} and B_{rs} are the support of Γ_{sr} and Γ_{rs}, respectively; and
3) β is the probability that two agents interact.

Note that using the fact that the integral with respect to v and the derivative with respect to t commute, the left-hand side of (11) can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{I_v} f_s(v,t) \phi(v) \mathrm{d}v. \tag{13}$$

Proper choices of the test function $\phi(v)$ can be used to study macroscopic properties of the system, as shown in the following section.

III. Analytic Study of Macroscopic Properties

In this section, we show how the weak form of the Boltzmann equation can be used to derive collective and asymptotic properties of considered multi-agent systems. First, we set $\phi(v) = 1$, so that the weak form of the Boltzmann equation with respect to such a test function becomes

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{I_v} f_s(v,t) \mathrm{d}v = 0 \qquad s \in \{1,2\}. \tag{14}$$

Recalling (7), it is possible to observe that the left-hand side of (14) represents the time derivative of the number of agents of class s. Hence, it can be concluded that the number of agents of any class $s \in \{1,2\}$ is constant, and, therefore, also the total number of agents in the system is constant.

Let us now consider the test function $\phi(v) = v$ in (12), which leads to simplify the right-hand side of (12) as

$$\begin{aligned} \sum_{r=1}^{2} \int_{I_v} \int_{I_v} \int_{B_{sr}} \int_{B_{rs}} & \beta \Theta_{sr}(\Gamma_{sr}) \Theta_{rs}(\Gamma_{rs}) \\ & \cdot f_s(v) f_r(w) \Gamma_{sr}(w - v) \mathrm{d}v \mathrm{d}w \mathrm{d}\Gamma_{rs} \mathrm{d}\Gamma_{sr}. \end{aligned} \tag{15}$$

Observe that (15) can be used to study the temporal evolution of the average opinion of agents of class s. Actually, recalling the definition of the distribution function $f_s(v,t)$, the average opinion of agents of class s at time t can be computed as

$$u_s(t) = \frac{1}{n_s} \int_{I_v} f_s(v,t) v \mathrm{d}v \qquad s \in \{1,2\}. \tag{16}$$

Note that the average opinion of the entire multi-agent system is computed as the following weighed sum of average opinions

$$u(t) = \frac{1}{n} \left(n_1 u_1(t) + n_2 u_2(t) \right). \tag{17}$$

Using (15) and (16), we can write

$$n_s \frac{\mathrm{d}}{\mathrm{d}t} u_s(t) = \beta \sum_{r=1}^{2} \overline{\Gamma}_{sr} \int_{{I_v}^2} f_s(v) f_r(w)(w - v) \mathrm{d}v \mathrm{d}w. \tag{18}$$

From (18), it is evident that also its right-hand side can be expressed in terms of average opinions. Simple algebraic manipulations show that the following equalities hold

$$\frac{\mathrm{d}}{\mathrm{d}t} u_s(t) = \beta \sum_{r=1}^{2} \overline{\Gamma}_{sr} n_r \left(u_r(t) - u_s(t) \right) \qquad s \in \{1,2\}. \tag{19}$$

The two equations (19) form a homogeneous system of first-order linear differential equations, which can be solved analytically. In order to simplify notation, let us introduce the following parameters

$$a_1 = \beta \overline{\Gamma}_{12} n_2 \qquad a_2 = \beta \overline{\Gamma}_{21} n_1. \tag{20}$$

Then, (19) can be written as

$$\begin{cases} \dot{u}_1(t) = -a_1(u_1(t) - u_2(t)) \\ \dot{u}_2(t) = a_2(u_1(t) - u_2(t)). \end{cases} \tag{21}$$

Introducing the auxiliary function

$$x(t) = u_1(t) - u_2(t) \tag{22}$$

and subtracting the second equation from the first equation in (21), we obtain the following differential equation

$$\dot{x}(t) = -(a_1 + a_2)x(t) \tag{23}$$

whose solution is

$$x(t) = C\mathrm{e}^{-(a_1+a_2)t} \tag{24}$$

where C is a constant. Recalling (22), the following relation between $u_1(t)$ and $u_2(t)$ can be found

$$u_1(t) = u_2(t) + C\mathrm{e}^{-(a_1+a_2)t}. \tag{25}$$

Using this result in the second equation of system (21), the following differential equation for $u_2(t)$ is found

$$\dot{u}_2(t) = Ca_2\mathrm{e}^{-(a_1+a_2)t} \tag{26}$$

and, hence, $u_2(t)$ can be expressed as

$$u_2(t) = -C\frac{a_2}{a_1+a_2}\mathrm{e}^{-(a_1+a_2)t} + K, \tag{27}$$

where K is a constant. Finally, inserting the expression of $u_2(t)$ in (25), we obtain

$$u_1(t) = C\frac{a_1}{a_1+a_2}\mathrm{e}^{-(a_1+a_2)t} + K \tag{28}$$

where K is the same constant used in (27). The two constants C and K can be found by imposing that initial conditions are satisfied. Simple algebraic manipulations show that

$$\begin{aligned} C &= u_1(0) - u_2(0) \\ K &= u_1(0)\frac{a_2}{a_1+a_2} + u_2(0)\frac{a_1}{a_1+a_2} \end{aligned} \tag{29}$$

where $\{u_s(0)\}_{s=1}^2$ are the initial average values of the opinions of the two classes of agents.

Therefore, it can be concluded that the solution of (21) can be expressed in closed form as

$$\begin{cases} u_1(t) = C\dfrac{a_1}{a_1+a_2}\mathrm{e}^{-(a_1+a_2)t} + K \\ u_2(t) = -C\dfrac{a_2}{a_1+a_2}\mathrm{e}^{-(a_1+a_2)t} + K \end{cases} \tag{30}$$

where C and K are computed in (29) using the initial distributions of the opinion of the two classes of agents. From (30), it can be observed that since $a_1 > 0$ and $a_2 > 0$, the following equalities hold

$$\lim_{t\to+\infty} u_1(t) = \lim_{t\to+\infty} u_2(t) = K. \tag{31}$$

Observe that, according to (20) and (29), K depends on the average initial opinions $\{u_s(0)\}_{s=1}^2$, on the number of agents $\{n_s\}_{s=1}^2$ in each class, and on $\bar{\Gamma}_{12}$ and $\bar{\Gamma}_{21}$.

TABLE I
THE CONSIDERED VALUES OF THE PARAMETERS FOR THE TWO CLASSES OF AGENTS IN SIMULATIONS: NUMBER OF AGENTS, n_1 AND n_2; INITIAL DISTRIBUTIONS OF THE OPINION, $f_1(v,0)$ AND $f_2(v,0)$; AND DISTRIBUTION OF RANDOM VARIABLES, Γ_{12} AND Γ_{21}.

n_1	n_2	$f_1(v,0)$	$f_2(v,0)$	Γ_{12}	Γ_{21}
900	100	$\mathcal{U}_{(-1,1)}$	$\mathcal{U}_{(3/4,1)}$	$\mathcal{U}_{(0,2/10)}$	$\mathcal{U}_{(0,2/100)}$
990	10	$\mathcal{U}_{(-1,1)}$	$\mathcal{U}_{(3/4,1)}$	$\mathcal{U}_{(0,2/10)}$	$\mathcal{U}_{(0,2/100)}$

IV. VERIFICATION OF RESULTS BY SIMULATION

In this section, we show analytic results obtained according to the framework outlined in previous sections for proper choices of parameters. In order to confirm the validity of such results, we compare them against those obtained by simulating a system composed of 10^3 agents, which interact according to (2). We remark that simulations are performed by randomly choosing two interacting agents at each step and by implementing interaction rules (2), independently of analytic results. Table I shows the values of the parameters relative to the two classes of agents that are considered to derive analytic and simulation results. In particular, different values of the parameters are considered for:

1) The number of agents $\{n_s\}_{s=1}^2$;
2) The initial distribution of the opinion; and
3) The distribution of random variables $\{\Gamma_{sr}\}_{s,r=1}^2$.

First, we consider the parameters shown in the first row of Table I. In this case, $n_1 = 900$ and $n_2 = 100$, meaning that 90% of the agents belong to class 1 and only 10% of the agents belong to class 2. Initial opinions of agents of class 1 are uniformly distributed in interval I_v, as shown in the third column. Therefore, the initial average opinion of agents of class 1 is 0. Initial opinions of agents of class 2, instead, are uniformly distributed in the smaller interval $(3/4; 1)$. This choice implies that agents of class 2 have extremal opinions and that their initial average opinion is $u_2(0) = 7/8$. Another feature that distinguishes the agents in the two classes concerns their inclination to change opinion. The distributions of random variables Γ_{12} and Γ_{21} are related to such an inclination. As shown in Table I, we assume that Γ_{12} has uniform distribution in interval $(0, 2/10)$, corresponding to an average value $\bar{\Gamma}_{12}$ of $1/10$, and that Γ_{21} has uniform distribution in interval $(0, 2/100)$, corresponding to an average value $\bar{\Gamma}_{21}$ of $1/100$. According to such choices, $\bar{\Gamma}_{12} = 10\bar{\Gamma}_{21}$, which means that the propensity of agents of class 2 to change their opinions in favor of those of agents of class 1 is much lower than the propensity of agents of class 1 to change their opinions in favor of those of agents of class 2. For this reason, agents of class 2 can be considered skeptical. Fig. 1 shows the average opinion $u_1(t)$ of the agents of class 1 (blue line) and the average opinion $u_2(t)$ of the agents of class 2 (red line). As expected from (31), $u_1(t)$ and $u_2(t)$ converge to the same value, which, according to this choice of parameters, corresponds to $K \simeq 0.46$. Fig. 1 also shows

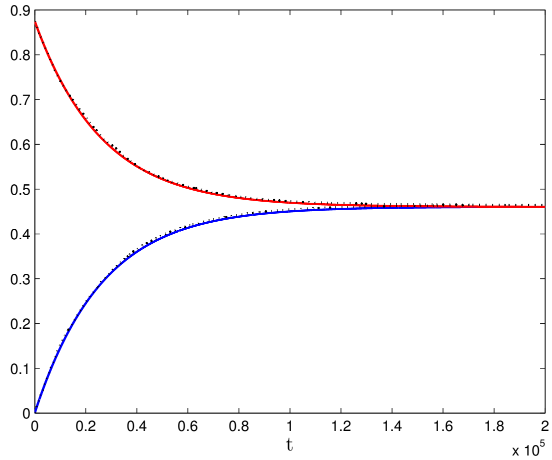

Fig. 1. Average opinions $u_1(t)$ (blue line) and $u_2(t)$ (red line) derived analytically, and average opinions $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ (dotted black lines) obtained by simulation, all computed with values in the first row of Table I.

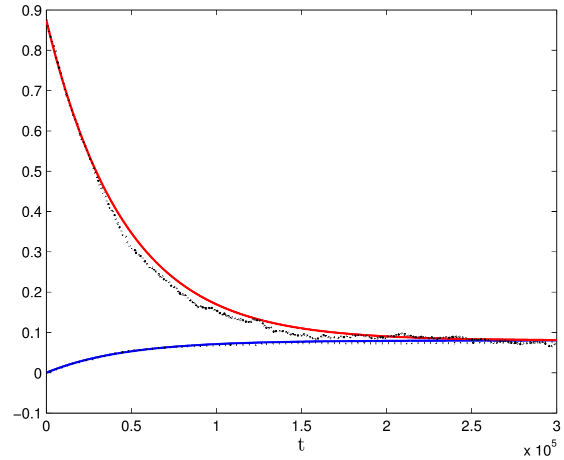

Fig. 2. Average opinions $u_1(t)$ (blue line) and $u_2(t)$ (red line) derived analytically, and average opinions $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ (dotted black lines) obtained by simulation, all computed with values in the second row of Table I.

the values of $\{\tilde{u}_s(t)\}_{s=1}^2$ obtained by simulation (dashed black lines), showing that analytic results are in agreement with those obtained by simulation.

We now consider the parameters shown in the second row of Table I, which differ from those in the first row only for $\{n_s\}_{s=1}^2$. The initial distributions of opinion $\{f_s(v,t)\}_{s=1}^2$ and, hence, the initial values of the average opinion of the two classes of agents, are the same as in the first scenario. This means that agents of class 2 still have extremal opinions. Also the distributions of random variable Γ_{12} and Γ_{21} are the same as in the first scenario, so that agents of class 2 can still be considered skeptical. The only difference is that, in this case, skeptical agents are only 1%. In fact, 990 agents, corresponding to 99% of the total number of agents, belong to class 1, and only 10 agents, corresponding to 1% of the total number of agents, belong to class 2. Fig. 2 shows the average opinion $u_1(t)$ of the agents of class 1 (blue line) and the average opinion $u_2(t)$ of the agents of class 2 (red line). As expected from (31), the values of $u_1(t)$ and $u_2(t)$ converge to the same value, which now corresponds to $K \simeq 0.08$. The values of $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ (dotted black lines) obtained by simulation with the parameters in the second row of Table I are also shown. As in the first scenario, simulation results are in agreement with analytic ones.

In order to improve the analysis of considered scenarios, Fig. 3 shows the distributions $f_1(v,t)$ (solid blue lines) and $f_2(v,t)$ (dashed red lines) of the opinions of the two classes of agents obtained by simulating the multi-agent system with the parameters in the first row of Table I. In detail:

1) Fig. 3(a) shows the distributions $f_s(v,t)$ after $5 \cdot 10^4$ interactions, which corresponds to 100 interactions per agent on average;
2) Fig. 3(b) shows the distributions $f_s(v,t)$ after 10^5 interactions, which corresponds to 200 interactions per agent on average;
3) Fig. 3(c) shows the distributions $f_s(v,t)$ after $1.5 \cdot 10^5$ interactions, which corresponds to 300 interactions per agent on average; and
4) Fig. 3(d) shows the distributions $f_s(v,t)$ after $2 \cdot 10^5$ interactions, which corresponds to 400 interactions per agent on average.

From Fig. 3 it can be observed that not only the average opinions $u_s(t)$ converge to the same value $K \simeq 0.46$, as already shown in Fig. 1, but also that consensus among agents is reached, since the opinions of all agents tend to the same value, as expected from the analytic model. Similar diagrams can be drawn using the parameters in the second row of Table I, obtaining similar results.

V. Conclusions

This paper presented an analytic model of opinion dynamics which assumes that the agents in the studied multi-agent system can be grouped into two classes on the basis of their characteristic parameters. Such classes can be used to model agents with different propensity to change their opinions after interactions, and they are used here to model the presence of skeptical agents in the multi-agent system. Among the sociological phenomena that can be used to describe the dynamics of the opinion, the presented model considers only compromise, which is the phenomenon that describes interactions that tend to consensus. The characteristic autonomy of agents is modeled in terms of random variables used at the microscopic level to perturb the classic model of compromise, which is deterministic. Analytic results ensure that the average opinion of the multi-agent system is conserved, and that consensus is always reached for a sufficiently large number of interactions. Simulations shown in the last part of the paper confirm such properties.

Ongoing research involves the extension of the presented model to account for major sociological phenomena (see,

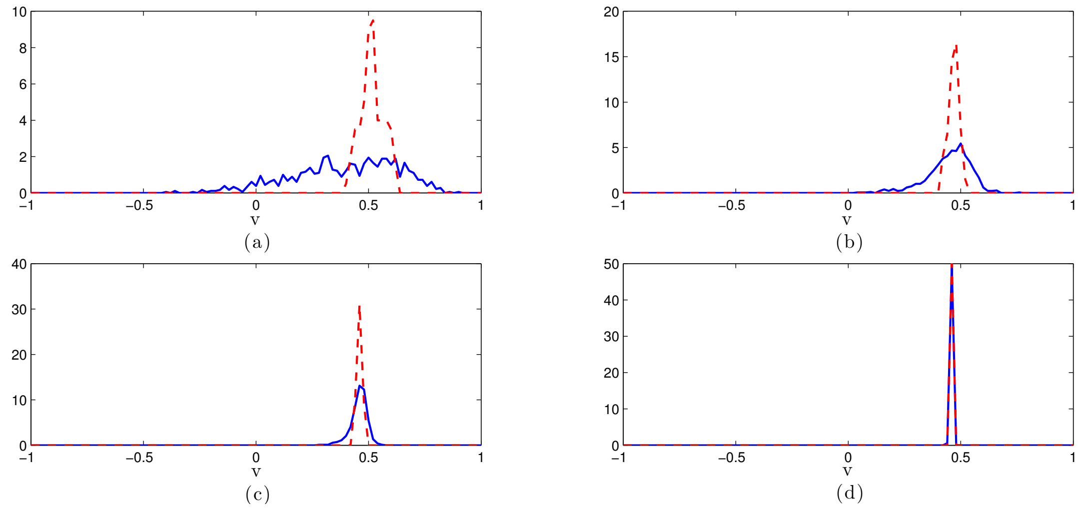

Fig. 3. The distributions $f_1(v,t)$ (blue line) and $f_2(v,t)$ (red line) obtained by simulation using the parameters in the first row of Table I for: (a) $t = 5 \cdot 10^4$; (b) $t = 10^5$; (c) $t = 1.5 \cdot 10^5$; (d) $t = 2 \cdot 10^5$.

e.g., [8]). Among considered phenomena, ongoing research includes stochastic models for:

- *Diffusion,* the phenomenon according to which the opinion of agents is influenced by the social context [13];
- *Homophily,* the process according to which agents interact only with those with similar opinions [14];
- *Negative influence,* the idea according to which agents evaluate their peers, and they only interact with those with positive scores [15];
- *Opinion noise,* the process according to which a random additive variable may lead to arbitrary opinion changes with small probability [16]; and
- *Striving for uniqueness,* the phenomenon according to which agents want to distinguish from others [17].

Preliminary results on the deterministic study of such phenomena for multi-agent systems with multiple classes of agents are encouraging (see, e.g., [9]), and they show that major collective and asymptotic properties of multi-agent systems can be fruitfully studied analytically.

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