Context-based defeasible subsumption for $d\mathcal{SROIQ}$

Katarina Britz
CSIR-SU CAIR, Stellenbosch University
South Africa
abritz@sun.ac.za

Ivan Varzinczak
CRIL, Univ. Artois & CNRS
France
varzinczak@cril.fr

Abstract

The description logic $d\mathcal{SROIQ}$ is a decidable extension of $\mathcal{SROIQ}$ that supports defeasible reasoning in the KLM tradition. It features a parameterised preference order on binary relations in a domain of interpretation, which allows for the use of defeasible roles in complex concepts, as well as in defeasible concept and role subsumption, and in defeasible role assertions. In this paper, we address an important limitation both in $d\mathcal{SROIQ}$ and in other defeasible extensions of description logics, namely the restriction in the semantics of defeasible concept subsumption to a single preference order on objects. We do this by inducing preference orders on objects from preference orders on roles, and use these to relativise defeasible subsumption. This yields a notion of contextualised defeasible subsumption, with contexts described by roles.

1 Introduction

$\mathcal{SROIQ}$ [Horrocks *et al.*, 2006] is an expressive, yet decidable Description Logic (DL) that serves as semantic foundation for the OWL 2 profile, on which several ontology languages of various expressivity are based. However, $\mathcal{SROIQ}$ still allows for meaningful, decidable extension, as new knowledge representation requirements are identified. A case in point is the need to allow for exceptions and defeasibility in reasoning over logic-based ontologies [Bonatti *et al.*, 2009; 2011; 2015; Britz *et al.*, 2011; 2013a; 2013b; Britz and Varzinczak, 2016a; Casini *et al.*, 2015; Casini and Straccia, 2010; 2013; Giordano *et al.*, 2013; 2015; Sengupta *et al.*, 2011]. Yet, $\mathcal{SROIQ}$ does not allow for the direct expression of and reasoning with different aspects of defeasibility.

Given the special status of subsumption in DLs in particular, and the historical importance of entailment in logic in general, past research efforts in this direction have focused primarily on accounts of defeasible subsumption and the characterisation of defeasible entailment. Semantically, the latter usually takes as point of departure orderings on a class of first-order interpretations, whereas the former usually assume a preference order on objects of the domain.

Recently, we proposed a decidable extension of $\mathcal{SROIQ}$ that supports defeasible knowledge representation and reasoning over defeasible ontologies [Britz and Varzinczak, 2016a; 2017]. Our proposal built on previous work to resolve two important ontological limitations of the preferential approach to defeasible reasoning in DLs — the assumption of a single preference order on all objects in the domain of interpretation, and the assumption that defeasibility is intrinsically linked to argument form [Britz and Varzinczak, 2013; 2016b].

We achieved this by extending $\mathcal{SROIQ}$ with nonmonotonic reasoning features in the concept language, in subsumption statements and in role assertions, via an intuitive notion of normality for roles. This parameterised the idea of preference while at the same time introducing the notion of defeasible class membership. Defeasible subsumption allows for the expression of statements of the form "C is usually subsumed by D", for example, "Chenin blanc wines *are usually* unwooded". In the extended language $d\mathcal{SROIQ}$, one can also refer directly to, for example, "Chenin blanc wines that *usually have* a wood aroma". We can also combine these seamlessly, as in: "Chenin blanc wines that *usually have* a wood aroma *are usually* wooded". This cannot be expressed in terms of defeasible subsumption alone, nor can it be expressed w.l.o.g. using a typicality operator on concepts. This is because the semantics of the expression is inextricably tied to the two distinct uses of the term 'usually'.

However, even this generalisation leaves open the question of different, possibly incompatible, notions of defeasibility in subsumption, similar to those studied in contextual argumentation [Amgoud *et al.*, 2000; Bikakis and Antoniou, 2010]. In the statement "Chenin blanc wines are usually unwooded", the context relative to which the subsumption is normal is left implicit – in this case, the style of the wine. In a different context such as consumer preference or origin, the most preferred (or normal, or typical) Chenin blanc wines may not correlate with the usual wine style. Wine x may be more exceptional than y in one context, but less exceptional in another context. This represents a form of inconsistency in defeasible knowledge bases arising from the presence of named individuals in the ontology. The example illustrates why a single ordering on individuals does not suffice. It also points to a natural index for relativised context, namely the use of preferential role names previously proposed for $d\mathcal{SROIQ}$.

In this paper, we therefore propose to induce preference orders on objects from preference orders on roles, and use these to relativise defeasible subsumption. This yields a notion of contextualised defeasible subsumption, with contexts described by roles.

The remainder of the paper is structured as follows: In Section 2 we present some DL background on $\mathcal{SROIQ}$. In Section 3 we introduce the syntax of the extended language $d\mathcal{SROIQ}$, and in Section 4 its semantics. The newly introduced defeasible language constructs are discussed in Section 5, where we also give examples to illustrate their semantics and use. Section 6 covers a number of rewriting and elimination results required for effective reasoning with $d\mathcal{SROIQ}$ knowledge bases.

We shall assume the reader's familiarity with the preferential approach to non-monotonic reasoning [Kraus *et al.*, 1990; Lehmann and Magidor, 1992; Shoham, 1988]. Whenever necessary, we refer the reader to the definitions and results in the relevant literature.

2 The description logic $\mathcal{SROIQ}$

In this section, we provide the basics of $\mathcal{SROIQ}$ [Horrocks *et al.*, 2006]. For space considerations, but also to avoid repetition, we defer many of its technicalities to the upcoming sections.

The language of $\mathcal{SROIQ}$ is built upon a finite set of atomic *concept names* C, of which N, the set of *nominals*, is a subset, a finite set of *role names* R and a finite set of *individual names* I such that C, R and I are pairwise disjoint. The *universal role* is denoted by u and the set of all roles is given by $\mathbf{R} := \mathsf{R} \cup \{r^- \mid r \in \mathsf{R}\} \cup \{u\}$, where r^- denotes the inverse of r. With $A, B, \ldots$ we denote atomic concepts, with $r, s, \ldots$ roles, and with $a, b, \ldots$ individual names. A nominal will also be denoted by o, possibly with subscripts.

The set of $\mathcal{SROIQ}$ *complex concepts* is the smallest set such that: $\top$, $\bot$ and every $A \in \mathsf{C}$ are concepts; if C and D are concepts, $r, s \in \mathbf{R}$, and $n \in \mathbb{N}$, then $\neg C$ (concept complement), $C \sqcap D$ (concept conjunction), $C \sqcup D$ (concept disjunction), $\forall r.C$ (value restriction), $\exists r.C$ (existential restriction), $\exists r.\mathsf{Self}$ (self restriction), $\geq ns.C$ (at-least restriction), $\leq ns.C$ (at-most restriction) are also concepts. With $C, D \ldots$ we denote complex $\mathcal{SROIQ}$ concepts. A more detailed description of the roles allowed in complex concept descriptions will be provided in Definitions 1 and 3.

If C and D are concepts, then $C \sqsubseteq D$ is a *general concept inclusion axiom* (GCI, for short), read "C is subsumed by D". $C \equiv D$ is an abbreviation for both $C \sqsubseteq D$ and $D \sqsubseteq C$.

Given C a concept, $r \in \mathbf{R}$ and $a, b \in \mathsf{I}$, an *individual assertion* is an expression of the form $a : C$, $(a, b) : r$, $(a, b) : \neg r$, $a = b$ or $a \neq b$.

A *role inclusion axiom* (RIA) is a statement of the form $r_1 \circ \cdots \circ r_n \sqsubseteq r$, where $r_1, \ldots, r_n, r \in \mathbf{R} \setminus \{\mathsf{u}\}$ and $r_1 \circ \cdots \circ r_n$ denotes the *composition* of $r_1, \ldots, r_n$. A *role assertion* is a statement of the form $\mathsf{Fun}(r)$ (functionality), $\mathsf{Ref}(r)$ (reflexivity), $\mathsf{Irr}(r)$ (irreflexivity), $\mathsf{Sym}(r)$ (symmetry), $\mathsf{Asy}(r)$ (asymmetry), $\mathsf{Tra}(r)$ (transitivity), and $\mathsf{Dis}(r, s)$ (role disjointness), where $r, s \neq \mathsf{u}$. A more detailed description of the roles allowed in RIAs will be given in Definition 2.

The semantics of $\mathcal{SROIQ}$ is in terms of the standard set theoretic Tarskian semantics. An *interpretation* is a structure $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty set called the *domain*, and $\cdot^{\mathcal{I}}$ is an *interpretation function* mapping concept names $A \in \mathsf{C}$ to subsets $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ (with $o^{\mathcal{I}}$ a singleton if $o \in \mathsf{N}$), role names $r \in \mathsf{R}$ to binary relations $r^{\mathcal{I}}$ over $\Delta^{\mathcal{I}}$, and individual names $a \in \mathsf{I}$ to elements of the domain $\Delta^{\mathcal{I}}$, i.e., $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. We extend $\cdot^{\mathcal{I}}$ from role names to roles by letting $\mathsf{u}^{\mathcal{I}} := \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and $(r^-)^{\mathcal{I}} := \{(y, x) \mid (x, y) \in r^{\mathcal{I}}\}$, and to role chains by setting $(r_1 \circ \cdots \circ r_n)^{\mathcal{I}} := r_1^{\mathcal{I}} \circ \cdots \circ r_n^{\mathcal{I}}$.

As an example, let $\mathsf{C} := \{A_1, A_2, A_3\}$, $\mathsf{R} := \{r_1, r_2\}$ and $\mathsf{I} := \{a_1, a_2, a_3\}$. Figure 1 depicts the interpretation $\mathcal{I}_1 = \langle \Delta^{\mathcal{I}_1}, \cdot^{\mathcal{I}_1} \rangle$, where $\Delta^{\mathcal{I}_1} = \{x_i \mid 1 \leq i \leq 9\}$, $A_1^{\mathcal{I}_1} = \{x_1, x_4, x_6\}$, $A_2^{\mathcal{I}_1} = \{x_3, x_5, x_9\}$, $A_3^{\mathcal{I}_1} = \{x_6, x_7, x_8\}$, $r_1^{\mathcal{I}_1} = \{(x_1, x_6), (x_4, x_8), (x_2, x_5)\}$, $r_2^{\mathcal{I}_1} = \{(x_4, x_4), (x_6, x_4), (x_5, x_8), (x_9, x_3)\}$, $a_1^{\mathcal{I}_1} = x_5$, $a_2^{\mathcal{I}_1} = x_1$, and $a_3^{\mathcal{I}_1} = x_2$.

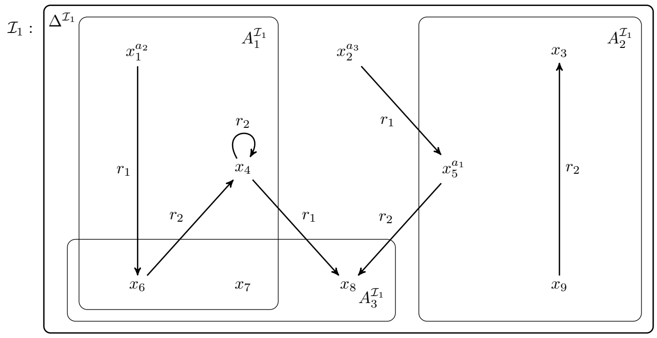

Figure 1: A $\mathcal{SROIQ}$ interpretation.

The notion of interpretation can be extended to interpret (complex) $\mathcal{SROIQ}$ concepts and to provide a notion of satisfaction of GCIs, RIAs, individual and role assertions in a way that will be made clear in Section 3.

3 Context-based defeasible $\mathcal{SROIQ}$

In this section, we present the syntax and semantics of an extension to $\mathcal{SROIQ}$ to represent defeasible complex concepts and subsumption. The logic presented here is an incremental extension of $d\mathcal{SROIQ}$, which was introduced recently [Britz and Varzinczak, 2017] to add defeasible reasoning features to $\mathcal{SROIQ}$. Previous work included various defeasible constructs on concepts based on preferential roles, but only a single preference order on objects. This was somewhat of an anomaly, as pointed out by some reviewers and colleagues. We address this anomaly by adding context-based orderings on objects that are derived from preferential roles. This turns out to be a remarkably seamless yet very useful refinement. Briefly, each preferential role r, interpreted as a strict partial order on the binary product space of the domain, gives rise to a context-based order on objects as detailed in Definition 6 below.

3.1 Defeasibility in RBoxes

Let $\mathsf{inv} : \mathbf{R} \longrightarrow \mathbf{R}$ be such that $\mathsf{inv} : r \mapsto r^-$, if $r \in \mathsf{R}$, $\mathsf{inv} : r \mapsto s$, if $r = s^-$, and $\mathsf{inv} : \mathsf{u} \mapsto \mathsf{u}$.

Let $r_1, \ldots, r_n, r \in \mathbf{R} \setminus \{\mathsf{u}\}$. A *classical role inclusion axiom* is a statement of the form $r_1 \circ \cdots \circ r_n \sqsubseteq r$. A *defeasible role inclusion axiom* has the form $r_1 \circ \cdots \circ r_n \mathrel{\underset{\sim}{\sqsubset}} r$, read "usually, $r_1 \circ \cdots \circ r_n$ is included in r". A finite set of (classical or defeasible) role inclusion axioms (RIAs) is called a *role hierarchy* and is denoted by $\mathcal{R}_h$.

Definition 1 ((Non-)Simple Role) *Let $r \in \mathbf{R}$ and let $\mathcal{R}_h$ be a role hierarchy. Then r is **non-simple** in $\mathcal{R}_h$ iff:*

1. *There is $r_1 \circ \cdots \circ r_n \sqsubseteq r$ or $r_1 \circ \cdots \circ r_n \mathrel{\underset{\sim}{\sqsubset}} r$ in $\mathcal{R}_h$ such that $n > 1$, or*
2. *There is $s \sqsubseteq r$ or $s \mathrel{\underset{\sim}{\sqsubset}} r$ in $\mathcal{R}_h$ such that s is non-simple, or*
3. $\mathsf{inv}(r)$ *is non-simple.*

*With $\mathbf{R}^n$ we denote the set of **non-simple** roles in $\mathcal{R}_h$. $\mathbf{R}^s := \mathbf{R} \setminus \mathbf{R}^n$ is the set of **simple** roles in $\mathcal{R}_h$.*

Intuitively, simple roles are those that are not implied by the composition of roles. They are needed to restrict the type of roles in certain concept constructors (see below), thereby preserving decidability [Horrocks *et al.*, 2006].

Definition 2 (Regular Hierarchy) *A role hierarchy $\mathcal{R}_h$ is **regular** if there is a strict partial order $<$ on $\mathbf{R}^n$ such that:*

1. *$s < r$ iff $\mathsf{inv}(s) < r$, for every r, s in $\mathbf{R}^n$, and*
2. *every role inclusion in $\mathcal{R}_h$ is of one of the forms:*
   (1a) $r \circ r \sqsubseteq r$, (1b) $r \circ r \mathrel{\underset{\sim}{\sqsubset}} r$,
   (2a) $\mathsf{inv}(r) \sqsubseteq r$, (2b) $\mathsf{inv}(r) \mathrel{\underset{\sim}{\sqsubset}} r$,
   (3a) $s_1 \circ \cdots \circ s_n \sqsubseteq r$, (3b) $s_1 \circ \cdots \circ s_n \mathrel{\underset{\sim}{\sqsubset}} r$,
   (4a) $r \circ s_1 \circ \cdots \circ s_n \sqsubseteq r$, (4b) $r \circ s_1 \circ \cdots \circ s_n \mathrel{\underset{\sim}{\sqsubset}} r$,
   (5a) $s_1 \circ \cdots \circ s_n \circ r \sqsubseteq r$, (5b) $s_1 \circ \cdots \circ s_n \circ r \mathrel{\underset{\sim}{\sqsubset}} r$, where $r \in \mathsf{R}$ (i.e., a role name), and $s_i < r$, for $i = 1, \ldots, n$.

(Regularity prevents a role hierarchy from inducing cyclic dependencies, which are known to lead to undecidability.)

A *classical role assertion* is a statement of the form $\mathsf{Fun}(r)$ (functionality), $\mathsf{Ref}(r)$ (reflexivity), $\mathsf{Irr}(r)$ (irreflexivity), $\mathsf{Sym}(r)$ (symmetry), $\mathsf{Asy}(r)$ (asymmetry), $\mathsf{Tra}(r)$ (transitivity), and $\mathsf{Dis}(r, s)$ (role disjointness), where $r, s \neq \mathsf{u}$. A *defeasible role assertion* is a statement of the form $\mathsf{dFun}(r)$ (r is usually functional), $\mathsf{dRef}(r)$ (r is usually reflexive), $\mathsf{dIrr}(r)$ (r is usually irreflexive), $\mathsf{dSym}(r)$ (r is usually symmetric), $\mathsf{dAsy}(r)$ (r is usually asymmetric), $\mathsf{dTra}(r)$ (r is usually transitive), and $\mathsf{dDis}(r, s)$ (r and s are usually disjoint), also with $r, s \neq \mathsf{u}$. With $\mathcal{R}_a$ we denote a finite set of role assertions.

Given a role hierarchy $\mathcal{R}_h$, we say that $\mathcal{R}_a$ is *simple* w.r.t. $\mathcal{R}_h$ if all roles r, s appearing in statements of the form $\mathsf{Irr}(r)$, $\mathsf{dIrr}(r)$, $\mathsf{Asy}(r)$, $\mathsf{dAsy}(r)$, $\mathsf{Dis}(r, s)$ or $\mathsf{dDis}(r, s)$ are simple in $\mathcal{R}_h$ (see Definition 1).

A $d\mathcal{SROIQ}$ RBox is a set $\mathcal{R} := \mathcal{R}_h \cup \mathcal{R}_a$, where $\mathcal{R}_h$ is a regular hierarchy and $\mathcal{R}_a$ is a set of role assertions which is simple w.r.t. $\mathcal{R}_h$.

3.2 Defeasibility in concepts

We now extend the set of $\mathcal{SROIQ}$ complex concepts via the definition of concept constructors allowing for the expression of defeasibility at the object level.

Definition 3 ($d\mathcal{SROIQ}$ Concepts) *The set of $d\mathcal{SROIQ}$ **complex concepts** is the smallest set such that $\top$, $\bot$ and every $A \in \mathsf{C}$ are concepts, and if C and D are concepts, $r \in \mathbf{R}$, $s \in \mathbf{R}^s$, and $n \in \mathbb{N}$, then $\neg C$ (concept complement), $C \sqcap D$ (concept conjunction), $C \sqcup D$ (concept disjunction), $\forall r.C$ (value restriction), $\exists r.C$ (existential restriction), $\tilde{\forall} r.C$ (**defeasible value restriction**), $\tilde{\exists} r.C$ (**defeasible existential restriction**), $\exists r.\mathsf{Self}$ (self restriction), $\tilde{\exists} r.\mathsf{Self}$ (**defeasible self restriction**), $\geq ns.C$ (at-least restriction), $\leq ns.C$ (at-most restriction), $\gtrsim ns.C$ (**defeasible at-least restriction**), $\lesssim ns.C$ (**defeasible at-most restriction**) are also concepts. With $\mathbf{C}$ we denote the set of all complex concepts.*

Note that every $\mathcal{SROIQ}$ concept is a $d\mathcal{SROIQ}$ concept, too. We shall use $C, D \ldots$, possibly with subscripts, to denote complex $d\mathcal{SROIQ}$ concepts.

3.3 Context-based defeasible subsumption

Given $C, D \in \mathbf{C}$, $C \sqsubseteq D$ is a *classical general concept inclusion*, read "C is subsumed by D". $C \equiv D$ is an abbreviation for both $C \sqsubseteq D$ and $D \sqsubseteq C$.

The extension of $d\mathcal{SROIQ}$ we propose here includes context-based defeasible subsumption statements in the *TBox*. Given $C, D \in \mathbf{C}$ and $r \in \mathbf{R}$, $C \mathrel{\underset{\sim}{\sqsubset}}_r D$ is a *defeasible general concept inclusion*, read "C is usually subsumed by D in the context r". A $d\mathcal{SROIQ}$ *TBox* $\mathcal{T}$ is a finite set of general concept inclusions (GCIs), whether classical or defeasible.

Before we present the semantics, we introduce the remaining components of $d\mathcal{SROIQ}$ ontologies. Recall I is a set of *individual names* disjoint from both C and R. Given $C \in \mathbf{C}$, $r \in \mathbf{R}$ and $a, b \in \mathsf{I}$, an *individual assertion* is an expression of the form $a : C$, $(a, b) : r$, $(a, b) : \neg r$, $a = b$ or $a \neq b$. A $d\mathcal{SROIQ}$ *ABox* $\mathcal{A}$ is a finite set of individual assertions.

Let $\mathcal{A}$ be an ABox, $\mathcal{T}$ be a TBox and $\mathcal{R}$ an RBox. A *knowledge base* (alias ontology) is a tuple $\mathcal{KB} := \langle \mathcal{A}, \mathcal{R}, \mathcal{T} \rangle$.

4 Preferential semantics

We shall anchor our semantic constructions in the well-known preferential approach to non-monotonic reasoning [Kraus *et al.*, 1990; Lehmann and Magidor, 1992; Shoham, 1988] and its extensions [Boutilier, 1994; Britz and Varzinczak, 2013; a; 2016b; b], especially those to the DL case [Britz *et al.*, 2011; Britz and Varzinczak, 2016a; Giordano *et al.*, 2009; Quantz and Royer, 1992].

Let X be a set and let $<$ be a strict partial order on X. With $\min_< X := \{x \in X \mid \text{there is no } y \in X \text{ s.t. } y < x\}$ we denote the *minimal elements* of X w.r.t. $<$. With $\#X$ we shall denote the *cardinality* of X.

Definition 4 (Ordered Interpretation) *An **ordered interpretation** is a tuple $\mathcal{O} := \langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}}, \ll^{\mathcal{O}} \rangle$ in which $\langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}} \rangle$ is a $\mathcal{SROIQ}$ interpretation with $A^{\mathcal{O}} \subseteq \Delta^{\mathcal{O}}$, for every $A \in \mathsf{C}$, $A^{\mathcal{O}}$ a singleton for every $A \in \mathsf{N}$, $r^{\mathcal{O}} \subseteq \Delta^{\mathcal{O}} \times \Delta^{\mathcal{O}}$, for all $r \in \mathbf{R}$, and $a^{\mathcal{O}} \in \Delta^{\mathcal{O}}$, for every $a \in \mathsf{I}$, and $\ll^{\mathcal{O}} := \langle \ll_1^{\mathcal{O}}, \ldots, \ll_{\#\mathsf{R}}^{\mathcal{O}} \rangle$, where $\ll_i^{\mathcal{O}} \subseteq r_i^{\mathcal{O}} \times r_i^{\mathcal{O}}$, for $i = 1, \ldots, \#\mathsf{R}$, and such that each $\ll_i^{\mathcal{O}}$ satisfies the smoothness condition [Kraus et al., 1990].*

As an example, let $\mathsf{C} := \{A_1, A_2, A_3\}$, $\mathsf{R} := \{r_1, r_2\}$, $\mathsf{I} := \{a_1, a_2, a_3\}$, and let the r-ordered interpretation $\mathcal{O}_1 = \langle \Delta^{\mathcal{O}_1}, \cdot^{\mathcal{O}_1}, \ll^{\mathcal{O}_1} \rangle$, where $\Delta^{\mathcal{O}_1} =$

$\Delta^{\mathcal{I}_1}$, $\cdot^{\mathcal{O}_1} = \cdot^{\mathcal{I}_1}$, and $\ll^{\mathcal{O}_1} = \langle \ll_1^{\mathcal{O}_1}, \ll_2^{\mathcal{O}_1} \rangle$, where $\ll_1^{\mathcal{O}_1} = \{(x_4x_8, x_2x_5), (x_2x_5, x_1x_6), (x_4x_8, x_1x_6)\}$ and $\ll_2^{\mathcal{O}_1} = \{(x_6x_4, x_4x_4), (x_5x_8, x_9x_3)\}$. (For the sake of readability, we shall henceforth sometimes write tuples of the form (x, y) as xy.) Figure 2 below depicts the r-ordered interpretation $\mathcal{O}_1$. In the picture, $\ll_1^{\mathcal{O}_1}$ and $\ll_2^{\mathcal{O}_1}$ are represented, respectively, by the dashed and the dotted arrows. (Note the direction of the $\ll^{\mathcal{O}}$-arrows, which point from more preferred to less preferred pairs of objects.) Also for the sake of readability, we shall omit the transitive $\ll^{\mathcal{O}}$-arrows.

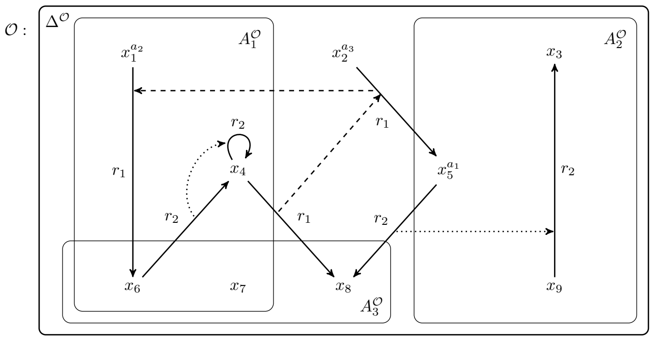

Figure 2: A $d\mathcal{SROIQ}$ ordered interpretation.

Given $\mathcal{O} = \langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}}, \ll^{\mathcal{O}} \rangle$, the intuition of $\Delta^{\mathcal{O}}$ and $\cdot^{\mathcal{O}}$ is the same as in a standard DL interpretation. The intuition underlying each of the orderings in $\ll^{\mathcal{O}}$ is that they play the role of *preference relations* (or *normality orderings*), in a sense similar to that introduced by Shoham [Shoham, 1988] with a preference on worlds in a propositional setting and as extensively investigated by Kraus et al. [Kraus *et al.*, 1990; Lehmann and Magidor, 1992] and others [Boutilier, 1994; Britz *et al.*, 2008; Giordano *et al.*, 2007]: the pairs (x, y) that are lower down in the ordering $\ll_i^{\mathcal{O}}$ are deemed as the most normal (or typical, or expected) in the context of (the interpretation of) r_i. Technically, the difference between our definitions and those in the aforementioned work lies on the fact that our $\ll_i^{\mathcal{O}}$ are orderings on binary relations on the domain $\Delta^{\mathcal{O}}$, instead of orderings on propositional valuations or on plain objects of $\Delta^{\mathcal{O}}$.

In the following definition we show how ordered interpretations can be extended to interpret the complex concepts of the language.

Definition 5 ($\mathcal{O}$ extended) *Let $\mathcal{O} = \langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}}, \ll^{\mathcal{O}} \rangle$. For any $r, r_1, r_2 \in \mathbf{R} \setminus \{\mathsf{u}\}$, $\mathcal{O}$ interprets **orderings on role inverses** and **on role compositions** as follows: $\ll_{r^-}^{\mathcal{O}} := \{((y_1, x_1), (y_2, x_2)) \mid ((x_1, y_1), (x_2, y_2)) \in \ll_r^{\mathcal{O}}\}$, and $\ll_{r_1 \circ r_2}^{\mathcal{O}} := \{((x_1, y_1), (x_2, y_2)) \mid$ for some z_1, z_2 $[((x_1, z_1), (x_2, z_2)) \in \ll_{r_1}^{\mathcal{O}}$ and $((z_1, y_1), (z_2, y_2)) \in \ll_{r_2}^{\mathcal{O}}]$, and for no z_1, z_2 $[((x_2, z_2), (x_1, z_1)) \in \ll_{r_1}^{\mathcal{O}}$ and $((z_2, y_2), (z_1, y_1)) \in \ll_{r_2}^{\mathcal{O}}]\}$. Moreover, let $r_i^{\mathcal{O}|x} := r_i^{\mathcal{O}} \cap (\{x\} \times \Delta^{\mathcal{O}})$ (i.e., the restriction of the domain of $r_i^{\mathcal{O}}$ to $\{x\}$). The interpretation function $\cdot^{\mathcal{O}}$ interprets $d\mathcal{SROIQ}$ concepts in the following way (whenever it is clear which component of $\ll^{\mathcal{O}}$ is used, we shall drop the subscript in $\ll_i^{\mathcal{O}}$):*

$\top^{\mathcal{O}} := \Delta^{\mathcal{O}}$; $\quad \bot^{\mathcal{O}} := \emptyset$; $\quad (\neg C)^{\mathcal{O}} := \Delta^{\mathcal{O}} \setminus C^{\mathcal{O}}$;
$(C \sqcap D)^{\mathcal{O}} := C^{\mathcal{O}} \cap D^{\mathcal{O}}$; $\quad (C \sqcup D)^{\mathcal{O}} := C^{\mathcal{O}} \cup D^{\mathcal{O}}$;
$(\forall r.C)^{\mathcal{O}} := \{x \mid r^{\mathcal{O}}(x) \subseteq C^{\mathcal{O}}\}$;
$(\mathbin{\rotatebox[origin=c]{180}{A}}\!\!\!\forall r.C)^{\mathcal{O}} := \{x \mid \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})(x) \subseteq C^{\mathcal{O}}\}$;
$(\exists r.C)^{\mathcal{O}} := \{x \mid r^{\mathcal{O}}(x) \cap C^{\mathcal{O}} \neq \emptyset\}$;
$(\exists\!\!\!\exists r.C)^{\mathcal{O}} := \{x \mid \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})(x) \cap C^{\mathcal{O}} \neq \emptyset\}$;
$(\exists r.\mathsf{Self})^{\mathcal{O}} := \{x \mid (x, x) \in r^{\mathcal{O}}\}$;
$(\exists\!\!\!\exists r.\mathsf{Self})^{\mathcal{O}} := \{x \mid (x, x) \in \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})\}$;
$(\geq n r.C)^{\mathcal{O}} := \{x \mid \# r^{\mathcal{O}}(x) \cap C^{\mathcal{O}} \geq n\}$;
$(\leq n r.C)^{\mathcal{O}} := \{x \mid \# r^{\mathcal{O}}(x) \cap C^{\mathcal{O}} \leq n\}$;
$(\gtrsim n r.C)^{\mathcal{O}} := \{x \mid \# \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})(x) \cap C^{\mathcal{O}}\} \geq n$;
$(\lesssim n r.C)^{\mathcal{O}} := \{x \mid \# \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})(x) \cap C^{\mathcal{O}} \leq n\}$.

It is not hard to see that, analogously to the classical case, $\forall\!\!\!\forall$ and $\exists\!\!\!\exists$, as well as $\gtrsim$ and $\lesssim$, are dual to each other.

Definition 6 (Satisfaction) *Let $\mathcal{O} = \langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}}, \ll^{\mathcal{O}} \rangle$ and let $r_1, \ldots, r_n, r, s \in \mathbf{R}$, $C, D \in \mathbf{C}$, and $a, b \in \mathsf{I}$. Let $\prec_r^{\mathcal{O}} := \{(x, y) \mid$ there is some $(x, z) \in r^{\mathcal{O}}$ such that for all $(y, v) \in r^{\mathcal{O}}$ $[((x, z), (y, v)) \in \ll_r^{\mathcal{O}}]\}$. The **satisfaction relation** $\Vdash$ is defined as follows:*

$\mathcal{O} \Vdash r \sqsubseteq s$ if $r^{\mathcal{O}} \subseteq s^{\mathcal{O}}$;
$\mathcal{O} \Vdash r \mathrel{\underset{\sim}{\sqsubset}} s$ if $\min_{\ll^{\mathcal{O}}} r^{\mathcal{O}} \subseteq s^{\mathcal{O}}$;
$\mathcal{O} \Vdash r_1 \circ \cdots \circ r_n \sqsubseteq r$ if $(r_1 \circ \cdots \circ r_n)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash r_1 \circ \cdots \circ r_n \mathrel{\underset{\sim}{\sqsubset}} r$ if $\min_{\ll^{\mathcal{O}}}(r_1 \circ \cdots \circ r_n)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{Fun}(r)$ if $r^{\mathcal{O}}$ is a function;
$\mathcal{O} \Vdash \mathsf{dFun}(r)$ if for all x, $\# \min_{\ll^{\mathcal{O}}}(r^{\mathcal{O}|x})(x) \leq 1$;
$\mathcal{O} \Vdash \mathsf{Ref}(r)$ if $\{(x, x) \mid x \in \Delta^{\mathcal{O}}\} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{dRef}(r)$ if for every $x \in \min_{\prec_{\mathsf{u}}^{\mathcal{O}}} \Delta^{\mathcal{O}}$, $(x, x) \in r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{Irr}(r)$ if $r^{\mathcal{O}} \cap \{(x, x) \mid x \in \Delta^{\mathcal{O}}\} = \emptyset$;
$\mathcal{O} \Vdash \mathsf{dIrr}(r)$ if for every $x \in \min_{\prec_{\mathsf{u}}^{\mathcal{O}}} \Delta^{\mathcal{O}}$, $(x, x) \notin r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{Sym}(r)$ if $\mathsf{inv}(r)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{dSym}(r)$ if $\min_{\ll^{\mathcal{O}}}(r^-)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{Asy}(r)$ if $r^{\mathcal{O}} \cap \mathsf{inv}(r)^{\mathcal{O}} = \emptyset$;
$\mathcal{O} \Vdash \mathsf{dAsy}(r)$ if $\min_{\ll^{\mathcal{O}}} r^{\mathcal{O}} \cap \min_{\ll^{\mathcal{O}}}(r^-)^{\mathcal{O}} = \emptyset$;
$\mathcal{O} \Vdash \mathsf{Tra}(r)$ if $(r \circ r)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{dTra}(r)$ if $\min_{\ll^{\mathcal{O}}}(r \circ r)^{\mathcal{O}} \subseteq r^{\mathcal{O}}$;
$\mathcal{O} \Vdash \mathsf{Dis}(r, s)$ if $r^{\mathcal{O}} \cap s^{\mathcal{O}} = \emptyset$;
$\mathcal{O} \Vdash \mathsf{dDis}(r, s)$ if $\min_{\ll^{\mathcal{O}}} r^{\mathcal{O}} \cap \min_{\ll^{\mathcal{O}}} s^{\mathcal{O}} = \emptyset$;
$\mathcal{O} \Vdash C \sqsubseteq D$ if $C^{\mathcal{O}} \subseteq D^{\mathcal{O}}$;
$\mathcal{O} \Vdash C \mathrel{\underset{\sim}{\sqsubset}}_r D$ if $\min_{\prec_r^{\mathcal{O}}} C^{\mathcal{O}} \subseteq D^{\mathcal{O}}$;
$\mathcal{O} \Vdash a : C$ if $a^{\mathcal{O}} \in C^{\mathcal{O}}$; $\mathcal{O} \Vdash (a, b) : r$ if $(a^{\mathcal{O}}, b^{\mathcal{O}}) \in r^{\mathcal{O}}$;
$\mathcal{O} \Vdash (a, b) : \neg r$ if $\mathcal{O} \not\Vdash (a, b) : r$;
$\mathcal{O} \Vdash a = b$ if $a^{\mathcal{O}} = b^{\mathcal{O}}$; $\mathcal{O} \Vdash a \neq b$ if $\mathcal{O} \not\Vdash a = b$.

*If $\mathcal{O} \Vdash \alpha$, then we say $\mathcal{O}$ **satisfies** α. $\mathcal{O}$ satisfies a set of statements or assertions X (denoted $\mathcal{O} \Vdash X$) if $\mathcal{O} \Vdash \alpha$ for every $\alpha \in X$, in which case we say $\mathcal{O}$ is a **model** of X. We say $C \in \mathbf{C}$ is **satisfiable** w.r.t. $\mathcal{KB} = \langle \mathcal{A}, \mathcal{R}, \mathcal{T} \rangle$ if there is a model $\mathcal{O}$ of $\mathcal{KB}$ s.t. $C^{\mathcal{O}} \neq \emptyset$, and **unsatisfiable** otherwise.*

A statement α is (classically) *entailed* by a knowledge base $\mathcal{KB}$, denoted $\mathcal{KB} \models \alpha$, if every model of $\mathcal{KB}$ satisfies α.

5 Modelling with $d\mathcal{SROIQ}$ ontologies

The motivation for $d\mathcal{SROIQ}$ is to represent defeasible knowledge, and to reason over defeasible ontologies. We now consider the different aspects of defeasibility that can be expressed in $d\mathcal{SROIQ}$. We first consider defeasible existential restriction:

$$\mathsf{Cheninblanc} \sqcap \precsim\!\!\exists\, \mathsf{hasAroma.Wood} \sqsubseteq \exists \mathsf{hasStyle.Wooded}$$

This statement is read: "Chenin blanc wines that normally have a wood aroma are wooded". That is, any Chenin blanc wine that has a characteristic wood aroma, has a wooded wine style. For an example of defeasible subsumption, consider the statement

$$\mathsf{Cheninblanc} \sqsubseteq\!\!\!\!\sim_{\mathsf{u}} \exists \mathsf{hasAroma.Floral}$$

which states that Chenin blanc wines usually have some floral aroma. That is, the most typical Chenin blanc wines all have some floral aroma. Similarly,

$$\mathsf{Cheninblanc} \sqsubseteq\!\!\!\!\sim_{\mathsf{u}} \forall \mathsf{hasOrigin.Loire}$$

states that Chenin blanc wines usually come only from the Loire Valley. Now suppose we have a Chenin blanc wine x, which comes from the Loire Valley but does not have a floral aroma, and another Chenin blanc wine y which has a floral aroma but comes from Languedoc. No model of this ontology can simultaneously have $x \prec_{\mathsf{u}} y$ w.r.t. origin and $y \prec_{\mathsf{u}} x$ w.r.t. aroma. There can therefore be no model that accurately models reality.

This is precisely the limitation imposed by having only a single ordering on objects, as usually assumed by preferential approaches to defeasible DLs [Britz *et al.*, 2008; 2011; Giordano *et al.*, 2007; 2009; 2013], and the motivation for introducing context-based defeasible subsumption. Although the two defeasible statements are not inconsistent, the presence of both rules out certain intended models. In contrast, with context-based defeasible subsumption, both subsumption statements can be expressed *and* x and y can have incompatible preferential relationships in the same model:

$$\mathsf{Cheninblanc} \sqsubseteq\!\!\!\!\sim_{\mathsf{hasAroma}} \exists \mathsf{hasAroma.Floral}$$
$$\mathsf{Cheninblanc} \sqsubseteq\!\!\!\!\sim_{\mathsf{hasOrigin}} \forall \mathsf{hasOrigin.Loire}$$

Note that this knowledge base cannot be changed to:

$$\mathsf{Cheninblanc} \sqsubseteq \precsim\!\!\exists \mathsf{hasAroma.Floral}$$
$$\mathsf{Cheninblanc} \sqsubseteq \precsim\!\!\forall \mathsf{hasOrigin.Loire}$$

as the latter states that every Chenin blanc wine has a characteristic floral aroma and is usually exclusive to the Loire Valley (ruling out the possibility of a Chenin blanc without a floral aroma, or one that comes only from Languedoc).

Lemma 1 below shows that every strict partial order on objects in the domain $\Delta^{\mathcal{O}}$ can be obtained from some strict partial order on $\Delta^{\mathcal{O}} \times \Delta^{\mathcal{O}}$ as in Definition 6. This justifies the use of $\sqsubseteq\!\!\!\!\sim_{\mathsf{u}}$ in defeasible subsumption statements where the context is universal. It also supports the definition of $\prec^{\mathcal{O}}_{\mathsf{u}}$, since it shows the more traditional preference ordering on all objects in the domain to be a special case of our proposal. Lemma 2 shows that the converse of Lemma 1 holds in the more general case of any context-based preference order $\ll^{\mathcal{O}}_r$.

Lemma 1 *Given domain $\Delta^{\mathcal{O}}$ and strict partial order $\prec$ on $\Delta^{\mathcal{O}}$, let $\ll^{\mathcal{O}}_{\mathsf{u}} := \{((x, z), (y, z)) \mid x \prec y\}$, and let $\prec^{\mathcal{O}}_{\mathsf{u}}$ be as in Definition 6. Then $\prec = \prec^{\mathcal{O}}_{\mathsf{u}}$.*

Lemma 2 *Let $\mathcal{O} = \langle \Delta^{\mathcal{O}}, \cdot^{\mathcal{O}}, \ll^{\mathcal{O}} \rangle$, and let $\prec^{\mathcal{O}}_r$ be as in Definition 6. Then $\prec^{\mathcal{O}}_r$ is a strict partial order on $\Delta^{\mathcal{O}}$.*

Corollary 1 *Let $\prec$ be a strict partial order on $\Delta^{\mathcal{O}}$, and let $\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim D$ iff $\min_{\prec} C^{\mathcal{O}} \subseteq D^{\mathcal{O}}$. Then universal defeasible subsumption $\sqsubseteq\!\!\!\!\sim_{\mathsf{u}}$ has the same semantics as $\sqsubseteq\!\!\!\!\sim$.*

Corollary 1 makes the intuition of universal defeasible subsumption clear. For the more general parameterised case, the intuition is essentially the same. Consider the role $\mathsf{hasOrigin}$, which links individual wines to origins. Wine x is considered more typical (or less exceptional) than y w.r.t. its origin if it has some origin link which is preferred to any such link from y.

Context-based defeasible subsumption $\sqsubseteq\!\!\!\!\sim_r$ can therefore also be viewed as defeasible subsumption based on a preference order on objects in the domain of $r^{\mathcal{O}}$, bearing in mind that in any given interpretation, it is dependent on $\ll^{\mathcal{O}}_r$. This raises the question whether a preference order on objects in the range of $r^{\mathcal{O}}$ could be considered as an alternative, but since role inverses are allowed in context-based defeasible subsumption, $\sqsubseteq\!\!\!\!\sim_{\mathsf{inv}(r)}$ achieves this.

The following result shows that context-based defeasible subsumption is indeed an appropriate notion of defeasible subsumption:

Lemma 3 *For every $r \in \mathsf{R}$, $\sqsubseteq\!\!\!\!\sim_r$ is a **preferential subsumption relation** on concepts in that, for every $\mathcal{O}$, the following properties hold:*

(Ref) $\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r C$

(LLE) $\dfrac{\mathcal{O} \Vdash C \equiv D,\ \mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r E}{\mathcal{O} \Vdash D \sqsubseteq\!\!\!\!\sim_r E}$

(And) $\dfrac{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r D,\ \mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r E}{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r D \sqcap E}$

(Or) $\dfrac{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r E,\ \mathcal{O} \Vdash D \sqsubseteq\!\!\!\!\sim_r E}{\mathcal{O} \Vdash C \sqcup D \sqsubseteq\!\!\!\!\sim_r E}$

(RW) $\dfrac{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r D,\ \mathcal{O} \Vdash D \sqsubseteq E}{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r E}$

(CM) $\dfrac{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r D,\ \mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r E}{\mathcal{O} \Vdash C \sqcap D \sqsubseteq\!\!\!\!\sim_r E}$

It is not hard to show that, moreover, if the ordering associated to a role r is modular, the defeasible subsumption $\sqsubseteq\!\!\!\!\sim_r$ it induces is also rational, i.e., it satisfies the following rational monotonicity property:

$$\text{(RM)}\ \frac{\mathcal{O} \Vdash C \sqsubseteq\!\!\!\!\sim_r D,\ \mathcal{O} \Vdash C \not\sqsubseteq\!\!\!\!\sim_r \neg C'}{\mathcal{O} \Vdash C \sqcap C' \sqsubseteq\!\!\!\!\sim_r D}$$

A further feature of context-based subsumption is therefore the ability to allow for both rational and preferential-only subsumption relations.

6 Eliminating ABoxes, classical GCIs and the universal role

As for classical $\mathcal{SROIQ}$ [Horrocks *et al.*, 2006, Lemma 7], it is possible to eliminate an ABox $\mathcal{A}$ by compiling all individual assertions in $\mathcal{A}$ as follows:

1. Let $\mathsf{N}' := \mathsf{N} \cup \{o_a \mid a$ appears in $\mathcal{A}\}$ (i.e., extend the signature with new nominals);
2. Let $\mathcal{A}' := \{a : C \in \mathcal{A}\} \cup \{a : \exists r.o_b \mid (a, b) : r \in \mathcal{A}\} \cup \{a : \forall r.\neg o_b \mid (a, b) : \neg r \in \mathcal{A}\} \cup \{a : \neg o_b \mid a \neq b \in \mathcal{A}\}$;
3. For every $C \in \mathbf{C}$, let $C' := C \sqcap \bigsqcap_{a:D \in \mathcal{A}'} \exists \mathsf{u}.(o_a \sqcap D)$.

It is then easy to see that C is satisfiable w.r.t. $\langle \mathcal{A}, \mathcal{R}, \mathcal{T} \rangle$ if and only if C' is satisfiable w.r.t. $\langle \emptyset, \mathcal{R}, \mathcal{T} \rangle$, which allows us to assume from now on and w.l.o.g. that ABoxes have been eliminated.

Next, in the same way that most of the classical role assertions can equivalently be replaced by GCIs or RIAs, under our preferential semantics, all of our defeasible role assertions, with the exception of $\mathsf{dAsy}(\cdot)$ and $\mathsf{dDis}(\cdot)$, can be reduced to defeasible RIAs in the following way. $\mathsf{dFun}(r)$ can be replaced by $\top \sqsubset\!\!\!\sim \lesssim 1r.\top$ — to be 'usually functional' means only non-normal arrows can break functionality. (Note that, since the number restriction is unqualified, r need not be simple.) $\mathsf{dRef}(r)$ and $\mathsf{dIrr}(r)$ can, respectively, be replaced with $\top \sqsubset\!\!\!\sim \exists r.\mathsf{Self}$ and $\top \sqsubset\!\!\!\sim \neg\exists r.\mathsf{Self}$. $\mathsf{dSym}(r)$ can be reduced to $r^- \sqsubset\!\!\!\sim r$ and $\mathsf{dTra}(r)$ to $r \circ r \sqsubset\!\!\!\sim r$. Furthermore, note that $\mathsf{dAsy}(r)$ can be reduced to $\mathsf{dDis}(r, r^-)$ (cf. Definition 6). Hence, from now on we can assume, w.l.o.g., that the set of role assertions $\mathcal{R}_a$ contains only statements of the form $\mathsf{Dis}(r, s)$ and $\mathsf{dDis}(r, s)$.

Finally, we can apply the same procedure for eliminating both all classical TBox statements and the universal role u defined for classical $\mathcal{SROIQ}$ [Horrocks *et al.*, 2006, Lemma 8][Schild, 1991], extended to the case of $d\mathcal{SROIQ}$ concepts. Hence, from now on we can assume that all classical concept subsumptions (as well as occurrences of u therein) have been eliminated.

The next theorem summarises the reduction outlined in this section:

Theorem 1 *Let $\mathcal{KB} = \langle \mathcal{A}, \mathcal{R}, \mathcal{T} \rangle$ and let $\mathcal{D} := \{C \sqsubset\!\!\!\sim_r D \mid C \sqsubset\!\!\!\sim_r D \in \mathcal{T}\}$. Satisfiability of $d\mathcal{SROIQ}$-concepts w.r.t. $\mathcal{KB}$ can be polynomially reduced to satisfiability of $d\mathcal{SROIQ}$-concepts w.r.t. $\langle \emptyset, \mathcal{R}, \mathcal{D} \rangle$ where all role assertions in $\mathcal{R}$ are of the form* $\mathsf{Dis}(r, s)$ *and* $\mathsf{dDis}(r, s)$.

We leave an investigation of a reduction of the axioms in $\mathcal{D}$ for future work.

7 Concluding remarks

In this paper, we have made a case for a context-based notion of defeasible concept inclusion in description logics. We have seen that roles can be used to provide a simple yet powerful context for such a notion. We have shown how the semantics of the resulting family of defeasible subsumption relations can be anchored to that of preferential roles, which we studied in previous work.

From the knowledge-representation standpoint, context-based subsumption provides the user with more flexibility in making defeasible statements in ontologies. From a modelling point of view, the semantic characterisation we propose here resolves an important limitation of many defeasible extensions of description logics, namely the restriction in the semantics of defeasible concept subsumption to a single preference order on objects.

The definitions and preliminary results reported in this paper raise a number of immediate questions:

- How to obtain the other direction of the KLM-style representation result (cf. Lemma 3)?
- How to reduce satisfiability w.r.t. defeasible TBoxes and RBoxes to satisfiability w.r.t. only defeasible RBoxes?
- How to extend the tableau system for $d\mathcal{SROIQ}$ to account for multi-defeasible subsumptions of the kind we introduced here?
- How to define and compute rational closure of a family of defeasible concept inclusions?
- Can a notion of context, together with our more expressive language, give rise to new KLM-style postulates characterising defeasible subsumption relations that are more powerful than the rational ones?

These are some of the questions that will drive future investigation of the topic.

Acknowledgements

This work is based on research supported in part by the National Research Foundation of South Africa (Grant Numbers 103345 and 85482).

References

[Amgoud *et al.*, 2000] L. Amgoud, S. Parsons, and L. Perrussel. An argumentation framework based on contextual preferences. In *Proceedings of the International Conference on Formal and Applied and Practical Reasoning (FAPR)*, pages 59–67, 2000.

[Bikakis and Antoniou, 2010] A. Bikakis and G. Antoniou. Defeasible contextual reasoning with arguments in ambient intelligence. *IEEE Transactions on Knowledge and Data Engineering*, 22(11):1492–1506, 2010.

[Bonatti *et al.*, 2009] P. Bonatti, C. Lutz, and F. Wolter. The complexity of circumscription in description logic. *Journal of Artificial Intelligence Research*, 35:717–773, 2009.

[Bonatti *et al.*, 2011] P. Bonatti, M. Faella, and L. Sauro. Defeasible inclusions in low-complexity DLs. *Journal of Artificial Intelligence Research*, 42:719–764, 2011.

[Bonatti *et al.*, 2015] P. Bonatti, M. Faella, I.M. Petrova, and L. Sauro. A new semantics for overriding in description logics. *Artificial Intelligence*, 222:1–48, 2015.

[Boutilier, 1994] C. Boutilier. Conditional logics of normality: A modal approach. *Artificial Intelligence*, 68(1):87–154, 1994.

[Britz and Varzinczak, a] K. Britz and I. Varzinczak. From KLM-style conditionals to defeasible modalities, and back. To appear.

[Britz and Varzinczak, b] K. Britz and I. Varzinczak. Preferential accessibility and preferred worlds. *Journal of Logic, Language and Information*. To appear.

[Britz and Varzinczak, 2013] K. Britz and I. Varzinczak. Defeasible modalities. In *Proceedings of the 14th Conference on Theoretical Aspects of Rationality and Knowledge (TARK)*, pages 49–60, 2013.

[Britz and Varzinczak, 2016a] K. Britz and I. Varzinczak. Introducing role defeasibility in description logics. In L. Michael and A.C. Kakas, editors, *Proceedings of the 15th European Conference on Logics in Artificial Intelligence (JELIA)*, number 10021 in LNCS, pages 174–189. Springer, 2016.

[Britz and Varzinczak, 2016b] K. Britz and I. Varzinczak. Preferential modalities revisited. In *Proceedings of the 16th International Workshop on Nonmonotonic Reasoning (NMR)*, 2016.

[Britz and Varzinczak, 2017] K. Britz and I. Varzinczak. Towards defeasible $d\mathcal{SROIQ}$. In *Proceedings of the 30th International Workshop on Description Logics*, volume 1879. CEUR Workshop Proceedings, 2017.

[Britz *et al.*, 2008] K. Britz, J. Heidema, and T. Meyer. Semantic preferential subsumption. In J. Lang and G. Brewka, editors, *Proceedings of the 11th International Conference on Principles of Knowledge Representation and Reasoning (KR)*, pages 476–484. AAAI Press/MIT Press, 2008.

[Britz *et al.*, 2011] K. Britz, T. Meyer, and I. Varzinczak. Semantic foundation for preferential description logics. In D. Wang and M. Reynolds, editors, *Proceedings of the 24th Australasian Joint Conference on Artificial Intelligence*, number 7106 in LNAI, pages 491–500. Springer, 2011.

[Britz *et al.*, 2013a] K. Britz, G. Casini, T. Meyer, K. Moodley, and I. Varzinczak. Ordered interpretations and entailment for defeasible description logics. Technical report, CAIR, CSIR Meraka and UKZN, South Africa, 2013.

[Britz *et al.*, 2013b] K. Britz, G. Casini, T. Meyer, and I. Varzinczak. Preferential role restrictions. In *Proceedings of the 26th International Workshop on Description Logics*, pages 93–106, 2013.

[Casini and Straccia, 2010] G. Casini and U. Straccia. Rational closure for defeasible description logics. In T. Janhunen and I. Niemelä, editors, *Proceedings of the 12th European Conference on Logics in Artificial Intelligence (JELIA)*, number 6341 in LNCS, pages 77–90. Springer-Verlag, 2010.

[Casini and Straccia, 2013] G. Casini and U. Straccia. Defeasible inheritance-based description logics. *Journal of Artificial Intelligence Research (JAIR)*, 48:415–473, 2013.

[Casini *et al.*, 2015] G. Casini, T. Meyer, K. Moodley, U. Sattler, and I. Varzinczak. Introducing defeasibility into OWL ontologies. In M. Arenas, O. Corcho, E. Simperl, M. Strohmaier, M. d'Aquin, K. Srinivas, P.T. Groth, M. Dumontier, J. Heflin, K. Thirunarayan, and S. Staab, editors, *Proceedings of the 14th International Semantic Web Conference (ISWC)*, number 9367 in LNCS, pages 409–426. Springer, 2015.

[Giordano *et al.*, 2007] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato. Preferential description logics. In N. Dershowitz and A. Voronkov, editors, *Logic for Programming, Artificial Intelligence, and Reasoning (LPAR)*, number 4790 in LNAI, pages 257–272. Springer, 2007.

[Giordano *et al.*, 2009] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato. $\mathcal{ALC} + T$: a preferential extension of description logics. *Fundamenta Informaticae*, 96(3):341–372, 2009.

[Giordano *et al.*, 2013] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato. A non-monotonic description logic for reasoning about typicality. *Artificial Intelligence*, 195:165–202, 2013.

[Giordano *et al.*, 2015] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato. Semantic characterization of rational closure: From propositional logic to description logics. *Artificial Intelligence*, 226:1–33, 2015.

[Horrocks *et al.*, 2006] I. Horrocks, O. Kutz, and U. Sattler. The even more irresistible $\mathcal{SROIQ}$. In P. Doherty, J. Mylopoulos, and C. Welty, editors, *Proceedings of the 10th International Conference on Principles of Knowledge Representation and Reasoning (KR)*, pages 57–67. Morgan Kaufmann, 2006.

[Kraus *et al.*, 1990] S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, 44:167–207, 1990.

[Lehmann and Magidor, 1992] D. Lehmann and M. Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55:1–60, 1992.

[Quantz and Royer, 1992] J. Quantz and V. Royer. A preference semantics for defaults in terminological logics. In *Proceedings of the 3rd International Conference on Principles of Knowledge Representation and Reasoning (KR)*, pages 294–305, 1992.

[Schild, 1991] K. Schild. A correspondence theory for terminological logics: Preliminary report. In *Proceedings of the 12th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 466–471, 1991.

[Sengupta *et al.*, 2011] K. Sengupta, A. Alfa Krisnadhi, and P. Hitzler. Local closed world semantics: Grounded circumscription for OWL. In L. Aroyo, C. Welty, H. Alani, J. Taylor, A. Bernstein, L. Kagal, N. Noy, and E. Blomqvist, editors, *Proceedings of the 10th International Semantic Web Conference (ISWC)*, number 7031 in LNCS, pages 617–632. Springer, 2011.

[Shoham, 1988] Y. Shoham. *Reasoning about Change: Time and Causation from the Standpoint of Artificial Intelligence*. MIT Press, 1988.