Reasoning about exceptions in ontologies: from the lexicographic closure to the skeptical closure

Laura Giordano[1] and Valentina Gliozzi[2]

[1] DISIT - Università del Piemonte Orientale, Alessandria, Italy, laura.giordano@uniupo.it
[2] Dipartimento di Informatica, Università di Torino, Italy, valentina.gliozzi@unito.it

Abstract. Reasoning about exceptions in ontologies is nowadays one of the challenges the description logics community is facing. The paper describes a preferential approach for dealing with exceptions in Description Logics, based on the rational closure. The rational closure has the merit of providing a simple and efficient approach for reasoning with exceptions, but it does not allow independent handling of the inheritance of different defeasible properties of concepts. In this work we outline a possible solution to this problem by introducing a variant of the lexicographical closure, that we call *skeptical closure*, which requires to construct a single base. A preliminary version of this work appeared in [22].

1 Introduction

Reasoning about exceptions in ontologies is nowadays one of the challenges the description logics community is facing, a challenge which is at the very roots of the development of non-monotonic reasoning in the 80s. Many non-monotonic extensions of Description Logics (DLs) have been developed incorporating non-monotonic features from most non-monotonic formalisms in the literature [2, 19, 25, 10, 21, 31, 8, 14, 37, 7, 20, 32, 13, 26, 30, 28], or defining new constructions and semantics such as in [6, 9].

The paper is based on a preferential approach for dealing with exceptions in description logics, where a typicality operator is used to select the typical (or most preferred) instances of a concept [25]. This approach, as the preferential approach in [10], has been developed along the lines of the preferential semantics introduced by Kraus, Lehmann and Magidor [33, 34].

We focus on the rational closure for DLs [14, 17, 13, 28, 12] and, in particular, on the construction developed in [28], which is semantically characterized by minimal (canonical) preferential models. While the rational closure provides a simple and efficient approach for reasoning with exceptions, exploiting polynomial reductions to standard DLs [27], the rational closure does not allow an independent handling of the inheritance of different defeasible properties of concepts[1] so that, if a subclass of C is exceptional for a given aspect, it is exceptional tout court and does not inherit any of the typical properties of C. This problem was called by Pearl [39] "the blocking of property inheritance problem", and it is an instance of the "drowning problem" in [5].

[1] By *properties* of a concept, here we generically mean characteristic features of a class of objects (represented by a set of inclusion axioms) rather than roles (properties in OWL [38]).

To cope with this problem Lehmann [35] introduced the notion of the lexicographic closure, which was extended to Description Logics by Casini and Straccia [16], while in [17] the same authors develop an inheritance-based approach for defeasible DLs. Other proposals to deal with this "all or nothing" behavior in the context of DLs are the logic of overriding, $\mathcal{DL}^N$, by Bonatti, Faella, Petrova and Sauro [6], a nonmonotonic description logic in which conflicts among defaults are solved based on specificity, and the work by Gliozzi [29], who develops a semantics for defeasible inclusions in which models are equipped with several preference relations.

In this paper we will consider a variant of the lexicographic closure. The lexicographic closure allows for stronger inferences with respect to rational closure, but computing the defeasible consequences in the lexicographic closure may require to compute several alternative *bases* [35], namely, consistent sets of defeasible inclusions which are maximal with respect to a (seriousness) ordering. We propose an alternative notion of closure, the *skeptical closure*, which can be regarded as a more skeptical variant of the lexicographic closure. It is a refinement of rational closure which allows for stronger inferences, but it is weaker than the lexicographic closure and its computation does not require to generate all the alternative maximally consistent bases. Roughly speaking, the construction is based on the idea of building a single base, i.e. a single maximal consistent set of defeasible inclusions, starting with the defeasible inclusions with highest rank and progressively adding less specific inclusions, when consistent, but excluding the defeasible inclusions which produce a conflict at a certain stage without considering alternative consistent bases.

Schedule of the paper is the following. In section 2 we recall the definition of rational closure for $\mathcal{ALC}$ in [28]. In section 3, we define the new closure and in Section 4 we conclude the paper with some discussion of related work.

2 The rational closure

We briefly recall the logic $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ which is at the basis of a rational closure construction proposed in [28] for $\mathcal{ALC}$, which extends to $\mathcal{ALC}$ the notion of rational closure introduced by Lehmann and Magidor [34]. The idea underlying $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ is that of extending the standard $\mathcal{ALC}$ with concepts of the form $\mathbf{T}(C)$, whose intuitive meaning is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C, to distinguish between the properties that hold for all instances of concept C ($C \sqsubseteq D$), and those that only hold for the typical such instances ($\mathbf{T}(C) \sqsubseteq D$). The $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ language is defined as follows: $C_R := A \mid \top \mid \bot \mid \neg C_R \mid C_R \sqcap C_R \mid C_R \sqcup C_R \mid \forall R.C_R \mid \exists R.C_R$, and $C_L := C_R \mid \mathbf{T}(C_R)$, where A is a concept name and R a role name. A KB is a pair $K = (\mathcal{T}, \mathcal{A})$, where the TBox $\mathcal{T}$ contains a finite set of concept inclusions $C_L \sqsubseteq C_R$ and the ABox $\mathcal{A}$ contains a finite set of assertions of the form $C_R(a)$ and $R(a, b)$, for a, b individual names.

The semantics of $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$ is defined in terms of rational models: ordinary models of $\mathcal{ALC}$ are equipped with a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements: $x < y$ means that x is more typical than y. The instances of $\mathbf{T}(C)$ are the instances of concept C that are minimal with respect to $<$. We refer to [28] for a detailed description of the semantics and we denote by $\models_{\mathcal{ALC}+\mathbf{T}_{\mathsf{R}}}$ entailment in $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$.

The rational closure construction assigns a rank to each concept of the KB (the highest the rank, the more specific is the concept). It is based on the notion of exceptionality. Roughly speaking $\mathbf{T}(C) \sqsubseteq D$ holds in the rational closure of K if C is less exceptional than $C \sqcap \neg D$. We shortly recall the construction of the rational closure w.r.t. TBox.

Definition 1 (Exceptionality of concepts and inclusions). *Let E be a TBox and C a concept. C is* exceptional *for E if and only if $E \models_{\mathcal{ALC}+\mathbf{T}_R} \mathbf{T}(\top) \sqsubseteq \neg C$.*[2] *An inclusion $\mathbf{T}(C) \sqsubseteq D$ is exceptional for E if C is exceptional for E. The set of inclusions in TBox which are exceptional for E will be denoted by $\mathcal{E}(E)$.*

Given a TBox $\mathcal{T}$, it is possible to define a sequence of non increasing subsets of TBox ordered according to the exceptionality of the elements $E_0 \supseteq E_1 \supseteq E_2 \ldots$ by letting $E_0 = \mathcal{T}$ and, for $i > 0$, $E_i = \mathcal{E}(E_{i-1}) \cup \{C \sqsubseteq D \in$ TBox s.t. $\mathbf{T}$ does not occurr in $C\}$. Observe that, being KB finite, there is an $n \geq 0$ such that, for all $m > n, E_m = E_n$ or $E_m = \emptyset$. A concept C has *rank* i (denoted $rank(C) = i$) for $\mathcal{T}$, iff i is the least natural number for which C is not exceptional for E_i. If C is exceptional for all E_i then $rank(C) = \infty$ (C has no rank). The rank of a typicality inclusion $\mathbf{T}(C) \sqsubseteq D$ is $rank(C)$. Rational closure builds on this notion of exceptionality:

Definition 2 (Rational closure of TBox [28]). *Let $K = (\mathcal{T}, \mathcal{K})$ be a DL knowledge base. A typicality inclusion $\mathbf{T}(C) \sqsubseteq D$ is in the rational closure of K w.r.t. TBox if either $rank(C) < rank(C \sqcap \neg D)$ or $rank(C) = \infty$.*

Exploiting the fact that entailment in $\mathcal{ALC} + \mathbf{T}_R$ can be polynomially encoded into entailment in $\mathcal{ALC}$, it is easy to see that deciding if an inclusion $\mathbf{T}(C) \sqsubseteq D$ belongs to the rational closure of TBox is a problem in EXPTIME [28].

Example 1. Let K be the knowledge base with the following TBox $\mathcal{T}$:
$\mathbf{T}(Student) \sqsubseteq \neg Pay\_Taxes$
$\mathbf{T}(WStudent) \sqsubseteq Pay\_Taxes$
$\mathbf{T}(Student) \sqsubseteq Young$
$WStudent \sqsubseteq Student$
stating that typical students do not pay taxes and are young, while typical working students (which are students) do pay taxes. We can see that $Student$ has rank 0, while $WStudent$ has rank 1 (as working students falsify the first default) and:
$E_0 = \mathcal{T}$; $\quad E_1 = \{\mathbf{T}(WStudent) \sqsubseteq Pay\_Taxes,\ WStudent \sqsubseteq Student\}$;
and the defeasible inclusions $\mathbf{T}(Student \sqcap Italian) \sqsubseteq \neg Pay\_Taxes$ and $\mathbf{T}(WStudent \sqcap Italian) \sqsubseteq Pay\_Taxes$ both belong, as expected, to the rational closure of K, as being Italian is irrelevant with respect to being or not a typical student. However, we cannot conclude that $\mathbf{T}(WStudent) \sqsubseteq Young$, as concept $WStudent$ is exceptional w.r.t. $Student$ concerning the property of paying taxes and, hence, it does not inherit *any* defeasible property of $Student$.

In this example the rational closure is too weak to infer that typical working students, as all typical students, are young. The lexicographic closure [35] strengthens the rational

[2] Observe that, as the instances of concept $\top$ are all the domain elements, $\mathbf{T}(\top)$ is the set of all the preferred domain elements w.r.t. $<$

closure by allowing to retain, roughly speaking, as many as possible of the defeasible properties, lgiving preference to the more specific properties. In the example, the property of students of being $Young$ would be inherited by working students, as it is consistent with all the other (strict or defeasible) properties of $WStudent$ (those in E_1). In the general case, there may be exponentially many alternative sets of defeasible inclusions (bases) which are maximal and consistent for a given concept and the lexicographic closure considers all of them to conclude that a defeasible inclusion is accepted. Besides specificity, the lexicographic closure also considers the number of defaults accepted, for each rank, in the alternative bases and gives preference to those bases maximizing the number of defaults with the highest rank. In the next section we propose an approach weaker than the lexicographic closure, which leads to the construction of a single base.

3 From the lexicographic to the skeptical closure

Given a concept B, one wants to identify the defeasible properties of the B-elements. Assume that the rational closure of the knowledge base K has already been constructed and that k is the rank of concept B in the rational closure. The typical B elements are clearly compatible with all the defeasible inclusions in E_k, but they might satisfy other defeasible inclusions with lower rank, i.e. those included in $E_0, E_1, \ldots, E_{k-1}$. In general, there may be alternative maximal sets of defeasible inclusions compatible with B, among which one would prefer those that maximize the number of defeasible inclusions with higher rank. This is indeed what is done by the lexicographic closure [35], which considers alternative maximally preferred sets of defaults called "bases", which, roughly speaking, maximize the number of defaults of higher ranks with respect to those with lower ranks (the so called degree of seriousness), and where situations which violate more defaults with a certain rank are considered to be less plausible than situations which violates less defaults with the same rank. As a difference, in the following, we aim at defining a construction which skeptically builds a single set of defeasible inclusions compatible with B.

Let S^B be the set of typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ in K which are *individually compatible with* B *w.r.t.* E_k, that is

$S^B = \{\mathbf{T}(C) \sqsubseteq D \in \text{TBox} \mid \; E_k \cup \{\mathbf{T}(C) \sqsubseteq D\} \not\models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq \neg B\}$.

Clearly, although each defeasible inclusion in S^B is compatible with B, it might be the case that overall set S^B *is not compatible with* B, i.e., $E_k \cup S^B \models_{\mathcal{ALC}+\mathbf{T_R}} \mathbf{T}(\top) \sqsubseteq \neg B$. When compatible with B, S^B is the unique maximal basis with respect to the *seriousness ordering* in [35] (as defined for constructing the lexicographic closure).

When S^B is not compatible with B, we cannot use all the defeasible inclusions in S^B to derive conclusions about typical B elements. In this case, we can either just use the defeasible inclusions in E_k, as in the rational closure, or we can additionally use a subset of the defeasible inclusions S^B. For instance, we can additionally use all the defeasible inclusions in S^B with rank $k-1$ (let us call this set S^B_{k-1}), provided they are (altogether) compatible with B and E_k. Then, we can, possibly, add all the defeasible inclusions with rank $k-2$ which are individually compatible with B w.r.t. $E_k \cup S^B_{k-1}$ (let us call them S^B_{k-2}), provided they are altogether compatible with B, E_k and S^B_{k-1}, and so on and so forth, for lower ranks. This leads to the construction below.

Definition 3. *Given two sets of defeasible inclusions S and S', S* is globally compatible with B w.r.t. S' *if* $S \cup S' \not\models_{\mathcal{ALC}+\mathbf{T}_R} \mathbf{T}(\top) \sqsubseteq \neg B$.

Definition 4. *Let B be a concept such that $rank(B) = k$. The* skeptical closure of K with respect to B *is the set of inclusions* $S^{sk,B} = E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_h$ *where:*
- $S^B_i \subseteq E_i - E_{i+1}$ is the set of defeasible inclusions with rank i which are individually compatible with B w.r.t. $E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_{i+1}$ (for each finite rank $i \leq k$);
- h is the least j (for $0 \leq j \leq k-1$) such that S^B_j is globally compatible with B w.r.t. $E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S^B_{j+1}$, if such a j exists; $S^{sk,B} = E_k$, otherwise.

Intuitively, $S^{sk,B}$ contains, for each rank j, all the defeasible inclusions having rank j which are compatible with B and with the more specific defeasible inclusions (with rank $> j$). As S^B_{h-1} is not included in the skeptical closure, it must be that $E_k \cup S^B_{k-1} \cup S^B_{k-2} \cup \ldots \cup S_h \cup S^B_{h-1} \models_{\mathcal{ALC}+\mathbf{T}_R} \mathbf{T}(\top) \sqsubseteq \neg B$ i.e., the set S^B_{h-1} contains conflicting defeasible inclusions which are not overridden by more specific ones. In this case, the inclusions in S^B_{h-1} (and all the defeasible inclusions with rank lower than $h-1$) are not included in the skeptical closure w.r.t. B. Let us now define entailment of a defeasible inclusion from the skeptical closure of TBox.

Definition 5. *Let $\mathbf{T}(B) \sqsubseteq D$ be a defeasible inclusion and let $k = rank(B)$ be the rank of concept B in the rational closure.* $\mathbf{T}(B) \sqsubseteq D$ is in the skeptical closure of TBox *if* $S^{sk,B} \models_{\mathcal{ALC}+\mathbf{T}_R} \mathbf{T}(\top) \sqsubseteq (\neg B \sqcup D)$.

After the rational closure of the TBox has been computed, the identification of the defeasible inclusions in $S^{sk,B}$ requires a number of entailment checks which is linear in the number of defeasible inclusions in TBox: the individual compatibility of a defeasible inclusion of rank i in TBox has to be checked only once to compute S^B_i; also, for each rank i of the rational closure (in the worst case), a (global) compatibility check is needed for S^B_i.

In Example 1 the inclusion $\mathbf{T}(\mathit{WStudent}) \sqsubseteq \mathit{Young}$ is in the skeptical closure of TBox, as $\mathit{WStudent}$ has rank 1 and inclusion $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$ in E_0 is compatible with $\mathit{WStudent}$. No other inclusions with rank 0 are compatible with E_1.

Example 2. Let us consider, instead, the knowledge base K' with TBox:
$\mathbf{T}(\mathit{Student}) \sqsubseteq \neg \mathit{Pay\_Taxes}$
$\mathbf{T}(\mathit{Worker}) \sqsubseteq \mathit{Pay\_Taxes}$
$\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$
$\mathit{WStudent} \sqsubseteq \mathit{Student} \sqcap \mathit{Worker}$
the inclusion $\mathbf{T}(\mathit{WStudent}) \sqsubseteq \mathit{Young}$ is not in the skeptical closure of TBox', as S_0^{WStudent} is not compatible with $\mathit{WStudent}$ (w.r.t. E_1), due to the conflicting defaults concerning tax payment for Worker and $\mathit{Student}$ (both with rank 0). Hence, the defeasible property that typical students are young is not inherited by typical working students.

Notice that, the property that typical working students are young is accepted in the lexicographic closure of K', as there are two bases (the one including $\mathbf{T}(\mathit{Student}) \sqsubseteq \neg \mathit{Pay\_Taxes}$ and the other $\mathbf{T}(\mathit{Worker}) \sqsubseteq \mathit{Pay\_Taxes}$), both containing $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{Young}$. The skeptical closure is indeed weaker than the lexicographic closure.

4 Conclusions and related work

We have introduced a weaker variant of the lexicographic closure [35, 16], which deals with the problem of "all or nothing" affecting the rational closure without generating alternative "bases". Other refinements of the rational closure, also deal with this limitation of the rational closure, are the relevant closure [11] and the inheritance-based rational closure [15, 17], In particular, in [15, 17], a new closure operation is defined by combining the rational closure with defeasible inheritance networks. The inheritance-based rational closure, in Example 2, is able to conclude that typical working students are young, relying on the fact that only the information related to the connection of $WStudent$ and $Young$ (and, in particular, only the defeasible inclusions occurring on the routes connecting $WStudent$ and $Young$ in the corresponding net) are used in the rational closure construction for answering the query.

Another approach which deals with the above problem of "inheritance with exceptions" has been proposed by Bonatti et al. in [6], where the logic $\mathcal{DL}^N$ captures a form of "inheritance with overriding": a defeasible inclusion is inherited by a more specific class if it is not overridden by more specific (conflicting) properties. In Example 2, our construction behaves differently from $\mathcal{DL}^N$, as in $\mathcal{DL}^N$ the concept $WStudent$ has an inconsistent prototype, as working students inherit two conflicting properties by superclasses: the property of students of paying taxes and the property of workers of paying taxes. In the skeptical closure one cannot conclude that $\mathbf{T}(WStudent) \sqsubseteq \bot$ and, using the terminology in [6], the conflict is "silently removed". In this respect, the skeptical closure appears to be weaker than $\mathcal{DL}^N$, although it shares with $\mathcal{DL}^N$ (and with lexicographic closure) a notion of overriding.

Bozzato et al. in [9] present an extension of the CKR framework in which defeasible axioms can be included in the global context and can be overridden by knowledge in a local context. Exceptions have to be justified in terms of semantic consequence. A translation of extended CHRs (with knowledge bases in $\mathcal{SROIQ}$-RL) into Datalog programs under the answer set semantics is also defined.

Concerning the multipreference semantics introduced in [29] (and further refined in [23]) to provide a semantic strengthening of the rational closure, we have shown in [23] that a variant of Lehmann's lexicographic closure (which does not take into account the number of defaults within the same level, but only their subset inclusion) provides a sound approximation of the multipreference semantics. We expect that the skeptical closure introduced in this work is still a sound, though weaker, approximation for the multipreference semantics in [23].

Detailed comparisons and the study of the semantics underlying the skeptical closure will be subject of future work. The relationships among the above variants of rational closure for DLs and the notions of rational closure for DLs developed in the contexts of fuzzy logic [18] and probabilistic logics [36] have to be investigated as well. As it has been show in [3] for the propositional logic case, KLM preferential logics and the rational closure [33, 34], the probabilistic approach [1], the system Z [39] and the possibilistic approach [4, 3] are all related with each other, and similar relations might be expected to hold for the non-monotonic extensions of description logics as well. Although the skeptical closure has been defined based on the preferential extension of

$\mathcal{ALC}$, the same construction could be adopted for more expressive description logics, provided the rational closure can be defined [24], as well as for the propositional case.

Acknowledgement: We thank the anonymous reviewers for their helpful comments.

References

1. E.W. Adams. *The logic of conditionals*. D. Reidel, Dordrecht, 1975.
2. F. Baader and B. Hollunder. Priorities on defaults with prerequisites, and their application in treating specificity in terminological default logic. *Journal of Automated Reasoning (JAR)*, 15(1):41–68, 1995.
3. S. Benferhat, D. Dubois, and H. Prade. Nonmonotonic reasoning, conditional objects and possibility theory. *Artificial Intelligence*, 92(1-2):259–276, 1997.
4. Salem Benferhat, Didier Dubois, and Henri Prade. Representing default rules in possibilistic logic. In *Proc. KR'92, Cambridge, MA, October 25-29, 1992.*, pages 673–684, 1992.
5. Salem Benferhat, Didier Dubois, and Henri Prade. Possibilistic logic: From nonmonotonicity to logic programming. In *Symbolic and Quantitative Approaches to Reasoning and Uncertainty, European Conference, ECSQARU'93, Granada, Spain, November 8-10, 1993, Proceedings*, pages 17–24, 1993.
6. P. A. Bonatti, M. Faella, I. Petrova, and L. Sauro. A new semantics for overriding in description logics. *Artif. Intell.*, 222:1–48, 2015.
7. P. A. Bonatti, M. Faella, and L. Sauro. Defeasible inclusions in low-complexity DLs. *J. Artif. Intell. Res. (JAIR)*, 42:719–764, 2011.
8. P. A. Bonatti, C. Lutz, and F. Wolter. The Complexity of Circumscription in DLs. *Journal of Artificial Intelligence Research (JAIR)*, 35:717–773, 2009.
9. L. Bozzato, T. Eiter, and L. Serafini. Enhancing context knowledge repositories with justifiable exceptions. *Artif. Intell.*, 257:72–126, 2018.
10. Katarina Britz, Johannes Heidema, and Thomas Meyer. Semantic preferential subsumption. In G. Brewka and J. Lang, editors, *Principles of Knowledge Representation and Reasoning: Proceedings of the 11th International Conference (KR 2008)*, pages 476–484, Sidney, Australia, September 2008. AAAI Press.
11. G. Casini, T. Meyer, K. Moodley, and R. Nortje. Relevant closure: A new form of defeasible reasoning for description logics. In *JELIA 2014*, LNCS 8761, pages 92–106. Springer, 2014.
12. G. Casini, T. Meyer, K. Moodley, U. Sattler, and I.J. Varzinczak. Introducing defeasibility into OWL ontologies. In *The Semantic Web - ISWC 2015 - 14th International Semantic Web Conference, Bethlehem, PA, USA, October 11-15, 2015, Proceedings, Part II*, pages 409–426, 2015.
13. G. Casini, T. Meyer, I. J. Varzinczak, , and K. Moodley. Nonmonotonic Reasoning in Description Logics: Rational Closure for the ABox. In *DL 2013, 26th International Workshop on Description Logics*, volume 1014 of *CEUR Workshop Proceedings*, pages 600–615. CEUR-WS.org, 2013.
14. G. Casini and U. Straccia. Rational Closure for Defeasible Description Logics. In T. Janhunen and I. Niemelä, editors, *Proc. JELIA 2010*, volume 6341 of *Lecture Notes in Artificial Intelligence*, pages 77–90, Helsinki, Finland, September 2010. Springer.
15. G. Casini and U. Straccia. Defeasible Inheritance-Based Description Logics. In Toby Walsh, editor, *Proc. IJCAI 2011*, pages 813–818, Barcelona, Spain, July 2011. Morgan Kaufmann.
16. G. Casini and U. Straccia. Lexicographic Closure for Defeasible Description Logics. In *Proc. of Australasian Ontology Workshop, vol.969*, pages 28–39, 2012.
17. G. Casini and U. Straccia. Defeasible inheritance-based description logics. *Journal of Artificial Intelligence Research (JAIR)*, 48:415–473, 2013.

18. G. Casini and U. Straccia. Towards rational closure for fuzzy logic: The case of propositional Gödel logic. In *Logic for Programming, Artificial Intelligence, and Reasoning - 19th International Conference, LPAR-19, Stellenbosch, South Africa, December 14-19, 2013. Proceedings*, pages 213–227, 2013.
19. F. M. Donini, D. Nardi, and R. Rosati. Description logics of minimal knowledge and negation as failure. *ACM Transactions on Computational Logic (ToCL)*, 3(2):177–225, 2002.
20. T. Eiter, G. Ianni, T. Lukasiewicz, and R. Schindlauer. Well-founded semantics for description logic programs in the semantic web. *ACM Trans. Comput. Log.*, 12(2):11, 2011.
21. T. Eiter, G. Ianni, T. Lukasiewicz, R. Schindlauer, and H. Tompits. Combining answer set programming with description logics for the semantic web. *Artif. Intell.*, 172(12-13):1495–1539, 2008.
22. L. Giordano. Reasoning about exceptions in ontologies: a skeptical preferential approach (extended abstract). In *Joint Proc. of ICTCS 2017 and CILC 2017, Naples, Italy, September 26-28, 2017*, volume 1949 of *CEUR Workshop Proceedings*, pages 6–10, 2017.
23. L. Giordano and V. Gliozzi. Reasoning about multiple aspects in dls: Semantics and closure construction. *CoRR*, abs/1801.07161, 2018.
24. L. Giordano, V. Gliozzi, and N. Olivetti. Towards a rational closure for expressive description logics: the case of $\int\langle\rangle$II. *Fundam. Inform.*, 159(1-2):95–122, 2018.
25. L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Preferential Description Logics. In Nachum Dershowitz and Andrei Voronkov, editors, *Proceedings of LPAR 2007*, volume 4790 of *LNAI*, pages 257–272, Yerevan, Armenia, October 2007. Springer-Verlag.
26. L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. A NonMonotonic Description Logic for Reasoning About Typicality. *Artificial Intelligence*, 195:165–202, 2013.
27. L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Rational Closure in SHIQ. In *DL2014*, volume 1193 of *CEUR Workshop Proceedings*, pages 1–13, 2014.
28. L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Semantic characterization of rational closure: From propositional logic to description logics. *Artificial Intelligence*, 226:1–33, 2015.
29. V. Gliozzi. Reasoning about multiple aspects in rational closure for DLs. In *Proc. AI*IA 2016, Genova, Italy, November 29 - December 1, 2016*, pages 392–405, 2016.
30. G. Gottlob, A. Hernich, C. Kupke, and T. Lukasiewicz. Stable model semantics for guarded existential rules and description logics. In *Proc. KR 2014*, 2014.
31. P. Ke and U. Sattler. Next Steps for Description Logics of Minimal Knowledge and Negation as Failure. In *DL 2008*, volume 353 of *CEUR Workshop Proceedings*, Dresden, Germany, May 2008. CEUR-WS.org.
32. M. Knorr, P. Hitzler, and F. Maier. Reconciling OWL and non-monotonic rules for the semantic web. In *ECAI 2012*, page 474479, 2012.
33. S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, 44(1-2):167–207, 1990.
34. D. Lehmann and M. Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55(1):1–60, 1992.
35. D. J. Lehmann. Another perspective on default reasoning. *Ann. Math. Artif. Intell.*, 15(1):61–82, 1995.
36. T. Lukasiewicz. Expressive probabilistic description logics. *Artif. Intell.*, 172:852–883, 2008.
37. B. Motik and R. Rosati. Reconciling Description Logics and rules. *Journal of the ACM*, 57(5), 2010.
38. P.F. Patel-Schneider, P.H. Hayes, and I. Horrocks. OWL Web Ontology Language; Semantics and Abstract Syntax. In *http: //www.w3.org/TR/owl-semantics/*, 2002.
39. J. Pearl. System Z: A natural ordering of defaults with tractable applications to nonmonotonic reasoning. In R. Parikh, editor, *TARK 1990*, pages 121–135, Pacific Grove, CA, USA, 1990. Morgan Kaufmann.