Muticriteria Model of Balanced Layout Problem of 3D-Objects

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Abstract: **The paper studies the optimal layout problem of 3D-objects. The problem takes into account placement constraints, as well as, behaviour characteristics of the mechanical system. We construct a mathematical model of the problem in the form of a multicriteria optimisation problem and call the problem Multicriteria Balanced Layout Problem (MBLP).**

Keywords: **Layout problem, Behaviour Constraints, Placement Constraints, Multicriteria Optimisation.**

I. Introduction

3D layout optimisation problems have a wide spectrum of practical applications. In particular, these problems arise in space engineering for rocketry design. Their distinctive feature consists of taking into account behaviour constraints of a satellite system. Behaviour constraints specify the requirements for system's mechanical properties such as equilibrium, inertia, and stability. Many publications analyze problems of the equipment layout in modules of spacecraft or satellites [1, 2]. These problems are NP-hard.

In the research we consider the balance layout problem in the following statement: arrange 3D-objects in a container taking into account special placement and behaviour constraints so that the objective function attains its extreme value [3].

We consider here an extension of the balanced layout of 3D-objects considered in [3,4]. The paper studies 3D optimisation balance layout problem taking into account minimal and maximal allowable distances. Classes of adjusted phi-functions and adjusted quasi-phi-functions are derived for analytical description of non-overlapping, containment and distance constraints. A circular cylinder, a paraboloid, or a truncated cone are taken as a container. We consider cylinders, spheres, tores, spherecylinders and straight convex prisms as the placement objects. An exact mathematical model of the problem in the form of NLP problem is provided.

The aim of this study is to develop a mathematical model of 3D layout optimisation problem taking into account behavior constraints in the form of multicriteria optimisation problem. We call the problem the Multicriteria Balanced Layout Problem (MBLP).

To describe placement constraints (non-overlapping of objects, containment of objects in a container with regard for the minimal and maximal allowable distances) analytically we employ phi-function technique [5]. We also formalise behaviour constraints (equilibrium, moments of inertia, and stability constraints) based on [3].

The variety of forms of objective functions and combinations of placement and behaviour constraints generates various variants of the MBLP problem.

II. Problem Formulation

Let $\Omega = \{(x, y, z) \in \mathbb{R}^3 : \ G(x, y, z) \geq 0\}$ be a container of given height H. We consider the following types of containers: 1) $\Omega \equiv \mathbf{C}$, $\mathbf{C}$ is a straight circular cylinder with a base of radius R, $G(x, y, z) = \min\{-x^2 - y^2 + R^2, -z + H, z\}$; 2) $\Omega \equiv \mathbf{\Lambda}$, $\mathbf{\Lambda}$ is a paraboloid of revolution with a base of radius $R = \sqrt{H}$, $G(x, y, z) = \min\{-z - x^2 - y^2 + H, z\}$; 3) $\Omega \equiv \mathbf{E}$, $\mathbf{E}$ is a straight circular blunted cone with lower and upper bases of radii R_1 and $R_2 < R_1$ respectively, $G(x, y, z) = \min\{-z - H(\sqrt{x^2 + y^2} - R_1)/(R_1 - R_2), -z + H, z\}$. Suppose that Ω is divided by circular racks S_k, $k = 1, 2, \ldots, m+1$, into subcontainers Ω^k, $k = 1, 2, \ldots, m$. We assume that S_1 is a base of Ω. Between racks S_k and S_{k+1} the distance t_k is given.

Family $A = \{A_i, i \in I_n\}$, $I_n = \{1, 2, .., n\}$, involves the following shapes of objects: solid spheres $\mathbb{S}_i$ of radius r_i; straight circular cylinders $\mathbb{C}_i$ of radius r_i and height $2h_i$; tori $\mathbb{T}_i$ with metric characteristics (r_i, h_i), where r_i is the distance from the center of generating circle to the axis of revolution, $2h_i$ is the height of $\mathbb{T}_i$, h_i is the radius of the generating circle; spherocylinders $\mathbb{S}_{\mathbb{C}i}$ with metric characteristics (l_i, r_i, h_i), where l_i is the height of ball segments, r_i is the radius and $2h_i$ is the height of cylinder; straight regular prisms and cuboids $\mathbb{K}_i$ with metric characteristics $(h_i, \tilde{v}_{il},)$, where $2h_i$ is the height of $\mathbb{K}_i$, $\tilde{v}_{il} = (\tilde{x}_{il}, \tilde{y}_{il})$, $l = 1, \ldots, s_i$, are vertices of the base of $\mathbb{K}_i$ (which is a convex polygon K_i), s_i is the number of vertices of K_i.

MBLP: Pack 3D-objects $A_i \in A$, $i \in I_n = \{1, 2, \ldots, n\}$, inside container Ω, so that the vector function attains its extreme value with regard for placement and behaviour constraints.

The placement constraints in the MBLP problem are generated by non-overlapping of objects A_i, A_j, $i > j \in I_n$, which have to be placed inside container Ω, and containment of object A_i in container Ω, $i \in I_n$. In addition, the minimal ρ_{ij}^- and maximal $\rho_{ij}^+ \geq \rho_{ij}^-$ allowable distances between objects A_i, A_j, $i > j \in I_n$, may be specified. Also, the minimal allowable distance ρ_i^- between object $A_i \in A$, $i \in I_n$, and the lateral surface of container Ω may be given. Without loss of generality we set $\rho_{ij}^- = 0$ (or $\rho_{ij}^+ = \varpi$) if a minimal (or a maximal) allowable distance between objects A_i and A_j is not given, $i > j \in I_n$. Here ϖ is a given sufficiently great number. In particular, the condition $\rho_{ij}^+ = \rho_{ij}^-$ provides the arrangment of objects A_i and A_j on the exact distance. We also set $\rho_i^- = 0$ if a minimal allowable distance between object A_i and the lateral surface of container Ω is not given.

Placement constraints in the MBLP problem may be presented as the following: $\rho_{ij}^- \leq \mathrm{dist}(A_i, A_j) \leq \rho_{ij}^+$, $i > j \in I_n$, and $\mathrm{dist}(A_i, \Omega^*) \geq \rho_i^-$, $i = 1, \ldots, n$, where $\Omega^* = \mathbb{R}^3 \setminus \mathrm{int}\,\Omega$.

To describe the placement constraints analytically we employ the phi-function technique [5–8].

Let us consider the constraints of mechanical characteristics of system Ω_A.

The equilibrium constaints are defined by the following system of inequalities:

$$\mu_{11}(u) = \min\{-(x_s(u) - x_e) + \Delta x_e, (x_s(u) - x_e) + \Delta x_e\} \geq 0$$

$$\mu_{12}(u) = \min\{-(y_s(u) - y_e) + \Delta y_e, (y_s(u) - y_e) + \Delta y_e\} \geq 0,$$

$$\mu_{13}(u) = \min\{-(z_s(u) - z_e) + \Delta z_e, (z_s(u) - z_e) + \Delta z_e\} \geq 0,$$

where (x_e, y_e, z_e) is the expected position of O_s, $(\Delta x_e, \Delta y_e, \Delta z_e)$ are admissible deviations from the point (x_e, y_e, z_e).

The constraints of moments of inertia are defined as the following:

$$\mu_{21}(u) = -J_X(u) + \Delta J_X \geq 0,$$

$$\mu_{22}(u) = -J_Y(u) + \Delta J_Y \geq 0,$$

$$\mu_{23}(u) = -J_Z(u) + \Delta J_Z \geq 0,$$

where $J_X(u), J_Y(u), J_Z(u)$ are the moments of inertia of the system Ω_A with respect to the axes of coordinate system $O_s XYZ$, $\Delta J_X, \Delta J_Y, \Delta J_Z$ are admissible values for $J_X(u), J_Y(u), J_Z(u)$, where

$$J_X(u) = J_{x_0} + \sum_{i=1}^{n} (J_{x_i} \cos^2 \theta_i + J_{y_i} \sin^2 \theta_i) + \sum_{i=1}^{n} (y_i^2 + z_i^2) m_i - M(y_s^2 + z_s^2),$$

$$J_Y(u) = J_{y_0} + \sum_{i=1}^{n} (J_{x_i} \sin^2 \theta_i + J_{y_i} \cos^2 \theta_i) + \sum_{i=1}^{n} (x_i^2 + z_i^2) m_i - M(x_s^2 + z_s^2)$$

$$J_Z(u) = \sum_{i=0}^{n} J_{z_i} + \sum_{i=1}^{n} (y_i^2 + z_i^2) m_i - M(x_s^2 + y_s^2),$$

$J_{x_0}, J_{y_0}, J_{z_0}$ are the moments of inertia of container Ω with respect to the axes of the coordinate system $Oxyz$, $J_{x_i}, J_{y_i}, J_{z_i}$, $i \in I_n$, are the moments of inertia of object A_i with respect to the axes of coordinate system $O_i x_i y_i z_i$.

The stability constraints are defined by the following system of inequalities:

$$\mu_{31}(u) = \min\{-J_{XY}(u) + \Delta J_{XY}, J_{XY}(u) + \Delta J_{XY}\} \geq 0,$$

$$\mu_{32}(u) = \min\{-J_{YZ}(u) + \Delta J_{YZ}, J_{YZ}(u) + \Delta J_{YZ}\} \geq 0,$$

$$\mu_{33}(u) = \min\{-J_{XZ}(u) + \Delta J_{XZ}, J_{XZ}(u) + \Delta J_{XZ}\} \geq 0,$$

where $J_{XY}(u), J_{YZ}(u), J_{XZ}(u)$ are the products of inertia of system Ω_A with respect to the axes of the coordinate system $O_s XYZ$, $\Delta J_{XY}, \Delta J_{YZ}, \Delta J_{XZ}$ are admissible values for $J_{XY}(u), J_{YZ}(u), J_{XZ}(u)$, respectively,

$$J_{XY}(u) = \frac{1}{2} \sum_{i=1}^{n} (J_{x_i} - J_{y_i}) \sin 2\theta_i + \sum_{i=1}^{n} x_i y_i m_i - M x_s y_s,$$

$$J_{YZ}(u) = \sum_{i=1}^{n} y_i z_i m_i - M y_s z_s,$$

$$J_{XZ}(u) = \sum_{i=1}^{n} x_i z_i m_i - M x_s z_s.$$

Behaviour constraints of the BLP problem we define as the system of inequalities

$$\mu_1(u) \geq 0, \mu_2(u) \geq 0, \mu_3(u) \geq 0,$$

where

$$\mu_1(u) = \min\{\mu_{11}(u), \mu_{12}(u), \mu_{13}(u)\}, \quad (1)$$

$$\mu_2(u) = \min\{\mu_{21}(u), \mu_{22}(u), \mu_{23}(u)\}, \quad (2)$$

$$\mu_3(u) = \min\{\mu_{31}(u), \mu_{32}(u), \mu_{33}(u)\}. \quad (3)$$

Here $O_s = (x_s, y_s, z_s)$ is the center of mass of system Ω_A,

$$x_s(u) = \frac{1}{M} \sum_{i=1}^{n} m_i x_i, \; y_s(u) = \frac{1}{M} \sum_{i=1}^{n} m_i y_i,$$

$z_s(u) = \frac{1}{M} \sum_{i=1}^{n} m_i z_i$, $M = \sum_{i=0}^{n} m_i$ is the mass of system Ω_A.

III. Mathematical Model

A mathematical model of the MBLP problem can be presented in the form

$$extrF(p, u) \text{ s.t. } (u, p) \in W \quad (4)$$

$$W = \{(u, p) \in \mathbb{R}^{\xi} : \Upsilon(u, p) \geq 0, \mu(u, p) \geq 0, \zeta \geq 0\}, \quad (5)$$

where $F(p,u) = (F_1(p,u), F_2(p,u), \ldots, F_k(p,u))$,
$\Upsilon(u, p)$ describes placement constraints,
$\Upsilon(\mathrm{u,p}) = \min\{\Upsilon_1(\mathrm{u}), \Upsilon_2(\mathrm{u,p})\}$,
$\Upsilon_1(\mathrm{u})$ is responsible for non-overlapping constraints,
$\Upsilon_2(\mathrm{u,p})$ is responsible for containment constraints,
$\mu(u) = \min\{\mu_s(u), s \in U_t\}$ is responsible for behavior constraints, $U_t \in P(U)$, $P(U)$ is the power set of $U = \{1,2,3\}$, functions $\mu_1(u), \mu_2(u), \mu_3(u)$ are given by (1)-(3), $\zeta \geq 0$ is the system of additional constraints of metric characteristics of container Ω and placement parameters of objects. If $s = \varnothing$, i.e. behavior constraints are not involved in (5), then our objective function $F(u)$ meets mechanical characteristics of system Ω_A.

Depending on the different combinations of objective functions $F_1(p,u), F_2(p,u), \ldots, F_k(p,u)$ different variants of mathematical model (4)-(5) can be generated. The most frequently occurring objective functions found in related publications are the following: 1) size of container Ω; 2) deviation of the center of mass of system Ω_A from a given point; 3) moments of inertia of system Ω_A [3,9-13].

Let us consider some of realisations of model (4) - (5):

- $F(p,u) = p$ s.t. $(p,u) \in W \subset \mathbb{R}^{\xi}$,

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu(p,u) \geq 0, \zeta \geq 0\}$$

- $F(u) = d$, $(p,u) \in W \subset \mathbb{R}^{\xi}$,

$$d = (x_s(u) - x_e)^2 + (y_s(u) - y_e)^2 + (z_s(u) - z_e)^2,$$

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_2(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\};$$

- $F(p,u) = (F_1(p,u) = p, F_2(p,u) = d)$, $(p,u) \in W \subset \mathbb{R}^{\xi}$,

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_2(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\};$$

- $F(p,u) = (F_1(p,u) = J_X(p,u), F_2(p,u) = J_Y(p,u), F_3(p,u) = J_Z(p,u))$

$$(p,u) \in W \subset \mathbb{R}^{\xi},$$

$$W = \{(p,u) \in \mathbb{R}^{\xi} : \Upsilon_1(u) \geq 0, \Upsilon_2(p,u) \geq 0, \mu_1(p,u) \geq 0, \mu_3(p,u) \geq 0, \zeta \geq 0\}.$$

III. CONCLUSIONS

In this paper we formulate the optimisation layout problem of 3D-objects into a container taking into account placement (non-overlapping, containment, distance) and behaviour (equilibrium, inertia and stability) constraints. We call the problem as Multicriteria Balance Layout Problem (MBLP). In order to describe placement constraints analytically we employ phi-function technique. A mathematical model of the problem in the form of multicriteria optimisation problem is proposed.

We also consider some variants of the MBLP problem depending on the forms of the objective functions, shapes of objects and containers, combinations of placement and behavior constraints.

IV. COMPUTATIONAL RESULTS

Instance 1. Let $\Omega \equiv \mathbf{E}$, $m = 2$, $H = 0.6$, $R_1 = 0.5$, $R_3 = 0.3$, $A = \{\mathbb{S}_1, \mathbb{S}_2, \mathbb{C}_3, \mathbb{C}_4, \mathbb{T}_5, \mathbb{T}_6, \mathbb{S}_{C7}, \mathbb{S}_{C8}, \mathbb{K}_9, \mathbb{K}_{10}\}$, $A_{-}^1 = \{\mathbb{S}_1, \mathbb{C}_3, \mathbb{T}_5, \mathbb{S}_{C7}, \mathbb{K}_9\}$, $A_{+}^2 = \{\mathbb{S}_2, \mathbb{C}_4, \mathbb{T}_6, \mathbb{S}_{C8}, \mathbb{K}_{10}\}$, $\rho_{ij}^{-} = 0.03$, $i < j \in I_{10}$, $\rho_{39}^{+} = 0.1$, $\rho_{26}^{+} = 0.08$, $(x_e, y_e, z_e) = (0, 0, 0.275)$, $t_1 = 0.3$, $n = 10$, $\{z_i, i = 1,\ldots,10\}$={0.19, 0.4, 0.19, 0.41, 0.24, 0.35, 0.19, 0.39, 0.18, 0.42}, $\{m_i, i = 1,\ldots,10\}$={27.8764, 20.944, 34.5575, 16.9332, 28.4245, 22.2066, 17.2159, 19.2265, 38.4, 19.9532}, $r_1 = 0.11$, $r_2 = 0.1$, $r_3 = 0.1$, $h_3 = 0.11$, $r_4 = 0.07$, $h_4 = 0.11$, $r_5 = 0.08$, $h_5 = 0.06$, $r_6 = 0.09$, $h_6 = 0.05$, $r_7 = 0.08$, $h_7 = 0.05$, $l_7 = 0.06$, $h_8 = 0.06$, $l_8 = 0.03$, $s_9 = 4$, $h_9 = 0.12$, $\tilde{v}_{91} = (0.08, 0.1)$, $\tilde{v}_{92}$ = (0.08,-0.1), $\tilde{v}_{93}$ =(-0.08,-0.1), $\tilde{v}_{94} = (-0.08, 0.1)$, $s_{10} = 6$, $h_{10} = 0.12$, $\tilde{v}_{(10)1} = (0.04, 0.07)$, $\tilde{v}_{(10)2} = (0.08, 0)$, $\tilde{v}_{(10)3}$, =(0.04,-0.07) $\tilde{v}_{(10)4}$ =(-0.04,-0.07), $\tilde{v}_{(10)5} = (-0.08, 0)$, $\tilde{v}_{(10)6} = (-0.04, 0.07)$.

The local-optimal solution found by NLP-solver in CAS Math 9 (Fig. 1) is $F(u^*, u'^*) = 1.12726 \times 10^{-6}$.

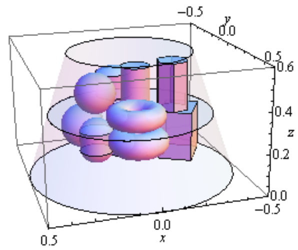

Fig. 1 The local optimal layout of 3D-objects in Instance 1

Instance 2. Let $\Omega \equiv \mathbf{C}$, $m = 3$, $H = 1$, $R = 0.45$, $t_2 = 0.35$, $n = 20$, $A = \{\mathbb{S}_i, i = 1,\ldots,4,$ $\mathbb{C}_i, i = 5,\ldots,8,$ $\mathbb{T}_i, i == 9\ldots12,$ $\mathbb{S}_{Ci}, i = 13,\ldots16,$ $\mathbb{K}_i, i = 17,\ldots,20\}$, $A_{+}^1 = \{\mathbb{S}_1, \mathbb{C}_5, \mathbb{C}_6, \mathbb{T}_9, \mathbb{S}_{C14}, \mathbb{P}_{17}\}$, $A_{+}^2 = \{\mathbb{S}_2, \mathbb{S}_3, \mathbb{C}_7, \mathbb{T}_{10}, \mathbb{S}_{C15}, \mathbb{P}_{18}, \mathbb{K}_{20}\}$, $A_{+}^3 = \{\mathbb{S}_4, \mathbb{C}_8, \mathbb{T}_{11}, \mathbb{T}_{12}, \mathbb{S}_{C16}, \mathbb{P}_{19}\}$, $U_t = \varnothing$, $\rho_{ij}^{-} = 0.02$, $i < j = 1,\ldots,20$, $(x_e, y_e, z_e) = (0, 0, 0.5)$, $\{z_i, i = 1,\ldots,20\}$={0.1, 0.44, 0.46, 0.81, 0.11, 0.12, 0.46, 0.78, 0.06, 0.425, 0.76, 0.77, 0.11, 0.13, 0.46, 0.81, 012, 0.47, 0.82, 0.46}, $\{m_i, i = 1,..,20\}$={20.944, 15.2681, 27.8764, 34.5575, 63.7115, 41.8146, 30.4106, 28.4245, 49.9649, 24.8714, 38.6888, 26.2637, 20.7764, 17.2159, 16.8756, 52.8, 52.8, 52.8, 23.1489}, $r_1 = 0.1$, $r_2 = 0.09$, $r_3 = 0.11$,

$r_4 = 0.11$, $r_5 = 0.1$, $h_5 = 0.11$, $h_6 = 0.12$, $r_7 = 0.11$, $r_8 = 0.11$, $h_8 = 0.08$, $r_9 = 0.08$, $h_9 = 0.07$, $r_{10} = 0.09$, $h_{10} = 0.075$, $r_{11} = 0.07$, $h_{11} = 0.06$, $r_{12} = 0.08$, $h_{12} = 0.07$, $r_{13} = 0.1$, $h_{13} = 0.05$, $l_{13} = 0.07$, $r_{14} = 0.05$, $h_{14} = 0.05$, $l_{14} = 0.08$, $r_{15} = 0.08$, $h_{15} = 0.05$, $l_{15} = 0.06$, $r_{16} = 0.08$, $h_{16} = 0.04$, $l_{16} = 0.07$, $s_i = 4$, $\tilde{v}_{i1} = (-0.11, -0.1)$, $\tilde{v}_{i2} = (0.11, -0.1)$, $\tilde{v}_{i3} = (0.11, 0.1)$, $\tilde{v}_{i4} = (-0.11, 0.1)$, $h_i = 0.12$, $i = 17, 18, 19$, $s_{20} = 6$, $\tilde{v}_{(20)1} = (0.045, 0.078)$, $\tilde{v}_{(20)2}$ =(0.09,0), $\tilde{v}_{(20)3}$ =(0.045,-0.078), $\tilde{v}_{(20)4} = (-0.045, -0.078)$, $\tilde{v}_{(20)5} = (-0.09, 0)$, $\tilde{v}_{(20)6} = (-0.045, 0.078)$, $h_{20} = 0.11$.

The local optimal solution found by NLP-solver in CAS Math 9 is $F(u^*, u'^*) = 0.001911$ (Fig. 2).

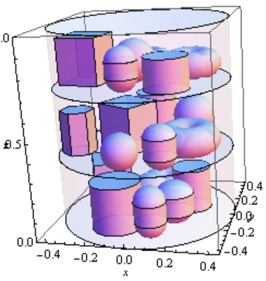

Fig. 2 The local optimal layout of 3D-objects in Instance 2

Instance 3. Let $\Omega \equiv \mathbf{C}$, $m = 2$, $H = 20$, $R = 8.8$, $t_1 = 10$, $n = 80$, $A = \{\mathbb{C}_i, i = 1, \dots 64, \mathbb{K}_i, i = 65, \dots, 80\}$, $A_+^1 = \{\mathbb{C}_i, i = 1, .., 16, \mathbb{K}_i, i = 65, .., 68\}$, $A_-^1 = \{\mathbb{C}_i,\ i = 17,$ $.., 32,\ \mathbb{K}_i, i = 69, \dots, 72\}$, $A_-^2 = \{\mathbb{C}_i, i = 49, \dots, 64,\ \mathbb{K}_i,$ i=77, $\dots, 80\}$, $A_+^2 = \{\mathbb{C}_i, i = 33, \dots, 48, \mathbb{K}_i, i = 73, \dots, 76\}$, $\{r_i, i = 1, \dots, 64\} =$ {2.0, 2.4, 0.8, 1.1, 1.3, 0.7, 0.7, 1.5, 2.4, 1.8, 1.5, 1.7, 1.7, 1.4, 1.6, 2.1, 2.0, 2.4, 0.8, 1.1, 1.3, 0.7, 0.7, 1.5, 2.4, 1.8, 1.5, 1.7, 1.7, 1.4, 1.6, 2.1, 2.0, 2.4, 0.8, 1.1, 1.3, 0.7, 0.7, 1.5, 2.4, 1.8, 1.5, 1.7, 1.7, 1.4, 1.6, 2.1, 2.0, 2.4, 0.8, 1.1, 1.3, 0.7, 0.7, 1.5, 2.4, 1.8, 1.5, 1.7, 1.7, 1.4, 1.6, 2.1}, $s_i = 4$, $i = 65, \dots, 80$, $\tilde{v}_{i1} = (-1.8, -1.8)$, $\tilde{v}_{i2} = (1.8, -1.8)$, $\tilde{v}_{i3} = (1.8, 1.8)$, $\tilde{v}_{i4} = (-1.8, 1.8)$, $i = 65, 69, 73, 77$, $\tilde{v}_{i1} = (-0.5, -0.5)$, $\tilde{v}_{i2} = (0.5, -0.5)$, $\tilde{v}_{i3} = (0.5, 0.5)$, $\tilde{v}_{i4} = (-0.5, 0.5)$, $i = 66, 70, 74, 78$, $\tilde{v}_{i1} = (-2.1, -2.1)$, $\tilde{v}_{i2} = (2.1, -2.1)$, $\tilde{v}_{i3} = (2.1, 2.1)$, $\tilde{v}_{i4} = (-2.1, 2.1)$, $i = 67, 71, 75, 79$, $\tilde{v}_{i1} = (-1.3, -1.3)$, $\tilde{v}_{i2} = (1.3, -1.3)$, $\tilde{v}_{i3} = (1.3, 1.3)$, $\tilde{v}_{i4} = (-1.3, 1.3)$, $i = 68, 72, 76, 80$, $\{h_i, i = 1, \dots, 62\}$={1.5, 1.5, 1.5, 1.5, 1.5, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 3.0, 1.5, 3.0, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 3.0, 1.5, 1.5, 3.0, 1.5, 1.5, 3.0, 1.5, 1.5, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 1.5, 1.5, 3.0, 3.0, 1.5, 1.5, 1.5, 3.0, 1.5, 3.0, 1.5, 1.5, 3.0, 1.5, 1.5, 3.0, 1.5, 1.5, 3.0, $\{h_i, i = 63, \dots, 80\}$={1.5}, $\{m_i, i = 1, \dots, 80\}$={86, 72, 81, 54, 29, 94, 92, 41, 57, 77, 40, 67, 31, 47, 39, 61, 73, 83, 11, 20, 86, 72, 81, 54, 29, 94, 92, 41, 57, 77, 40, 67, 31, 47, 39, 61, 73, 83, 11, 20, 86, 72, 81, 54, 29, 94, 92, 41, 57, 77, 40, 67, 31, 47, 39, 61, 73, 83, 11, 20, 86, 72, 81, 54, 29, 94, 92, 41, 57, 77, 40, 67, 31, 47, 39, 61, 73, 83, 11, 20}.

The local optimal solution found by IPOPT is $F(u^*, u'^*) = 0.000000$ (Fig. 3).

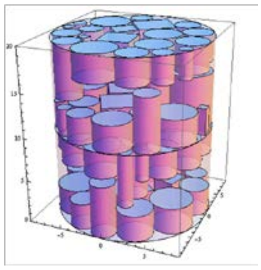

Fig. 3 The local optimal layout of 3D-objects in Instance 3

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