Finite Satisfiability of Unary Negation Fragment with Transitivity (extended abstract) ⋆

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Searching for attractive fragments of first-order logic is an important theme in theoretical computer science. Many seminal fragments of this kind, like the *two-variable fragment*, FO^2, [11] and the *guarded fragment*, GF, [2], embed, via the so-called *standard translation*, some modal and description logics. A more recent proposal in this vein is the *unary negation fragment*, UNFO, restricting the use of negation to subformulas with at most one free variable [4]. UNFO is defined by the following grammar: $\varphi = B\bar{x} \mid x = y \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \exists x \varphi \mid \neg\varphi(x)$, where, in the first clause, B represents any relational symbol, and, in the last clause, φ has no free variables besides (at most) x.

UNFO may be seen as a generalization of basic modal and description logics to contexts with relations of arbitrary arity, alternative and orthogonal in expressive power to GF. As GF, UNFO has many good algorithmic and model theoretic properties, including the finite model property, a tree-like model property and the decidability of the satisfiability (= finite satisfiability) problem [4]. Besides modal and description logics, UNFO embeds also, *inter alia*, unions of *conjunctive queries* and *frontier-one TGDs* [3], which makes it attractive for the database and knowledge representation communities. Its serious weakness is, on the other hand, that it cannot express transitivity. This weakness is shared by FO^2 and GF, both of which, moreover, become undecidable when extended by transitive relations [9, 8]. UNFO is an exception here: its satisfiability problem remains decidable in the presence of arbitrarily many transitive relations. This has been explicitly stated in [10], as a corollary from the decidability of UNFO with regular path expressions, and follows also from [1], where some extensions of the *guarded negation fragment* are considered. From both papers the 2-ExpTime-completeness of UNFO with transitivity, UNFO+$\mathcal{S}$, can be inferred.

The results from [10] and [1] are obtained by employing tree-like model properties of the logics and then using some automata techniques. Since tree-like unravelings of models are infinite, such approach works only for general satisfiability, and gives little insight into the decidability/complexity of finite satisfiability, which in the case of UNFO+$\mathcal{S}$ is evidently a different problem, since, *e.g.*, the formula $\forall x \exists y Txy \wedge \forall x \neg Txx$, with transitive T, is satisfiable but has only infinite models. Our main contribution is demonstrating the decidability of finite satisfiability for UNFO+$\mathcal{S}$ and establishing its 2-ExpTime-completeness.

UNFO, via the above-mentioned standard translation, embeds the logic $\mathcal{ALC}$, as well as its extension by inverse roles ($\mathcal{I}$) and role intersections ($\sqcap$). Thus, being able to express conjunctive queries, we can use our results to solve, for some

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description logics, DLs, the so-called *(finite) ontology mediated query answering* problem, (F)OMQA: given a conjunctive query (or a union of conjunctive queries) and a knowledge base specified in a DL, check whether the query holds in every (finite) model of this knowledge base. While there are quite a lot of results for OMQA, not much is known about FOMQA. In particular, for DLs with transitive roles ($\mathcal{S}$) the only positive results we are aware of are the ones obtained recently in [7], where the decidability and 2-ExpTime-completeness of FOMQA for the logics $\mathcal{SOI}$, $\mathcal{SIF}$ and $\mathcal{SOF}$ is shown.

In this context it is an interesting question if our decidability result can be extended to capture some more expressive DLs. Unfortunately, we cannot hope for *number restrictions* ($\mathcal{Q}$ or $\mathcal{N}$) or even *functional roles* ($\mathcal{F}$), as (finite) satisfiability for UNFO with functional relations is undecidable (implicit in [4]). On the positive side, we show the decidability and 2-ExpTime-completeness of finite satisfiability for UNFO+$\mathcal{SOH}$, extending UNFO+$\mathcal{S}$ by constants (corresponding to *nominals* ($\mathcal{O}$)) and inclusions of binary relations (capturing *role hierarchies* ($\mathcal{H}$)). This is sufficient, in particular, to imply the decidability of FOMQA for the logic $\mathcal{SHOI}^{\sqcap}$, which, up to our knowledge, is a new result.

To show our basic result for UNFO+$\mathcal{S}$ we introduce a natural notion of tree-like structures and a measure associating with transitive paths of such structures their so-called *ranks*. Intuitively, for a transitive relation T and a T-path π, the T-rank of π is the number of one-directional T-edges in π. Then we show that having the following forms of models is equivalent for any formula φ:

(f1) finite
(f2) tree-like, with bounded ranks of transitive paths
(f3) tree-like, with ranks of paths bounded doubly exponentially in $|\varphi|$
(f4) tree-like, with ranks of paths bounded doubly exponentially in $|\varphi|$, and regular (with doubly exponentially many non-isomorphic subtrees)
(f5) finite, of size triply exponential in $|\varphi|$.

The most difficult transitions are those from (f2) to (f3) and from (f4) to (f5). The former is obtained in an expected way, by a kind of tree-pruning; our pruning strategy is, however, rather intricate, due to possible interactions among different transitive relations. The latter is done by an inductive construction, building some bigger and bigger substructures of the final model, in which some transitive relations are total. The induction goes, roughly speaking, by the number of non-total transitive relations in the substructure. (We used a similar small model construction to show the finite model property for UNFO with equivalence relations [6]). Having proved the equivalence of the (fi), we design a natural AExpSpace(=2-ExpTime)-algorithm looking for models of the form (f3).

To show that the equivalence of the (fi) holds in the case of UNFO+$\mathcal{SOH}$, we start with some preprocessing on the input formula and then reuse our model transformations for UNFO+$\mathcal{S}$, treating them as black boxes. We then apply additional *shrinking* and *pseudotransitive closure* operations tailored to deal, resp., with constants and the inclusions.

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