An Overview of Ateb-Theory Mathematical Apparatus for Data Confidentiality in Medical Computer Networks

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Abstract. In the paper, a new mathematical apparatus for confidential data transmission in medical computer networks is proposed. It is based on the use of Ateb algebra mathematical apparatus. It is shown, that traffic simulation software was previously developed for Ateb-functions parameters. The proposed Ateb-transform algebra and developed software traffic analyzer can be used for further testing and investigation of medical computer networks traffic samples, collected from other computer network topologies. The proposed approach can be used in the fields of medicine, economics, materials science, service sciences etc., for future implementation of investigated results.

Keywords: Ateb-transform algebra, Ateb-function, medical confidentiality, software solution, traffic flow analyzer.

1 Introduction

For the proper functioning of healthcare facilities, it is important to ensure the confidentiality of data, transmitted through computer networks of healthcare facilities. In order to solve this problem, this article proposes a method of converting signals into the traffic of computer networks of medical institutions based on Ateb-transform, which suppresses the usual Fourier transform [1-5,12]. This approach can be implemented based on special software that is installed on all computers of the health care facility. The use of the proposed software guarantees the confidentiality of the information, transmitted on the network, because in case of theft information will not be readable.

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2 Traffic simulation software

2.1 Previous results description

For effective network monitoring, a traffic analyzer has been developed, one of the components of which is software for analyzing the operation of a computer network. The purpose of the software is to provide automated collection of information from network devices and implementation of modeling values of traffic samples. As a result of the software operation, the traffic parameters are visually displayed on the charts and the appropriate redistribution of network load is given.

A network traffic analyzer consists of server (PHP, HTML, CSS, JS) and client (C ++ / QT) parts. The server side deals with data analysis and storage. The client, in turn, collects, processes network data and displays results. This software is created using C ++ and Haskell programming languages [6].

The main features of the developed traffic flow analyzer are reading samples of flow of traffic from the nodal equipment of the computer network with the ability to record the read results in a special file format * .pcap, analysis of previously recorded flow of traffic. The processed traffic flow data is stored in a relational database with a resolution of 1 second.

The primary purpose of a computer network traffic analyzer is to analyze samples of that traffic, with the subsequent ability to predict the amount of traffic using Ateb-theory. Developed software interface is shown in Figure 1. The next traffic modeling investigation results are shown in Figures 2-9.

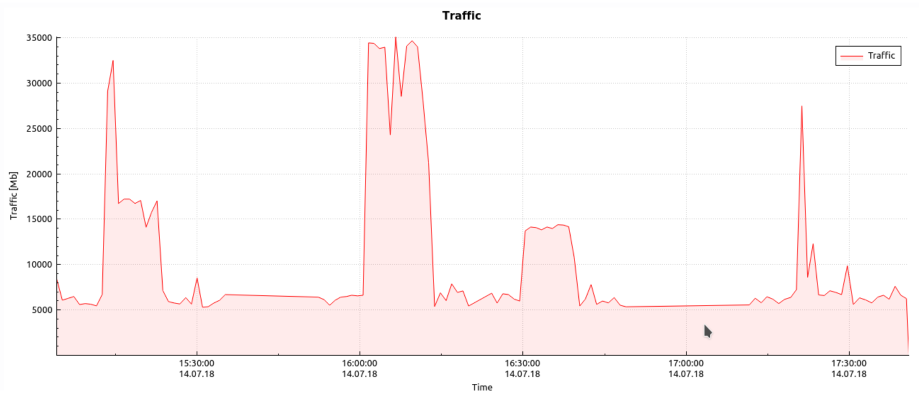

Fig. 1. Program window of developed traffic analyzer.

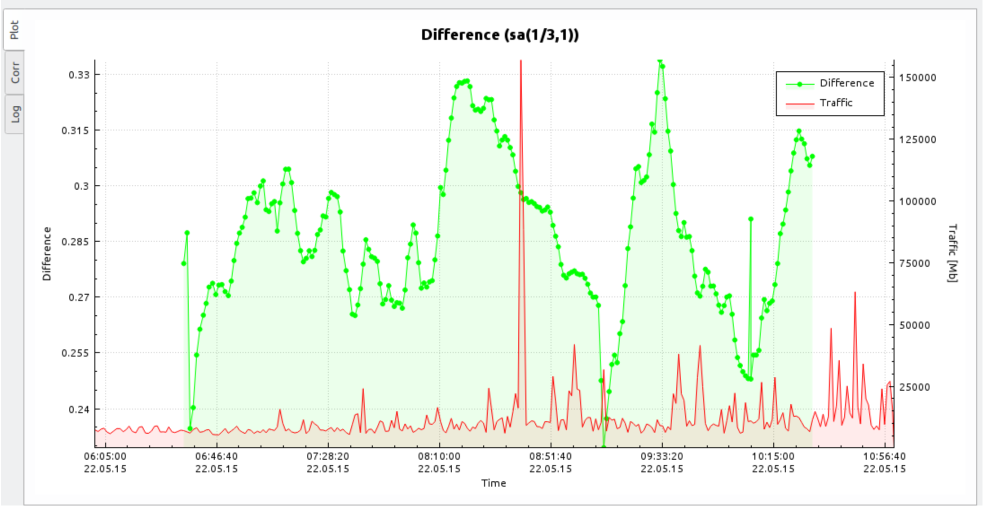

Fig. 2. Investigation of real and modeled traffic samples similarity based on using of Ateb-sine with (1/3,1) parameters.

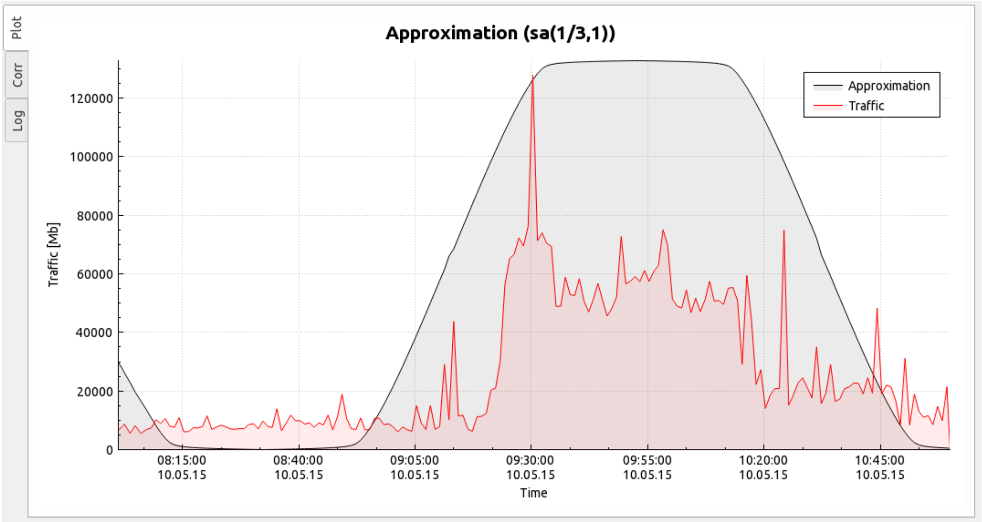

Fig. 3. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1/3,1) parameters.

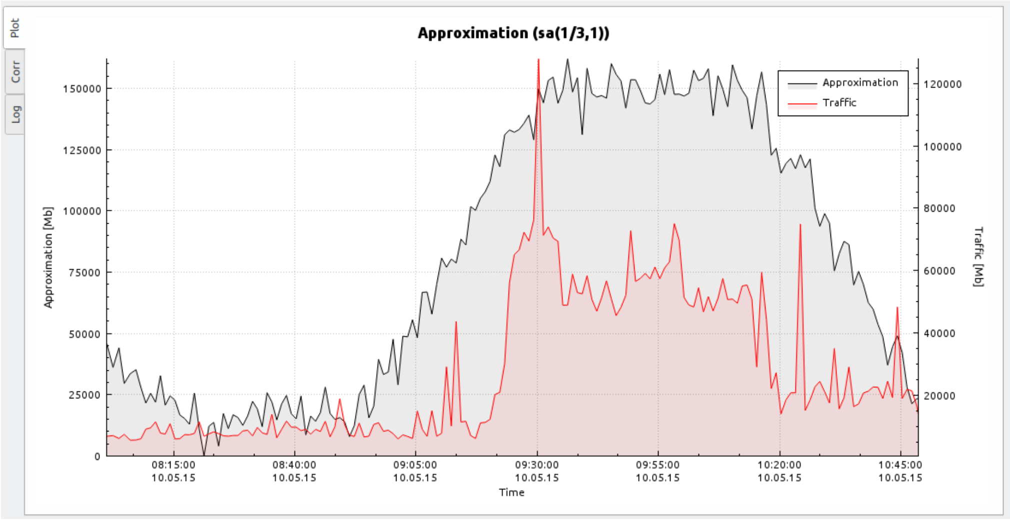

Fig. 4. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1/3,1) parameters and Dirac function.

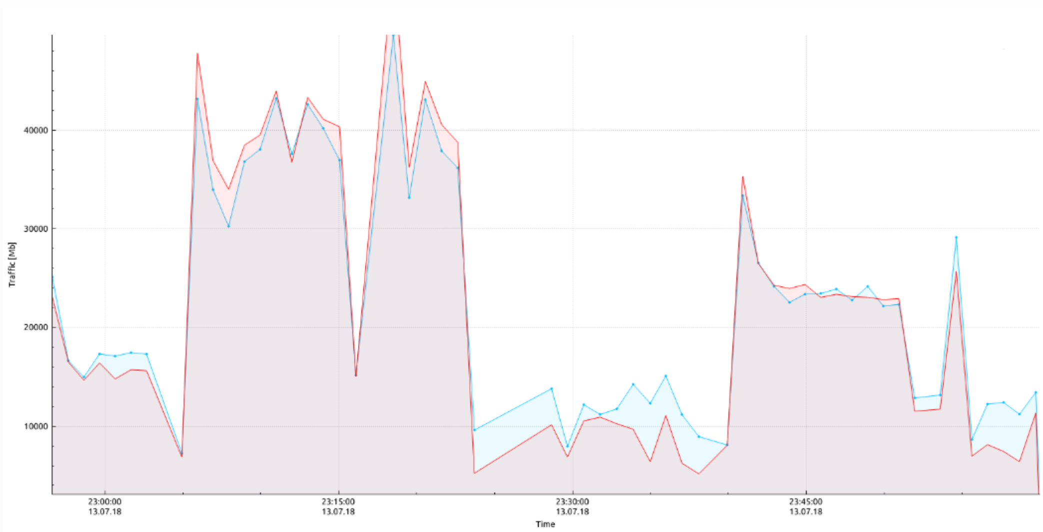

Fig. 5. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (0.01,0.1) parameters.

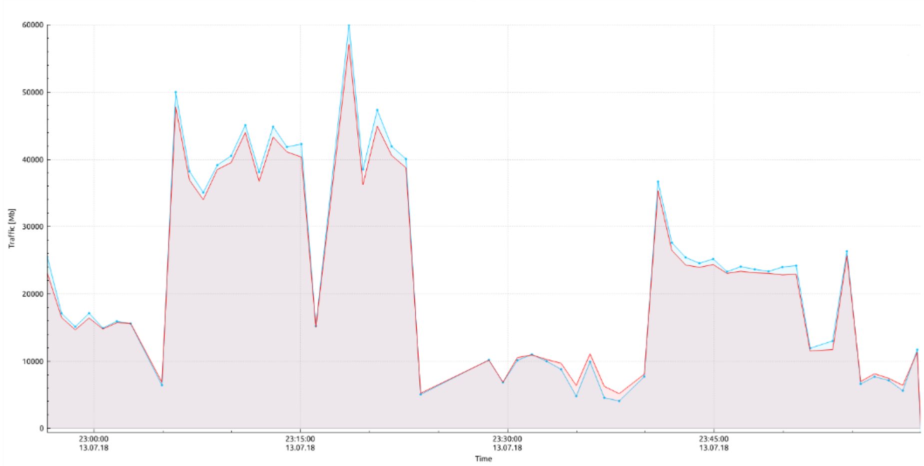

Fig. 6. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1, 1/3) parameters.

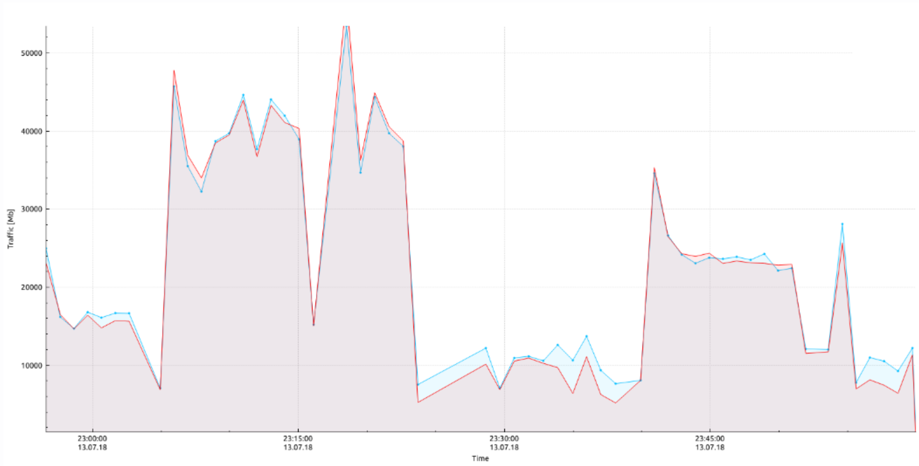

Fig. 7. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1, 1/7) parameters.

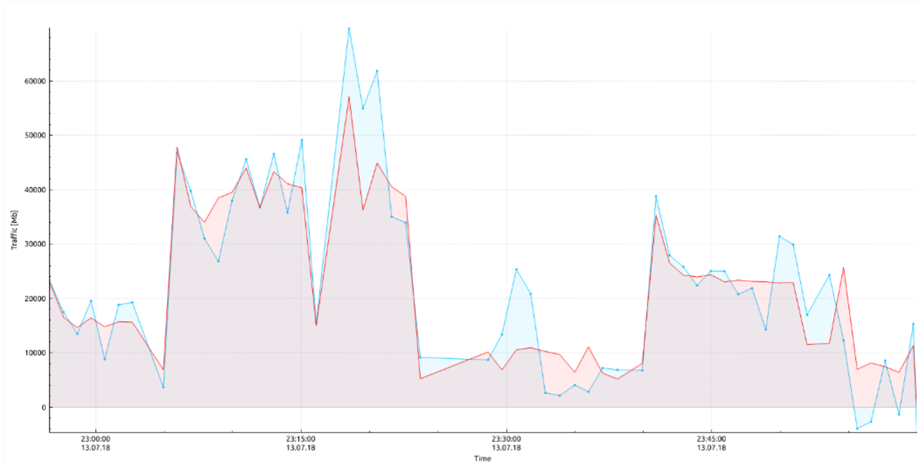

Fig. 8. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1/3, 1) parameters.

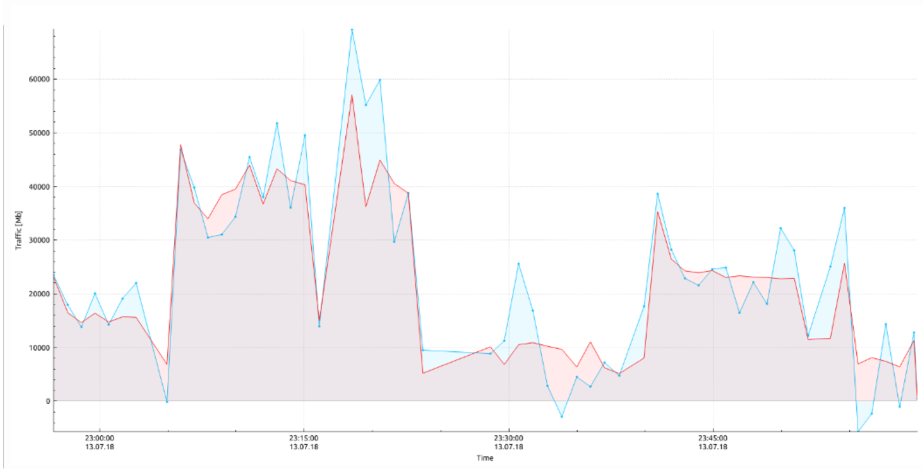

Fig. 9. Investigation of real and modeled traffic samples approximation based on using of Ateb-sine with (1/5, 1) parameters.

3 Review and problem statement

The introduction of the concept of T-classes became the basis for the classification of types of properties of signals and converters in the time domain and the justification of mathematical means of their analysis. It is also known that using a generalized shift operator, it is possible to construct a system of signal conversion and processing based on any orthogonal system of functions, not just a multiplication [7, 8]. This fact is the basis for providing a new method other than harmonic analysis, signal pro-

cessing and transmission in wireless networks due to the ability to adapt signal processing. The system of Ateb functions is complete and orthonormal in the functional Hilbert space and can therefore serve as a basis [8]. To process and transmit signals, it is necessary to have a toolkit to ensure correct addition and multiplication operations. For this purpose, an Ateb transform algebra was constructed by the introduction of an external convolution operation.
Here is an algebraic interpretation of the idea of T-classes with respect to Ateb functions as elements of the bases of signal spaces for special communication problems. An overview of the general state-of-the-art of the theory of generalized shift operators, hyper-complex systems, and hyper groups used to construct the Ateb-transform algebra is presented in [9].

4 Ateb-transform algebra

Related operator for $L_{m,n,t} : H \to C$ for Ateb-fuctions is a differential operator in a form

$$L_{m,n,t} = \frac{d^2}{dt^2}(\bullet) + c_\theta^2 (\bullet)|\bullet|^{\theta-1} , \tag{1}$$

де H- Hilbert functional space, C – set of complex numbers, c_θ - some constant, θ - the degree of nonlinearity of the operator. Ateb functions are the inherent functions of this operator $ca(m,n,t), sa(n,m,t)$, where m,n are dependent from θ .

Now lets construct the algebra using the generalized shift operator U^s -such, that $U^s f(t) = f(t+s),$ where s - the value of a function argument change, interpreted as a left shift of its function, and is therefore a family parameter $\{U^s, s \in R\}$ of shift operators. Lets denote the set of operators of Ateb-transform by $\Lambda_{m,n} = \{L_{m,n,t}\}$. Therefore, the algebra of Ateb transform can be represented as $A_{m,n} = \langle \Lambda_{m,n}; \Omega \rangle,$ where $\Lambda_{m,n}$ - set of operators of Ateb-transform, Ω - algebra`s signature $A_{m,n}$. The set Ω contains "add" and "multiply". The native operator's own functions are valid, so the convolution operation can be defined as

$$(g * f)(t) = \int g(s) L^s_{m,n,t} f(t) ds. \tag{2}$$

Lets accept the specified convolution for the multiplication operation in the corresponding hyper group. Adding is determined by the normal addition of features. Thus, a linear system on the Hilbert function space is constructed, which completes within the hyper group algebra the construction of the apparatus of spectral-temporal analysis of functional spaces based on bases of Ateb functions. Parameters m,n can be used as management and adaptation capabilities in today's communications networks.

4.1 Class of invariant Ateb-transforms

At this point, based on the foregoing considerations, an invariant class with respect to the introduced transforms is constructed, which allows to close the system and show the integrity of the constructed system.

Definition [8, 10]. The T-class is called the set of linear operators and second-order random processes as mathematical models of systems (transducers) and signals, respectively, such that the change (variability) in the time of their characteristics is determined (embodied) by the operator T of generalized shift (GSO).

An analog of this definition is the fact that the term encompasses and generalizes the notion of an invariant class when the characteristics are constant in time or formally invariant with respect to a group of displacements on the time axis. R {Us, s∈R}, because Us·Uv = Us+v, and a basis of spaces are constructing harmonics eitλ of frequencies λ∈R, which are the own functions of the shift operator by virtue of the fact: Us·eitλ= ei(t+s)λ = eitλ·eisλ, here s – parameter (shift value), and harmonic schedules give a classic Fourier transform. In the generalizations, the concept of a hypergroup is used instead of a group.

Lets consider D as a subspace of the space of real numbers. Then the GSO hypergroup{Ts, s∈D} above the linear space of functions f: D→H ($D \subseteq R$)are created by operators whose eigenvalues are equal for each parameter value s GSO, by value of own function φ(•, λ) a native operator of basis L {φ(•, λ), λ∈Λ}, such as Lφ(•,λ) = λ·φ(•, λ), that is, Tsφ(•,λ) = φ(s,λ)·φ(•, λ), λ∈Λ – function index, or, in short: Ts = φ(s,L). The basis must be orthonormal in sense $\int \varphi(t,\lambda)\overline{\varphi(s,\lambda)}\,m(d\lambda) = \delta(t-s)$, here m(•) – measure on space D, the dash over the expression in this formula is a complex conjugation, a δ(•) – Dirac delta function.

Hypergroup GSO {Ts, s∈D} in *-algebra (where * means involutive-convolutional algebra) sets the external multiplication as a convolution by the formula

$$f * g(t) = \int f(s)T^s g(t)dt \tag{3}$$

In a linear space of functions f: D→H lets define the operator of Ateb transformation $L_{m,n} : H \to C$ in the form (1). Ateb functions are the inherent functions of this operator $ca(m,n,t),\ sa(n,m,t)$ The system of Ateb functions at certain values of the parameters m, n is complete and ortho-normalized in the functional Hilbert space, and therefore can serve as a basis. To process and transmit signals, it is necessary to have the tools to ensure that the addition and multiplication operations are performed correctly. In the case of periodic Ateb-functions, the convolution is given by (3). Lets denote the set of operators of Ateb transformation by $\Lambda_{m,n} = \{L_{m,n}\}$. Therefore, the algebra of Ateb transforms can be represented as $A_{m,n} = \langle \Lambda_{m,n}; \Omega \rangle$, where $\Lambda_{m,n}$ – a set of Ateb transform operators, Ω– algebra`s signature $A_{m,n}$. A Ω set contains "Addition" and "multiplication”. The multiplication above is a convolution, and the addition is the usual addition of functions. Thus the totality $\{A_{m,n};f\}$ forms an Ateb class that is invariant to Ateb transforms.

Lets consider F – vector space of complex-valued functions given on the set of real numbers. It is known [8] that the linear operator L: F → F will be a generalized shift operator if the following properties are satisfied:

1) the associativity axiom $LxRy = RyLx$;
2) there is an element e, the so-called neutral element, which is identical to equation $Le = I$, where I – identical operator.

The associative axiom can be replaced by a stronger requirement, namely the commutative axiom $LxLy = LyLx$, then the generalized shift operator is called the commutative generalized shift operator [8, 11].

If consider that the Ateb-transform operator is given by equation $L_{m,n}x = [A(m,n,\omega) - iB(n,m,\omega)]x$ then it is is a commutative generalized shift operator.

To show it, it is needed to show that fulfillment of the axiom of commutativity and the existence of a neutral element.

1. $L_{m,n}xL_{m,n}y = L_{m,n}yL_{m,n}x$.

Proof of this property directly follows from the commutativity of multiplication.

2. A neutral element is a function with zero values for all argument values: $L_{m,n}\mathbf{O} = \mathbf{O}$.

From the shown theorem, an important consequence follows. Since the Ateb transform operator is a commutative GSO and it is proved in [8, 11] that the set of GSO creates an algebra, hence the set of Ateb transform operators also form an algebra.

Lets denote the set of operators of Ateb transform by $\Lambda = \{L_{m,n}\}$.

So, set Λ creates algebra A under space F.

The proof follows from the previous theorem and from the statement that the set of GSO forms an algebra [8, 10, 11].

5 Conclusions

The proposed method is new and belongs to the methods of confidential information and data protecting based on generalized Fourier transform. The novelty is the use of discrete Ateb-transform, which is based on Ateb-functions. In addition, a method of protection has been developed to increase the level of data security in medical computer networks in order to prevent violation of the integrity of information to ensure the appropriate level of information transmission security.

The scope of the proposed method is quite wide; it covers different types of traffic in various computer networks. The proposed method has been tested on traffic files. It indicates the relevance and practical significance of the proposed method. However, the mathematical apparatus should be improved and tested on wide samples on medical traffic data.

References

1. Tkachenko, R., Izonin, I., Vitynskyi, P., Lotoshynska, N., Pavlyuk, O.: Development of the Non-Iterative Supervised Learning Predictor Based on the Ito Decomposition and SGTM Neural-Like Structure for Managing Medical Insurance Costs, Data, vol. 3, no. 4, p. 46, (2018).
2. Medykovskyy, M.O., Pasyeka, M.S., Pasyeka, N.M., Turchyn, O.B. Scientific research of life cycle perfomance of information technology Proceedings of the 12th International Scientific and Technical Conference on Computer Sciences and Information Technologies, CSIT 2017, Volume 1, 6 November 2017, Pages 425-428, Polyana, 2015. — P. 42 – 44.
3. Gupta, V. ,Mittal, M. A Comparison of ECG Signal Pre-processing Using FrFT, FrWT and IPCA for Improved Analysis IRBM, Volume 40, Issue 3, June 2019, Pages 145-156
4. Kumar, A., Ranganatham, R., Komaragiri, R. et al. Biomed. Eng. Lett. (2019) 9: 145. https://doi.org/10.1007/s13534-018-0087-y
5. Praveena, G & Southry, S & Raju, Anand. (2019). Band Limited Noise Cancellation Using Adaptive Filters. Journal of Advanced Research in Dynamical and Control Systems. 11. 1222-1228.
6. Dronyuk I., Fedevych O., Lizanets D. Researching measured and modeled traffic with self-similar properties for Ateb-modeling method improvement Computer Networks - 25th International Conference, CN 2018, Gliwice, Poland, June 19-22, 2018, Proceedings. Communications in Computer and Information Science 860, pp.13-25, Springer 2018.
7. Dronyuk I, Nazarkevych M. A study on Ateb transform as a generalization of Fourier transform|/ Editors: Mityushev, Vladimir, Ruzhansky, Michael V. // Series Trends in Mathematics Current trends in analysis and its applications, Springer Switzerland, V.XVI, 2015.-pp 723-731.
8. Yaroslav Dragan, Ivanna Dronyuk. System Analisys and Grounding for the Data Processing Means and Tecnologies based on Optimization of the Computer Network work based on Ateb-functions /Ya.P. Dragan, I.M. Droniuk // Proceedings of the 12-th International Scientific and Technical Conference CSIT 2017, 05-08 September 2017, Lviv, Ukraine, p.272 - 275.
9. Steven B. Damelin; Willard Miller, Jr (2012). The Mathematics of Signal Processing. Cambridge University Press. ISBN 978-1-107-01322-3
10. Ivanna Dronyuk, Olga Fedevych and Bogdan Demyda. Signals and Images Protection Based on Ateb-Transforms in Infocommunication Systems IEEE International Scientific-Practical Conference «Problems of Infocommunications. Science and Technology» (PIC S&T'2018) 9 - 12 October, Kharkiv pp.461-465, 2018
11. Ivanna Droniuk, Olga Fedevych. Construction of Noise-like signals Based on Ateb – Functions / I.M. Droniuk, O.Yu. Fedevych // CADSM 2015, February 24-27. — Lviv-
12. Bagchi, S.: Formulating analytical solution of network ODE systems based on input excitations. In: Journal of Information Processing Systems, Vol. 14, Issue 2, Pages 455-468. South Korea,(2018). DOI: 10.3745/JIPS.03.0092