The Identity of Property Particulars

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Abstract. A property particular is a particular that characterizes the satisfaction of a property by an object, for example the 'redness' of a specific rose. We consider three theories of identity for property particulars, illustrating them on a common example. We discuss their strengths and weaknesses, and whether they can be interpreted in an epistemic or realist perspective. We introduce some first steps of a theory of reification of property types and discuss the benefits that it could bring.

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1. Introduction

Properties are one of the most basic building block of ontologies. A variety of kinds of properties has been proposed by ontologists: qualities (e.g. the redness of a rose), realizable entities like dispositions (e.g. the fragility of a glass), functions (e.g. the function of a heart to pump blood), roles (e.g. a doctor role), etc. Several foundational ontologies introduce into the domain of quantification the *individualization* of properties, e.g., the redness of a particular rose or the weight of a given car, which are specific of (they *inhere in*) the rose and the car, respectively. For instance, BFO [1] considers *dependent continuants*, DOLCE [2] *individual qualities*, GFO [3] *property individuals*, and UFO [4] *moments*. We focus on individualizations of properties, called here *property particulars* (PPs), corresponding to the satisfaction of a property by an object. In a philosophical perspective, PPs are interpetrable in several ways, e.g., as *tropes* [5] or *truth-makers* [6], and they are intertwined with more complex entities like *states of affairs* [7] or *facts* [8]. The introduction of PPs into the domain of quantification brings a variety of representational advantages, such as analyzing the relations between simple PPs and the complex ones they compose (enabling, for instance, a finer analysis of causality as in [9]), or modeling some meta-properties via properties of PPs.

However, the identity conditions of PPs have been little studied (but see [10] for an investigation into the identity of dispositional properties). This question if deeply intertwined with the epistemic or realist position one takes to interpret an ontological framework. Since PPs are intuitively interconnected with propositions, they might be epistemically interpreted as pieces of information about objects, observations or cognitive

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categorizations. A theory could then admit a PP corresponding to the red character of a rose without introducing a PP corresponding to its specific shade of red, e.g., its specific scarletness. This would represent the situation where the only information at hand concerns the fact that the rose is red, and where no information is available about its specific shade of red. On the other hand, a realist framework might impose that the redness of a rose would be identical with (or at least would depend on) its scarletness. An important theoretical work would therefore be to clarify the identity conditions for PPs and their adequacy to capture a realist vs. epistemic stance.

The question of the identity of PPs is linked with two other questions that are still matter of debate and that we will not discuss here: first, whether and how PPs can persist through time; second, which kinds of properties are associated with PPs, and which kinds are not. Instead, we will analyze identity criteria in a static scenario where a given set of properties (and associated PP-types, see Sect. 2) are taxonomically organized.

2. Property Particulars

Suppose one wants to represent the situation where 'the object $\mathfrak{a}$ has the property P', shortly, '$\mathfrak{a}$ is P' or, as usually stated in the philosophical literature, '$\mathfrak{a}$ instantiates P'. In first order logic (FOL) one would typically represent the object $\mathfrak{a}$ by the individual constant $\mathtt{a}$, the property P by the unary predicate $\mathtt{P}$, and the instantiation of the property P by the object $\mathfrak{a}$ by the proposition $\mathtt{P}(\mathtt{a})$. For example, the property *being red* would be represented by a predicate $\mathtt{RED}$, and the red character of a rose $\mathfrak{r}$ would be represented by the proposition $\mathtt{RED}(\mathtt{r})$.[2]

An alternative way to represent (also in FOL) this kind of situations relies on *property particulars*. In the above situation, on top of the object $\mathfrak{a}$, we have an additional individual, a PP *inhering* in $\mathfrak{a}$ that intuitively stands for '$\mathfrak{a}$*'s being* P'. The inherence relation between PPs and objects, a kind of specific existential dependence, is represented by the primitive $\mathtt{INH}$ that is minimally characterized by axioms (a1)-(a3), where $\mathrm{PP}(p)$ stands for 'p is a *property particular*' and $\mathrm{OB}(a)$ for 'a is an *object*'. Because a PP always inheres in a unique object (a2)—but an object may have several PPs inhering in it—we note $\mathtt{p}_{\mathtt{a}}$ a PP inhering in $\mathtt{a}$.[3]

a1 $\mathtt{INH}(p,a) \rightarrow \mathrm{PP}(p) \wedge \mathrm{OB}(a)$
a2 $\mathtt{INH}(p,a) \wedge \mathtt{INH}(p,b) \rightarrow a = b$
a3 $\mathrm{PP}(p) \rightarrow \exists a(\mathtt{INH}(p,a))$

In a framework containing PPs, the property P can be associated with a type of PPs represented by the predicate $\bar{\mathtt{P}}$. Therefore, '$\mathfrak{a}$ is P' is represented by $\exists p(\mathtt{INH}(p,\mathtt{a}) \wedge \bar{\mathtt{P}}(p))$. For instance, the fact that the rose $\mathfrak{r}$ is red can be represented by $\exists p(\mathtt{INH}(p,\mathtt{r}) \wedge \overline{\mathtt{RED}}(p))$, i.e., *being red* is read as 'having an inherent $\overline{\mathtt{RED}}$-PP'. Given the fact that PPs can be interpreted in several ways, we introduce $\bar{\mathtt{P}}$ without making any further commitment (e.g., theories of tropes usually reduce $\bar{\mathtt{P}}$ to an equivalence class of *similar* PPs).

[2] We assume here that the rose $\mathfrak{r}$ has a uniform color.

[3] To improve the readability of formulas we assume the following conventions: (*i*) individual constants are noted using the `typewriter typestyle` font; (*ii*) variables are noted in *italic*; (*iii*) variables/constants ranging over objects are noted $a, b, \ldots \mathtt{a}, \mathtt{b}, \ldots$; (*iv*) variables/constants ranging over PPs are noted $p, q, \ldots, \mathtt{p}, \mathtt{q}, \ldots$

3. Specialization and Covering

We consider a scenario where the user (that could have a realist or epistemic stance) wants to represent the *taxonomical* organization of a finite set $\mathscr{P}$ of properties of objects using a FOL-framework. We assume that the user commits to the set of properties $\mathscr{P}$ and to their taxonomical organization, and we want to analyze the adequacy of alternative FOL-frameworks (especially the ones based on PPs) for representing this commitment. This scenario may be extended to denumerable sets of properties and relations, but here we stick to the finite case of properties.

Classically, each property in $\mathscr{P}$ would be represented by a unary predicate that applies to objects. We note $\mathbb{P}$ the set of FOL-predicates defined on objects representing the properties in $\mathscr{P}$. Alternatively, as seen in Sect. 2, properties can be represented by means of unary predicates that applies to PPs. Calling $\bar{\mathbb{P}}$ the set of FOL-predicates defined on PPs representing the properties in $\mathscr{P}$, we assume one-to-one mappings between $\mathscr{P}$, $\mathbb{P}$, and $\bar{\mathbb{P}}$. Given the property $P \in \mathscr{P}$, we note $\mathtt{P} \in \mathbb{P}$ the corresponding predicate defined on objects and $\bar{\mathtt{P}} \in \bar{\mathbb{P}}$ the one defined on PPs. For example, the property *RED* (*being red*) in $\mathscr{P}$ would be represented by the predicate $\mathtt{RED}$ in $\mathbb{P}$ or by the predicate $\mathtt{R\bar{E}D}$ in $\bar{\mathbb{P}}$. As said, the redness of the rose $\mathfrak{r}$ can be formally represented by $\mathtt{RED}(\mathtt{r})$ ("the rose $\mathfrak{r}$ is of type red") or by $\exists p(\mathtt{INH}(\mathtt{p},\mathtt{r}) \wedge \mathtt{R\bar{E}D}(\mathtt{p}))$ ("the rose $\mathfrak{r}$ has a property particular of type redness inhering in it"). Notice that, in principle, some of the predicates in $\mathbb{P}$ and $\bar{\mathbb{P}}$ could be FOL-definable in terms of other predicates. We do not consider this interesting aspect that requires a deeper analysis of the links between the considered properties, here we focus only on taxonomical relations between properties.

3.1. Specialization

Specialization, e.g., the property *being red* specializes the property *being colored* (*COL*), is one of the main taxonomical relation. Suppose one wants to represent the fact that the property $P \in \mathscr{P}$ *specializes* the property $Q \in \mathscr{P}$, i.e., intuitively, the fact that an object being P is also Q. We analyze below different options to model this fact and, in Sect. 4.2, how these options are intertwined with the question of the identity of PPs.

In FOL specialization is usually represented by material implication, i.e., one would consider the predicates $\mathtt{P},\mathtt{Q} \in \mathbb{P}$ corresponding to the properties P and Q and introduce an implication axiom from $\mathtt{P}$ to $\mathtt{Q}$. Let $\mathscr{S}$ be the set of couples of properties in $\mathscr{P}$ that the user assumes to be linked by a specialization relation[4] and $\mathbb{S}$ the set of couples of predicates in $\mathbb{P}$ corresponding to the couples in $\mathscr{S}$ (similarly for $\bar{\mathbb{S}}$). The specializations relations in $\mathscr{S}$ can be represented by introducing the axiom (cspec), stating that if A specializes B, i.e., $(A,B) \in \mathscr{S}$, then an object that is A is also B.

(cspec) $\bigwedge_{(\mathtt{A},\mathtt{B})\in\mathbb{S}} \forall a(\mathtt{A}(a) \rightarrow \mathtt{B}(a))$

Having PPs in the domain of quantification opens new options. One can start from a basic requirement where the specialization is seen as a *generic existential dependence*

[4]Note that we will not address in this article the question of determining which properties should enter in $\mathscr{P}$, and in particular which predicates should be linked by a specialization relation (that is, which couple of predicates should be included in $\mathscr{S}$); in particular, we will not address the question of determining whether we can associate a predicate to each predicative linguistic expression, and whether a PP should be associated to each predicate. In addition, we assume here that specialization is at least a partial order relation with no loops.

between PPs inhering in the same object, i.e., one can introduce the axiom (spec) (where p and q range over PPs), stating that if there is a PP p of type $\bar{\mathtt{A}}$ inhering in an object a, then there is a PP q (which might be different from p) of type $\bar{\mathtt{B}}$ inhering in a.

$$\text{(spec)}\ \bigwedge_{(\bar{\mathtt{A}},\bar{\mathtt{B}})\in\bar{\mathbb{S}}} \forall pa(\bar{\mathtt{A}}(p) \wedge \mathtt{INH}(p,a) \rightarrow \exists q(\bar{\mathtt{B}}(q) \wedge \mathtt{INH}(q,a)))$$

(spec) is very permissive on the existence of PPs. Suppose from now on that $(\bar{\mathtt{P}},\bar{\mathtt{Q}}) \in \bar{\mathbb{S}}$. (spec) is compatible with the facts that either (i) $\bar{\mathtt{P}}$-PPs are *included* in $\bar{\mathtt{Q}}$-PPs or (ii) $\bar{\mathtt{P}}$-PPs are *disjoint* from $\bar{\mathtt{Q}}$-PPs (it is also compatible with a proper subset of $\bar{\mathtt{P}}$-PPs being included in $\bar{\mathtt{Q}}$-PPs, but we will not consider this option here). (spec) can then be refined by adopting one of the two (incompatible) options, i.e., the specializations relations in $\mathscr{S}$ can be represented by introducing the axiom (incl·spec) or, alternatively, the axiom (disj·spec) where $\bar{\mathbb{S}}^*$ is the set of proper specializations, i.e., $\bar{\mathbb{S}}^* = \bar{\mathbb{S}} \setminus \{(\bar{\mathtt{A}},\bar{\mathtt{A}}) \mid \bar{\mathtt{A}} \in \bar{\mathbb{P}}\}$.

$$\text{(incl·spec)}\ \bigwedge_{(\bar{\mathtt{A}},\bar{\mathtt{B}})\in\bar{\mathbb{S}}} \forall p(\bar{\mathtt{A}}(p) \rightarrow \bar{\mathtt{B}}(p))$$
$$\text{(disj·spec)}\ \text{(spec)} \wedge \bigwedge_{(\bar{\mathtt{A}},\bar{\mathtt{B}})\in\bar{\mathbb{S}}^*} \neg\exists p(\bar{\mathtt{A}}(p) \wedge \bar{\mathtt{B}}(p))$$

According to (incl·spec), if a PP is associated with a property, then it is also associated with all the properties it specializes, e.g., a $\bar{\mathtt{P}}$-PP is also a $\bar{\mathtt{Q}}$-PP. This amounts to require the identity between p and q in (spec). In the rose example, all $\mathtt{R\bar{E}D}$-PPs are also $\mathtt{C\bar{O}L}$, but it is possible to have $\mathtt{C\bar{O}L}$-PPs that are not $\mathtt{R\bar{E}D}$.

According to (disj·spec), PPs are specific to a given property: if P (properly) specializes Q, a $\bar{\mathtt{P}}$-PP is not a $\bar{\mathtt{Q}}$-PP. In the rose example, it is not possible to have PPs that are instances of both $\mathtt{R\bar{E}D}$ and $\mathtt{C\bar{O}L}$. However, (disj·spec) (as well as (incl·spec)) allows for the existence of several $\mathtt{R\bar{E}D}$-PPs and of several $\mathtt{C\bar{O}L}$-PPs inhering in the rose.[5]

An interesting extension of the proposed framework would be to assume that the user distinguishes specialization from *correlation*. These two relations could then be represented differently, for instance, one could consider (incl·spec) for specialization and (disj·spec) for correlation.

3.2. Covering

A second important taxonomical relation is *covering*. While specialization concerns the way more specific properties are related to more generic properties, covering is intended to cope with the opposite mechanism. Intuitively, when a property Q is *covered* by n properties P_i (with $i \in \{1,\ldots,n\}$) then all the objects having the property Q also have (at least) a P_i-property. For simplicity, consider the case with $n = 2$, i.e., Q is covered by P_1 and P_2 both different from Q. For example, in the case of the rose, assume that *being colored* is covered by *being red* and *being yellow*. Similarly to the case of specialization, $\mathscr{C}$ represents the set of triples of properties linked by the covering relation, while $\mathbb{C}$ and $\bar{\mathbb{C}}$ are the corresponding sets of triples of properties in, respectively, $\mathbb{P}$ and $\bar{\mathbb{P}}$.

Classically, one could represent the being covered of Q by P_1 and P_2 by considering the predicates $\mathtt{Q},\mathtt{P}_1,\mathtt{P}_2 \in \mathbb{P}$ and by adding an implication from $\mathtt{Q}$ to the disjunction of $\mathtt{P}_1$ and $\mathtt{P}_2$, i.e., one would add the following axiom (ccov) (where a ranges over objects).

[5] An additional interesting possibility is to have a *specific existential dependence* $\mathtt{SD}$ between PPs: $\mathtt{SD}(p,q)$ stands for 'the PP p *specifically depends* on the PP q', where $\mathtt{SD}(p,q) \wedge \mathtt{SD}(p,r) \rightarrow q = r$. One could then introduce $\bigwedge_{(\bar{\mathtt{A}},\bar{\mathtt{B}})\in\bar{\mathbb{S}}^*} \forall pa(\bar{\mathtt{A}}(p) \wedge \mathtt{INH}(p,a) \rightarrow \exists q(q \neq p \wedge \mathtt{SD}(q,p) \wedge \bar{\mathtt{B}}(q) \wedge \mathtt{INH}(q,a)))$. This would allow to specify the links between the different PPs.

(ccov) $\bigwedge_{(\mathtt{A},\mathtt{B}_1,\mathtt{B}_2)\in\mathbb{C}} \forall a(\mathtt{A}(a) \to (\mathtt{B}_1(a) \vee \mathtt{B}_2(a)))$

Note that this does not mean that all the material implications with form $\mathtt{A}(a) \to (\mathtt{B}_1(a) \vee \mathtt{B}_2(a))$ are cases of covering. As a matter of fact, covering has an intensional aspect that is not captured by (ccov) (indeed, this is also true in the case of (cspec) for the specialization relation). For instance, one could think that *being colored* is not covered by *being 1kg heavy* and *not being 1kg heavy*.

Considering PPs, similarly to specialization, covering can be modeled by introducing a generic existential dependence, i.e., by introducing (cov) according to which the existence in the object a of a PP associated with a given property implies the existence of a PP inhering in a associated with one of its covering properties:

(cov) $\bigwedge_{(\mathtt{A},\mathtt{B}_1,\mathtt{B}_2)\in\bar{\mathbb{C}}} \forall pa(\bar{\mathtt{A}}(p) \wedge \mathtt{INH}(p,a) \to \exists q((\bar{\mathtt{B}}_1(q) \vee \bar{\mathtt{B}}_2(q)) \wedge \mathtt{INH}(q,a)))$

Similarly to (spec), (cov) can be refined in the two following ways: (*i*) (incl·cov) guarantees that a PP associated with a property is also associated with one of its covering properties; (*ii*) (disj·cov) assures that the existence of a PP associated with a property implies the existence of a *different* PP associated with one of its covering properties:

(incl·cov) $\bigwedge_{(\mathtt{A},\mathtt{B}_1,\mathtt{B}_2)\in\bar{\mathbb{C}}} \forall p(\bar{\mathtt{A}}(p) \to (\bar{\mathtt{B}}_1(p) \vee \bar{\mathtt{B}}_2(p)))$
(disj·cov) (cov) $\wedge \bigwedge_{(\mathtt{A},\mathtt{B}_1,\mathtt{B}_2)\in\bar{\mathbb{C}}} \forall p(\neg\exists p(\bar{\mathtt{A}}(p) \wedge (\bar{\mathtt{B}}_1(p) \vee \bar{\mathtt{B}}_2(p))))$

In the example of the rose:

– according to (incl·cov), when we have $\bar{\mathtt{COL}}(\mathtt{p_r})$ we also have $\bar{\mathtt{RED}}(\mathtt{p_r}) \vee \bar{\mathtt{YEL}}(\mathtt{p_r})$, i.e. $\mathtt{p_r}$ represents also either the *being red* of $\mathfrak{r}$ or its *being yellow*.
– (disj·cov) allows for general PPs that satisfy $\bar{\mathtt{COL}}$ but neither $\bar{\mathtt{RED}}$ nor $\bar{\mathtt{YEL}}$; that is, PPs associated only with the general colored character of the rose. However, from the existence of this general PP, we can deduce the existence of a different, more specific PP associated with either the redness or the yellowness of the rose.

3.3. Combining Specialization and Covering

Suppose now to represent the situation where Q is covered by P_1 and P_2 and where P_1 and P_2 are both specializations of Q. For the predicates defined on objects, by combining (cspec) with (ccov), we obtain $\forall a(\mathtt{Q}(a) \leftrightarrow (\mathtt{P}_1(a) \vee \mathtt{P}_2(a)))$.

For the predicates defined on PPs we obtain two consistent options:

– {(incl·spec), (incl·cov)} reduces to $\bigwedge_{(\mathtt{A},\mathtt{B}_1,\mathtt{B}_2)\in\bar{\mathbb{C}}} \forall p(\bar{\mathtt{A}}(p) \leftrightarrow (\bar{\mathtt{B}}_1(p) \vee \bar{\mathtt{B}}_2(p)))$,
– {(disj·spec), (disj·cov)} reduces to a mutual existential dependence between the PPs that are instances of a property and the (different) PPs that are instances of one of the covering properties.

These positions have several important features. First, in all the positions considered, independently of the level of specificity of the PPs, it is possible to have different PPs (inherent in the same object) instantiating the same PP-predicate, i.e., there could be p, q, and a such that $\mathtt{INH}(p,a) \wedge \mathtt{INH}(q,a) \wedge \bar{\mathtt{A}}(p) \wedge \bar{\mathtt{A}}(q) \wedge p \neq q$.

Second, (incl·spec) guarantees that the PPs that instantiate a given PP-predicate $\bar{\mathtt{A}}$ also instantiate all the PP-predicates specialized by $\bar{\mathtt{A}}$, i.e., all the predicates $\bar{\mathtt{B}}$ such that $(\bar{\mathtt{A}},\bar{\mathtt{B}}) \in \bar{\mathbb{S}}$. However, it allows for PPs that instantiate a given predicate without instantiating any more specific PP-predicate. The addition of (incl·cov) rules out this last possibility for those PPs associated with a covered property.

Third, (disj·spec) does not allow a PP to be an instance of two different PP-predicates one specializing the other. Actually, (disj·spec) does not even guarantee that the number of the instances of a given PP-predicate is greater than the number of the instances of its specializations: it is possible to have different specific PPs for only one generic PP, e.g., there could be different $\mathtt{R\bar{E}D}$-PPs and only one $\mathtt{C\bar{O}L}$-PP (inhering in the same object). To avoid that, one could specialize (disj·spec) by introducing a specific existential dependence as done in footnote 5. Similar considerations hold for (disj·cov).

All the options discussed are compatible with a quite strong multiplicativism of PPs—that is, many PPs can be associated with the same pair $(object, property)$. The next section will therefore discuss which principle of restriction on the identity of PPs could be endorsed to limit this multiplicativism.

4. Restrictions on the Identity of Property Particulars

4.1. A Strong Principle of Identity

As observed at the end of Sect. 3, nothing prevents the possibility to have different PPs (inherent in the same object) all instantiating the same predicate in $\bar{\mathbb{P}}$. However, intuitively, one could think that any object can satisfy any property in at most one way, i.e., there is a maximum of one PP for each couple $(object, property)$. To formalize this intuition one could introduce the axiom (idpp).

(idpp) $(\mathtt{INH}(p,a) \wedge \mathtt{INH}(q,a) \wedge \bigvee_{\bar{\mathtt{A}} \in \bar{\mathbb{P}}} (\bar{\mathtt{A}}(p) \wedge \bar{\mathtt{A}}(q))) \rightarrow p = q$

In general, (idpp) is consistent with all the options introduced in Sect. 3. When combined with (incl·spec), it prevents the possibility to have several PPs inherent in the same object at different levels of specificity. In the rose example, if we suppose $\mathtt{R\bar{E}D}(\mathtt{p_r})$, it follows that $\mathtt{C\bar{O}L}(\mathtt{p_r})$ and, by (idpp), $\mathtt{p_r}$ is the unique $\mathtt{C\bar{O}L}$-PP inhering in $\mathtt{r}$, i.e., in that case it is impossible to have a PP associated with the colored character of $\mathtt{r}$ that would not be associated with the redness of $\mathtt{r}$.[6]

In the case of the red rose $\mathtt{r}$ we are then left with two options:

(1) in {(incl·spec), (idpp)} there is only one PP $\mathtt{p_r}$ that satisfies $\mathtt{R\bar{E}D}(\mathtt{p_r}) \wedge \mathtt{C\bar{O}L}(\mathtt{p_r})$,
(2) in {(disj·spec), (idpp)} there are two PPs $\mathtt{p_r} \neq \mathtt{q_r}$ such that $\mathtt{R\bar{E}D}(\mathtt{p_r}) \wedge \mathtt{C\bar{O}L}(\mathtt{q_r})$.

In (1) $\mathtt{p_r}$ concerns both properties *being colored* and *being red*. Consequently, (idpp) forces a one-to-one mapping between PPs and $(object, property)$ couples only in the presence of (disj·spec). Another constraint brought by (1) concerns the relation of specialization. Assume, for example, that *having a mass* is represented by $\mathtt{M\bar{A}SS}$ and that there exists a predicate $\mathtt{\bar{M}C}$ such that $\mathtt{M\bar{A}SS}$ and $\mathtt{C\bar{O}L}$ specialize $\mathtt{\bar{M}C}$, that is, $(\mathtt{M\bar{A}SS}, \mathtt{\bar{M}C}), (\mathtt{C\bar{O}L}, \mathtt{\bar{M}C}) \in \bar{\mathbb{S}}$. If $\mathtt{C\bar{O}L}$ and $\mathtt{M\bar{A}SS}$ are disjoint (as they should intuitively be), there could not be two PPs c_a and m_a inhering in the same object a such that $\mathtt{C\bar{O}L}(c_a)$ and

[6] It is logically possible to have $\mathtt{C\bar{O}L}(\mathtt{p_r}) \wedge \neg\mathtt{R\bar{E}D}(\mathtt{p_r})$, but this implies that the rose has no $\mathtt{R\bar{E}D}$-PP, i.e., $\neg\exists p(\mathtt{R\bar{E}D}(p) \wedge \mathtt{INH}(p, \mathtt{r}))$, and this ontology would not formalize adequately the redness of the rose. Adding (incl·cov) would even worsen the situation because $\mathtt{C\bar{O}L}(\mathtt{p_r}) \wedge \neg\mathtt{R\bar{E}D}(\mathtt{p_r})$ would imply $\mathtt{Y\bar{E}L}(\mathtt{p_r})$.

$\bar{\mathtt{MASS}}(m_a)$: otherwise, we would deduce by (incl·spec) that $\bar{\mathtt{MC}}(c_a)$ and $\bar{\mathtt{MC}}(m_a)$, and by (idpp) we would have $c_a = m_a$. A suggestion could be that the relation of specialization, as well as the relation of covering, should be reserved for pairs of PP-types that are of kind determinate/determinable [11], an interesting point left for future work.

4.2. A Leibnizian Principle of Identity

An alternative view to (idpp) would be to allow the existence of multiple instances (inhering in the same object) of a given PP-predicate only when such predicate is specialized by other PP-predicates. In the case of the red rose, one would have a single $\bar{\mathtt{RED}}$-PP $\mathtt{p_r}$, that would also be a $\bar{\mathtt{COL}}$-PP; and a second, different $\bar{\mathtt{COL}}$-PP $\mathtt{q_r}$, that would not be an instance of any specialization of $\bar{\mathtt{COL}}$.[7] This kind of ontology would be compatible with the axiom (idall) below, a sort of restricted Leibniz principle that assures that two PPs (inhering in the same object) are identical only when they are indistinguishable by means of PP-predicates.

(idall) $(\mathtt{INH}(p,a) \wedge \mathtt{INH}(q,a) \wedge \bigwedge_{\bar{\mathtt{A}}\in\bar{\mathbb{P}}}(\bar{\mathtt{A}}(p) \leftrightarrow \bar{\mathtt{A}}(q))) \to p = q$

First note that (idall) is weaker than (idpp): (idall) (but not (idpp)) allows for two different PPs $\mathtt{p_r}$ and $\mathtt{q_r}$ such that $\bar{\mathtt{COL}}(\mathtt{p_r})$, $\bar{\mathtt{RED}}(\mathtt{p_r})$, $\bar{\mathtt{COL}}(\mathtt{q_r})$, $\neg\bar{\mathtt{RED}}(\mathtt{q_r})$, and $\mathtt{p_r} \neq \mathtt{q_r}$.

Second, in general (idpp) does not imply (idall). Consider PPs p and q and object a such that $\mathtt{INH}(p,a)$, $\mathtt{INH}(p,a)$, $p \neq q$, and $\bigwedge_{\bar{\mathtt{A}}\in\bar{\mathbb{P}}}(\neg\bar{\mathtt{A}}(p) \wedge \neg\bar{\mathtt{A}}(q))$. In this case (idpp) vacuously hold, but (idall) does not hold. However in the reasonable assumption that all PPs are classified under at least one predicate in $\bar{\mathbb{P}}$, (idpp) implies (idall). Consider PPs p and q and object a such that $\mathtt{INH}(p,a)$, $\mathtt{INH}(q,a)$ and $\bigwedge_{\bar{\mathtt{A}}\in\bar{\mathbb{P}}}(\bar{\mathtt{A}}(p) \leftrightarrow \bar{\mathtt{A}}(q))$. By our assumption, all PPs are classified under at least one predicate in $\bar{\mathbb{P}}$. Suppose $\bar{\mathtt{A}}(p)$ holds. From the equivalence just mentioned, we can deduce $\bar{\mathtt{A}}(q)$. And from $\bar{\mathtt{A}}(p)$ and $\bar{\mathtt{A}}(q)$, we deduce by (idpp) that $p = q$. Thus, we have proved (idall).

Third, $\{$(disj·spec), (idall)$\}$ is equivalent to $\{$(disj·spec), (idpp)$\}$ when all the predicates in $\bar{\mathbb{P}}$ are disjoint and each PP is classified under at least one predicate.

To summarize, we can distinguish at least four theories: $T_1 = \{$(incl·spec), (idpp)$\}$, $T_2 = \{$(disj·spec), (idpp)$\}$, $T_3 = \{$(incl·spec), (idall)$\}$, and $T_4 = \{$(disj·spec), (idall)$\}$ to each of which we can add an axiom to deal with the cases of covering, (incl·cov) or (disj·cov). For our discussion about the adequacy of these theories for modeling the epistemic vs. realist view, we will not consider T_4, which is quite close from T_2.

5. Realist vs. Epistemic Interpretations of the Three Theories of Identity

To further illustrate T_1-T_3, suppose that the rose $\mathfrak{r}$ is not only red, but more precisely it is scarlet (*SCA*) and that $(\bar{\mathtt{RED}}, \bar{\mathtt{COL}}), (\bar{\mathtt{SCA}}, \bar{\mathtt{RED}}) \in \bar{\mathbb{S}}$. To represent this situation, a theory should accept at least one PP associated with its colored character, one PP associated with its redness, and one PP associated with its scarletness. The next question is whether some of those PPs are identical or not. According to T_1, there can be only one PP $\mathtt{s}^1_\mathtt{r}$

[7] A weaker version would be to accept a first $\bar{\mathtt{COL}}$-PP $\mathtt{q_r}$ that would be specifically existentially dependent on $\mathtt{p_r}$, in the sense discussed in footnote 5; and a second $\bar{\mathtt{COL}}$-PP $\mathtt{q}'_\mathtt{r}$ that would not be specifically existentially dependent on any instance of any specialization of $\bar{\mathtt{COL}}$.

characterizing at the same time the scarletness, the redness and the colored character of $\mathfrak{r}$. According to T_2, there must be three different PPs: $\mathtt{s}_\mathtt{r}^2$ characterizing its scarletness only (but neither its redness nor its colored character), $\mathtt{r}_\mathtt{r}^2$ characterizing its redness only, $\mathtt{c}_\mathtt{r}^2$ characterizing its colored character only. Finally, T_3 is compatible with the existence of three PPs: $\mathtt{s}_\mathtt{r}^3$ characterizing its scarletness, redness and colored character, $\mathtt{c}_\mathtt{r}^3$ characterizing its redness and colored character, $\mathtt{r}_\mathtt{r}^3$ characterizing its colored character only (it is also compatible with a more economical ontology including only $\mathtt{s}_\mathtt{r}^3$, but it would then collapse to the same ontology as the one implied by T_1).

There are at least two broad perspectives to interpret PP-constants: an informational or epistemic perspective where PP-constants are intended to refer to pieces of information, observations or cognitive categorizations; and a realist perspective, where PP-constants are intended to refer to entities existing in the world (most of which are not informational, although some of them may be so). We will not discuss in detail the distinction between the two approaches (see, e.g., [12,13] for a related debate), but instead comment on which theory seems to fit with which perspective, acknowledging that definitive conclusions require more extensive investigations.

T_1 seems to make more sense when interpreted in a realist perspective than in an epistemic perspective: $\mathtt{s}_\mathtt{r}^1$ would represent the unique color of the rose, in its full specificity. T_2 may fit more naturally an epistemic perspective: $\mathtt{s}_\mathtt{r}^2$, $\mathtt{r}_\mathtt{r}^2$ and $\mathtt{c}_\mathtt{r}^2$ would represent pieces of information on the color of an object acquired by measurement instruments with different resolutions. The inference from the existence of $\mathtt{s}_\mathtt{r}^2$ to the existence of $\mathtt{r}_\mathtt{r}^2$ could amount to the deduction of the information that the rose if red from the information that the rose is scarlet. It might also be possible to interpret T_2 in a realist perspective, if one sees a statement like $\bar{\mathtt{RED}}(\mathtt{r}_\mathtt{r}^2)$ as expressing that $\mathtt{r}_\mathtt{r}^2$ is the *most specific* entity warranting the inference that the rose is red: in this ontology, the two other PPs $\mathtt{s}_\mathtt{r}^2$ and $\mathtt{c}_\mathtt{r}^2$ would be respectively the most specific entity warranting the inference that the rose is scarlet, and the most specific entity warranting the inference that the rose is colored. T_3 seems to be interpretable in both perspectives. In a realist perspective, $\mathtt{c}_\mathtt{r}^3$ could be viewed as the general structure responsible for the colored character of $\mathfrak{r}$, whereas $\mathtt{r}_\mathtt{r}^3$ (resp. $\mathtt{s}_\mathtt{r}^3$) could be viewed as the more specific structure responsible for its red (resp. scarlet) character, which is also responsible for its colored (resp. red and colored) character. In epistemic terms, $\mathtt{c}_\mathtt{r}^3$ could represent the information collected by a device that could only detect the colored character of an object, $\mathtt{r}_\mathtt{r}^3$ to the information collected by a device resolving *being red* and categorizing the redness as a color, and $\mathtt{s}_\mathtt{r}^3$ to the information collected by a device that would resolve *being scarlet*, and categorize it as a redness (and as a color).

As we said, covering axioms can be added to those theories. Suppose that we add (incl·cov) to T_3 and that $(\mathtt{C\bar{O}L},\mathtt{R\bar{E}D},\mathtt{Y\bar{E}L}),(\mathtt{R\bar{E}D},\mathtt{S\bar{C}A},\mathtt{C\bar{R}I}) \in \bar{\mathbb{C}}$ (including the color crimson *CRI*). It is easy to see that, in the rose example, there is only one possible configuration of PPs, namely a single PP $\mathtt{s}_\mathtt{r}^1$ characterizing at the same time the scarletness, the redness and the colored character of $\mathfrak{r}$, i.e., $\mathtt{S\bar{C}A}(\mathtt{s}_\mathtt{r}^1)$, $\mathtt{R\bar{E}D}(\mathtt{s}_\mathtt{r}^1)$ and $\mathtt{C\bar{O}L}(\mathtt{s}_\mathtt{r}^1)$. That is, in presence of (incl·cov) and covering axioms between the relevant properties, the ontology compatible with T_3 will collapse to the ontology compatible with T_1.

6. The Reification of PP-types

We will now sketch an alternative representation method that reifies PP-predicates and is able to represent *direct* vs. *indirect* classification under a property. In an epistemic view, this method allows to clearly separate the information acquired from measurement devices from the one deduced from such information by using some general knowledge.

Up to now, the PPs corresponding to a property P has been collected by means of the PP-predicate $\bar{\mathtt{P}}$. We now *reify* PP-types into the domain of quantification. For this, we introduce a new kind of entities—$\textsc{pt}(t)$ stands for "t is a *PP-type*"—and we assume that there is a one-to-one correspondence between the properties in $\mathscr{P}$ and the $\textsc{pt}$-constants in the set $\mathbb{P}_{\mathrm{PT}}$. We note $\mathtt{p} \in \mathbb{P}_{\mathrm{PT}}$ the PP-type corresponding to $P \in \mathscr{P}$.[8]

The general idea is that PPs are classified by PP-types. In particular we consider the primitive relation of *direct classification* between PPs and PP-types: $\mathtt{dCF}(p,t)$ stands for "the PP p is *directly classified* under the PP-type t" (a4). We assume that PPs may have a unique direct PP-type, i.e., they can be directly classified under a single PP-type (a5).

a4 $\mathtt{dCF}(p,t) \to \textsc{pp}(p) \wedge \textsc{pt}(t)$
a5 $\mathtt{dCF}(p,t) \wedge \mathtt{dCF}(p,u) \to t = u$

The specialization relation between properties is represented by the *partial order* $\sqsubseteq$ defined between PP-types: $t \sqsubseteq u$ stands for "the PP-type t is a *specialization* of the PP-type u". The specialization relations in $\mathscr{S}$ are modeled by introducing ($\mathsf{pt}_{\mathsf{spec}}$) where $\mathbb{S}_{\mathrm{PT}}$ is the set of couples of $\textsc{pt}$-constants corresponding to the couples in $\mathscr{S}$.

($\mathsf{pt}_{\mathsf{spec}}$) $\bigwedge_{(\mathsf{a},\mathsf{b}) \in \mathbb{S}_{\mathrm{PT}}} \mathsf{a} \sqsubseteq \mathsf{b}$

Crucially, in this framework it is possible to make the difference between the above-mentioned direct classification and the *indirect* classification ($\mathtt{iCF}$) defined in (d1).

d1 $\mathtt{iCF}(p,t) \triangleq \exists u(\mathtt{dCF}(p,u) \wedge u \sqsubseteq t)$

The general case where '$\mathtt{a}$ is P' is then modeled using indirect, rather that direct, classification, i.e., by $\exists q(\mathtt{INH}(q,\mathtt{a}) \wedge \mathtt{iCF}(q,\mathtt{p}))$, where $\mathtt{p}$ represents the property P.[9] In the example of the rose, $\mathtt{dCF}(\mathtt{p_r},\mathtt{red})$ together with $\mathtt{red} \sqsubseteq \mathtt{col}$ imply $\mathtt{iCF}(\mathtt{p_r},\mathtt{col})$. The indirect classification $\mathtt{iCF}$ allows then to *explicitly* represent classifications grounded on knowledge about the specialization relation. First note that $\mathtt{iCF}$ does not require the existence of any additional PP. Second, this framework is compatible with the existence of PPs $\mathtt{p_r}$ and $\mathtt{q_r}$ (both inhering in the rose) such that $\mathtt{iCF}(\mathtt{p_r},\mathtt{col})$ and $\mathtt{dCF}(\mathtt{q_r},\mathtt{col})$, i.e., the existence of a PP directly classified under a given PP-type does not exclude the possibility to have a different PP directly classified under a more general PP-type.

This theory is similar to T_3 where the previous situation may be represented by $\bar{\mathtt{RED}}(\mathtt{p_r}) \wedge \bar{\mathtt{COL}}(\mathtt{p_r})$ together with $\neg\bar{\mathtt{RED}}(\mathtt{q_r}) \wedge \bar{\mathtt{COL}}(\mathtt{q_r})$. Note however that in T_3 we don't really have a notion of indirect classification, we just have the PPs $\mathtt{p_r}$ and $\mathtt{q_r}$ that are both *standardly* classified under $\bar{\mathtt{COL}}$ while $\mathtt{p_r}$, but not $\mathtt{q_r}$, is also *standardly* classified under $\bar{\mathtt{RED}}$. Here all the atomic assertions have the same 'status' and one can establish the level of resolution of a given assertion only by looking at the other assertions present in the theory. On the other hand, by reifying PP-types, it is possible to direclty manage different kinds of classification of PPs (different classification modalities) by means of several relations defined between PPs and $\textsc{pt}$-instances. Furthermore, the fact that the specialization relation is also represented by a relation ($\sqsubseteq$) leaves space for a more intensional

[8] Variables/constants ranging over P-types are noted $t,u,\ldots,\mathtt{t},\mathtt{u},\ldots$

[9] Given the reflexivity of $\sqsubseteq$, $\mathtt{dCF}(p,t) \to \mathtt{iCF}(p,t)$, i.e., direct classification is a limit case of indirect classification.

characterization of this relation. Future work should investigate the possible benefits of the reification strategy.

7. Conclusion

We have thus identified three theories of identity among PP, and characterized with which perspective (epistemic or realist) they appear to be compatible. We have presented a reification of property-types that enables to formalize how PPs can be directly or indirectly classified under property-types. Future work should investigate more closely various theories about 1) what counts as a property 2) which of those properties are linked by a specialization relation and 3) how the distinction determinate/determinable can shed light on the application of our formalization to various theories of properties and specialization. It should also investigate further the benefits that the reification of property types could bring. The question of the diachronic identity of PPs will also need to be addressed: PPs are generally considered as continuants, that persist in time by being fully present at each instant; but what are the criteria for two PPs at different times to be the same entity? This investigation on the identity of PPs should also be refined into more specific investigations about the identity of complex properties such as roles, functions and dispositions.

References

[1] R. Arp, B. Smith, and A. D. Spear. *Building ontologies with basic formal ontology*. Mit Press, 2015.

[2] C. Masolo, S. Borgo, A. Gangemi, N. Guarino, and A. Oltramari. Wonderweb deliverable d18. Technical report, CNR, 2003.

[3] H. Herre, B. Heller, P. Burek, R. Hoehndorf, F. Loebe, and H. Michalek. General formal ontology (gfo): A foundational ontology integrating objects and processes. part i: Basic principles. Technical Report Version 1.0, Onto-Med Report Nr. 8, 01.07.2006, University of Leipzig, 2006.

[4] G. Guizzardi. *Ontological Foundations for Structural Conceptual Models*. Phd, University of Twente, 2005.

[5] D. Ehring. *Tropes: Properties, Objects, and Mental Causation*. Oxford Univeristy Press, 2011.

[6] F. MacBride. Truthmakers. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2019 edition, 2019.

[7] M. Textor. States of affairs. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.

[8] K. Mulligan and F. Correia. Facts. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2017 edition, 2017.

[9] A. Barton, L. Jansen, and J.-F. Ethier. A taxonomy of disposition-parthood. In A. Galton and F. Neuhaus, editors, *Proceedings of the Joint Ontology Workshops 2017*, volume 2050, pages 1–10. CEUR Workshop Proceedings, 2018.

[10] A. Barton, O. Grenier, L. Jansen, and J.-F. Ethier. The identity of dispositions. In S. Borgo, P. Hitzler, and O. Kutz, editors, *Formal Ontology in Information Systems: Proceedings of the 10th International Conference (FOIS 2018)*, volume 306, pages 113–126. IOS Press, 2018.

[11] D. H. Sanford. Determinates vs. determinables. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Spring 2013 Edition.

[12] G. H. Merrill. Ontological realism: Methodology or misdirection? *Applied Ontology*, 5(2):79–108, 2010.

[13] B. Smith and W. Ceusters. Ontological realism: A methodology for coordinated evolution of scientific ontologies. *Applied ontology*, 5(3-4):139–188, 2010.