Some Considerations on Mixing Semantics in Abstract Argumentation

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Abstract. This paper discusses the issue of mixing different argumentation semantics in a single Dung's argumentation framework. The general notion of combination schema is defined to model a specific way of mixing argumentation semantics, and several properties that may be desirable for a combination schema are introduced at an abstract level. A specific combination schema is then evaluated in the light of such properties, showing that there are several interesting challenges both from a conceptual and a technical perspective still to be tackled.

Keywords: Abstract Argumentation, Semantics, Hybrid Approach

1 Introduction

Dung's model of abstract argumentation focuses on evaluating the justification of a set of arguments related by an attack relation [7]. To this purpose, arguments are simply represented as nodes of a directed graph, called *abstract argumentation framework*, and binary attacks between them correspond to the edges of the graph. In this general setting, several *argumentation semantics* have been defined to determine the justification state of the arguments. Different semantics reflect different intuitions and are meant to satisfy specific properties and/or fulfill some desired behavior in problematic examples [5, 2]. Moreover, they feature different degrees of skepticism, i.e. they make more or less committed choices about argument justification, and exhibit different computational complexities.

With the exception of [10], all proposals are intended to apply a single argumentation semantics at a global level, under the assumption that the involved attacks between arguments are essentially homogeneous. However, in [10, 8, 4] various motivations have been presented to mix different semantics in the same argumentation framework. First, in a complex argumentation framework different parts may model different application contexts, thus a specific semantics may be suitable for each of these parts. Examples involving conflicts of heterogeneous nature include approaches to integrate epistemic and practical arguments [9], and modelling choices underlying specific assumptions on the treatment of conflicts between arguments [4]. Moreover, in multi-agent systems different reasoning attitudes may be adopted by individual agents, requiring multiple semantics to model the interactions between agents. Finally, practical considerations, such as the cost of evaluation errors and the computational complexity of semantics, can lead to the adoption of different semantics for different sets of arguments.

An approach to mix different argumentation semantics has been first proposed in [10] and then recasted as an instance of a framework for combining argumentation semantics based on decomposability [1], as sketched in [8]. In [4], we have shown how this approach is able to manage epistemic and practical arguments in several examples inspired from [9].

In this paper we adopt a more general perspective, by providing the following contributions:

- We introduce the definition of *combination schema* as a general model of approaches to mix argumentation semantics, and we define a number of properties to characterize such approaches.
- We analyze the approach introduced in [10, 8] in the light of these properties, also providing some preliminary results.
- We discuss several perspectives for further research opened by this preliminary analysis.

After some background in Section 2, the previous points are dealt with in Sections 3, 4 and 5, respectively. Proofs of the results are omitted due to space limitations.

2 Background

We follow the traditional definition of argumentation framework introduced by Dung [7] and define its restriction to a subset of arguments.

Definition 1. *An* argumentation framework *is a pair $AF = (Ar, att)$ in which Ar is a finite and non empty set*[1] *of arguments and $att \subseteq Ar \times Ar$. An argument A attacks an argument B, denoted as $A \to B$, if $(A, B) \in att$. An argument A such that for no B $B \to A$ is called* initial. *An argument B such that $(B, B) \in att$ is called* self-attacking. *Given a set $Args \subseteq Ar$, we denote as $Args^+$ the set of arguments attacked by $Args$, i.e. $Args^+ = \{A \mid \exists B \in Args, (B, A) \in att\}$. The set $Args$ is conflict-free iff $Args \cap Args^+ = \emptyset$. Given a set $Args \subseteq Ar$, the restriction of AF to $Args$, denoted as $AF{\downarrow_{Args}}$, is the argumentation framework $(Args, att \cap (Args \times Args))$.*

In this paper we adopt the labelling-based approach to the definition of argumentation semantics. A labelling assigns to each argument of an argumentation framework a label taken from the set $\{\texttt{in}, \texttt{out}, \texttt{undec}\}$, where the label `in` means that the argument is accepted, the label `out` means that the argument is rejected, and the label `undec` means that the status of the argument is undecided. For technical reasons, we define labellings both for argumentation frameworks and for arbitrary sets of arguments.

[1] In the general definition, the set of arguments may be infinite or empty.

Definition 2. *Given a set of arguments Args, a* labelling *of Args is a total function* $Lab : Args \longrightarrow \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$. *The set of all* labellings *of Args is denoted as* $\mathfrak{L}_{Args}$. *Given an argumentation framework* $AF = (Ar, att)$, *a* labelling *of AF is a labelling of Ar. The set of all* labellings *of AF is denoted as* $\mathfrak{L}(AF)$. *Given a labelling Lab, we write* $\mathtt{in}(Lab)$ *for* $\{A \mid Lab(A) = \mathtt{in}\}$, $\mathtt{out}(Lab)$ *for* $\{A \mid Lab(A) = \mathtt{out}\}$ *and* $\mathtt{undec}(Lab)$ *for* $\{A \mid Lab(A) = \mathtt{undec}\}$. *For a labelling Lab of Args, the restriction of Lab to a set of arguments* $Args' \subseteq Args$, *denoted as* $Lab\downarrow_{Args'}$, *is defined as* $Lab \cap (Args' \times \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\})$. *We extend this notation to set of labellings, i.e. given a set of a labellings* $\mathfrak{L} \subseteq \mathfrak{L}_{Args}$, $\mathfrak{L}\downarrow_{Args'} \triangleq \{Lab\downarrow_{Args'} \mid Lab \in \mathfrak{L}\}$.

A labelling-based semantics prescribes a set of labellings for each argumentation framework.

Definition 3. *Given an argumentation framework* $AF = (Ar, att)$, *a labelling-based semantics* $\mathbf{S}$ *associates with AF a subset of* $\mathfrak{L}(AF)$, *denoted as* $\mathbf{L_S}(AF)$.

Two notions of justification, i.e. skeptical and credulous, can then be introduced with respect to a semantics.

Definition 4. *Given a labelling-based semantics* $\mathbf{S}$ *and an argumentation framework AF, an argument A is* skeptically justified *under* $\mathbf{S}$ *if* $\forall Lab \in \mathbf{L_S}(AF)$ $Lab(A) = \mathtt{in}$; *an argument A is* credulously justified *under* $\mathbf{S}$ *if* $\exists Lab \in \mathbf{L_S}(AF) : Lab(A) = \mathtt{in}$.

In general, a semantics encompasses a set of alternative labellings for a single argumentation framework. If a semantics $\mathbf{S}$ is defined in such a way that such a set is always non empty, i.e. $\forall AF, \mathbf{L_S}(AF) \neq \emptyset$, then $\mathbf{S}$ is said to be *universally defined*. Moreover, a semantics may be defined so that a unique labelling is always prescribed, i.e. for every argumentation framework AF, $|\mathbf{L_S}(AF)| = 1$. In this case the semantics is said to be *single-status*, while in the general case it is said to be *multiple-status*. Note that according to the previous definitions a single-status semantics is universally defined.

In the labelling-based approach, a semantics definition relies on some *legality* constraints relating the label of an argument to those of its attackers.

Definition 5. *Let Lab be a labelling of the argumentation framework* (Ar, att). *An* `in`*-labelled argument is said to be* legally `in` *iff all its attackers are labelled* `out`. *An* `out`*-labelled argument is said to be* legally `out` *iff it has at least one attacker that is labelled* `in`. *An* `undec`*-labelled argument is said to be* legally `undec` *iff not all its attackers are labelled* `out` *and it does not have an attacker that is labelled* `in`.

We now introduce the definitions of labellings corresponding to traditional conflict-free, admissible and complete semantics.

Definition 6. *Let* $AF = (Ar, att)$ *be an argumentation framework. A* conflict-free labelling *is a labelling Lab where every* `in`*-labelled argument does not have an attacker that is* `in`*-labelled, and every* `out`*-labelled argument is legally* `out`.

Definition 7. *Let $AF = (Ar, att)$ be an argumentation framework. An* admissible labelling *is a labelling Lab where every* `in`*-labelled argument is legally* `in` *and every* `out`*-labelled argument is legally* `out`*.*

Definition 8. *A* complete labelling *is a labelling where every* `in`*-labelled argument is legally* `in`*, every* `out`*-labelled argument is legally* `out` *and every* `undec`*-labelled argument is legally* `undec`*.*

On this basis, the labelling-based definitions of several argumentation semantics can be introduced. The semantics are defined by referring to the commitment relation between labellings [2].

Definition 9. *Let Lab_1 and Lab_2 be two labellings. We say that Lab_2 is* more or equally committed *than Lab_1 ($Lab_1 \sqsubseteq Lab_2$) iff $\mathtt{in}(Lab_1) \subseteq \mathtt{in}(Lab_2)$ and $\mathtt{out}(Lab_1) \subseteq \mathtt{out}(Lab_2)$.*

Definition 10. *Let $AF = (Ar, att)$ be an argumentation framework. A* stable labelling *of AF is a complete labelling without* `undec`*-labelled arguments. The* grounded labelling *of AF is the minimal (w.r.t. $\sqsubseteq$) labelling among all complete labellings. A* preferred labelling *of AF is a maximal (w.r.t. $\sqsubseteq$) labelling among all complete labellings. The* ideal labelling *of AF is the maximal (under $\sqsubseteq$) complete[2] labelling Lab that is less or equally committed than each preferred labelling of AF (i.e. for each preferred labelling Lab_P it holds that $Lab \sqsubseteq Lab_P$).*

The uniqueness of the grounded and the ideal labelling has been proved in [6]. Accordingly, grounded and ideal semantics are single-status, the other semantics are multiple-status. Admissible, complete, stable, grounded, preferred, ideal semantics are denoted in the following as **AD**, **CO**, **ST**, **GR**, **PR** and **ID**, respectively. Since **AD** and **CO** are mainly used as a fundamental notions rather than as semantics for argument evaluation, in the following we will focus on **ST**, **GR**, **PR** and **ID**.

Argumentation semantics can be compared w.r.t. the tendency of making more or less committed choices about argument justification. In particular, since argument justification has been introduced according to a skeptical and credulous perspective, two corresponding skepticism relations can be defined between semantics.

Definition 11. *Given two labelling-based semantics $\mathbf{S}_1$ and $\mathbf{S}_2$, $\mathbf{S}_1 \preceq^S_{SK} \mathbf{S}_2$ iff for any AF where $\mathbf{L}_{\mathbf{S}_1}(AF) \neq \emptyset$ and $\mathbf{L}_{\mathbf{S}_2}(AF) \neq \emptyset$, it holds that $\forall Lab_2 \in \mathbf{L}_{\mathbf{S}_2}(AF)\ \exists Lab_1 \in \mathbf{L}_{\mathbf{S}_1}(AF)$ such that $Lab_1 \sqsubseteq Lab_2$.*

Intuitively, according to the skeptical viewpoint on argument justification, $\mathbf{S}_1 \preceq^S_{SK} \mathbf{S}_2$ indicates that $\mathbf{S}_1$ is not more committed (or, equivalently, not less skeptical) than $\mathbf{S}_2$, since every labelling in $\mathbf{L}_{\mathbf{S}_2}(AF)$ has a more skeptical counterpart in $\mathbf{L}_{\mathbf{S}_1}(AF)$, while $\mathbf{L}_{\mathbf{S}_1}(AF)$ can include additional unrelated labellings

[2] Literally, the original definition refers to an admissible labelling rather than a complete labelling. However, the definition adopted here is equivalent to the original one, since it can be shown that the ideal labelling is a complete labelling [6].

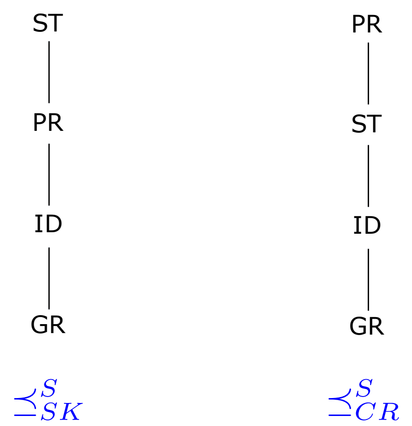

Fig. 1. Hasse diagrams of the semantics according to the skeptical relations.

that can only lead to less committed choices w.r.t. skeptical justification of arguments.

Definition 12. *Given two labelling-based semantics* $\mathbf{S}_1$ *and* $\mathbf{S}_2$*,* $\mathbf{S}_1 \preceq^{S}_{CR} \mathbf{S}_2$ *iff for any* AF *where* $\mathbf{L}_{\mathbf{S}_1}(AF) \neq \emptyset$ *and* $\mathbf{L}_{\mathbf{S}_2}(AF) \neq \emptyset$*, it holds that* $\forall Lab_1 \in \mathbf{L}_{\mathbf{S}_1}(AF)\ \exists Lab_2 \in \mathbf{L}_{\mathbf{S}_2}(AF)$ *such that* $Lab_1 \sqsubseteq Lab_2$*.*

According to the credulous viewpoint on argument justification, $\mathbf{S}_1 \preceq^{S}_{CR} \mathbf{S}_2$ indicates that $\mathbf{S}_1$ is not more committed (or, equivalently, not less skeptical) than $\mathbf{S}_2$, since every labelling in $\mathbf{L}_{\mathbf{S}_1}(AF)$ has a more committed counterpart in $\mathbf{L}_{\mathbf{S}_2}(AF)$, and $\mathbf{L}_{\mathbf{S}_2}(AF)$ can include additional unrelated labellings that can potentially lead to more committed choices w.r.t. the credulous justification of arguments.

The Hasse diagrams of the semantics considered in this paper according to the skeptical and the credulous perspective are shown in Figure 1. Basically, arcs connect pairs of comparable semantics, and lower semantics are less committed than higher ones.

With abuse of notation, we extend the previous notions to sets of labellings, i.e. given two sets of labellings $\mathfrak{L}_1$ and $\mathfrak{L}_2$ of $\mathfrak{L}(AF)$:

- $\mathfrak{L}_1 \preceq^{S}_{SK} \mathfrak{L}_2$ iff $\mathfrak{L}_1 \neq \emptyset$, $\mathfrak{L}_2 \neq \emptyset$, and $\forall Lab_2 \in \mathbf{L}_{\mathbf{S}_2}(AF)\ \exists Lab_1 \in \mathbf{L}_{\mathbf{S}_1}(AF)$ such that $Lab_1 \sqsubseteq Lab_2$
- $\mathfrak{L}_1 \preceq^{S}_{CR} \mathfrak{L}_2$ iff $\mathfrak{L}_1 \neq \emptyset$, $\mathfrak{L}_2 \neq \emptyset$, and $\forall Lab_1 \in \mathbf{L}_{\mathbf{S}_1}(AF)\ \exists Lab_2 \in \mathbf{L}_{\mathbf{S}_2}(AF)$ such that $Lab_1 \sqsubseteq Lab_2$

3 Combination schema and relevant properties

The notion of *mixing* models an argumentation framework partitioned into a set of subframeworks each managed by an associated semantics.

Definition 13. *A* mixing m *is a tuple* $(AF, \mathcal{P}, \mathbf{SEM}, S)$*, where* AF *is an agumentation framework* (Ar, att)*,* $\mathcal{P} = \{P_1, \ldots, P_n\}$ *is a partition*[3] *of* Ar*,* $\mathbf{SEM}$ *is a set of semantics, and* $S : \mathcal{P} \to \mathbf{SEM}$ *is a total function associating a semantics to each element of the partition such that* $\forall \mathbf{S} \in \mathbf{SEM}$*,* $\exists P_i \in \mathcal{P}$ *such that* $S(P_i) = \mathbf{S}$*. Given a mixing* m*, the elements in* m *are denoted as* AF_m*,* $\mathcal{P}_m$*,* $\mathbf{SEM}_m$*,* S_m*, respectively. Moreover, the set of all possible mixings is denoted as* MIX*.*

In general there are many possible ways to combine the semantics, each corresponding to a specific *combination schema* as in the following definition.

Definition 14. *A combination schema* cs *is a function which, given a mixing* $m \in MIX$*, returns an element of* $2^{\mathfrak{L}(AF_m)}$*, i.e. a set of labellings of the argumentation framework associated to* m*.*

In a sense, the notion of combination schema generalizes the notion of semantics. Whereas a semantics identifies a way to assign a set of labellings to an argumentation framework, a combination schema identifies a way to assign a set of labellings to a mixing, on the basis of the argumentation semantics associated to the elements of the partition.

In the following, several properties of combination schemas will be introduced to characterize different proposals for combining argumentation semantics. These properties are not meant to be mandatory, but they may allow one to know whether a combination schema is suitable to a specific application context.

In general, a property of a combination schema cs requires the set of labellings $cs(m)$ (with m possibly subject to specific constraints) to satisfy a set of conditions, where these conditions may depend in turn on the characteristics of the input mixing m. As it will be shown in the following, some properties do not hold in general, but they do if the set of semantics in the input mixing is restricted and/or the partition of the arguments in the input mixing satisfies specific constraints.

As to partitions, the relevant constraints can be expressed by the notion of *partition selector* of [1] here recalled.

Definition 15. *A partition selector* $\mathcal{F}$ *is a function receiving as input an argumentation framework* $AF = (Ar, att)$ *and returning a set of partitions of* Ar*.*

In this paper three restrictions on partitions are worth considering. The first one restricts the partitions to those including one set only, i.e. the whole set of arguments of the argumentation framework.

Definition 16. *For any argumentation framework* $AF = (Ar, att)$*,* $\mathcal{F}_{\mathtt{WH}}(AF) \triangleq \{\{Ar\}\}$*.*

The other two restrictions, as in [1], are based on the notion of strongly connected components.

[3] i.e. $\bigcup_{i \in \{1, \ldots, n\}} P_i = Ar$ and $i \neq j \to P_i \cap P_j = \emptyset$

Definition 17. *Given an argumentation framework* $AF = (Ar, att)$*, the set of strongly connected components of* AF*, denoted as* SCCS_{AF}*, consists of the equivalence classes of arguments induced by the binary relation of path-equivalence, i.e. the relation* $\rho(A, B)$ *defined over* $Ar \times Ar$ *such that* $\rho(A, B)$ *holds if and only if* $A = B$ *or there are directed paths from* A *to* B and *from* B *to* A *in* AF*.*

In particular, we consider the partition coinciding with the strongly connected components SCCS_{AF}, identified by the partition selector $\mathcal{F}_{\mathtt{SCC}}$, and the partitions where each element is the union of some strongly connected components, identified by the partition selector $\mathcal{F}_{\cup\mathtt{SCC}}$.

Definition 18. *For any argumentation framework* $AF = (Ar, att)$*,* $\mathcal{F}_{\mathtt{SCC}}(AF) \triangleq \{\mathrm{SCCS}_{AF}\}$*,* $\mathcal{F}_{\cup\mathtt{SCC}}(AF) \triangleq \{\{P_1, \ldots, P_n\} \mid \{P_1, \ldots, P_n\}$ *is a partition of* Ar *and* $\forall i \in \{1, \ldots n\}$ $((S \in \mathrm{SCCS}_{AF} \wedge P_i \cap S \neq \emptyset) \rightarrow S \subseteq P_i\})$*.*

Summing up, as to property restrictions we say that:

- a property is satisfied by a combination schema *cs under* a given set of semantics **SSEM** if it holds for any mixing m such that $\mathbf{SEM}_m \subseteq \mathbf{SSEM}$;
- a property is satisfied by a combination schema *cs w.r.t.* a partition selector $\mathcal{F}$ if it holds for any mixing m such that $\mathcal{P}_m \in \mathcal{F}(AF_m)$.

In the following the properties considered in this paper are introduced.

3.1 Properties related to Semantics Dependence and Independence

First, an expected requirement for any reasonable combination schema is that it should actually depend on the semantics assigned to the partition elements of the input mixing. This requirement can however be enforced at different levels of strictness.

At the lowest level, if a semantics **S** is applied to the whole argumentation framework (i.e. the partition includes just one set coinciding with the set of arguments Ar), then one can require the labellings returned by the combination schema to coincide with those returned by **S**.

Definition 19. *A combination schema cs is reasonable if* $\forall m \in MIX$*, if* $\mathcal{P}_m = \{Ar\}$ *then* $cs(m) = \mathbf{L}_{S_m(Ar)}(AF_m)$*.*

A more demanding requirement is to extend this property to all cases where a unique semantics **S** is assigned to the partition elements.

Definition 20. *A combination schema cs is adherent to a unique semantics if* $\forall m \in MIX$*, if* $\mathbf{SEM}_m = \{\mathbf{S}\}$ *then* $cs(m) = \mathbf{L}_{\mathbf{S}}(AF_m)$*.*

It is immediate to see that a combination schema is reasonable iff it is adherent to a unique semantics w.r.t. $\mathcal{F}_{\mathtt{WH}}$.

One can view the property of adherence to a unique semantics as a conjunction of two partial properties, namely *top-down* and *bottom-up* adherence to a unique semantics.

Definition 21. *A combination schema cs is top-down adherent to a unique semantics if* $\forall m \in MIX$*, if* $\mathbf{SEM}_m = \{\mathbf{S}\}$ *then* $\mathbf{L}_{\mathbf{S}}(AF_m) \subseteq cs(m)$.

Definition 22. *A combination schema cs is bottom-up adherent to a unique semantics if* $\forall m \in MIX$*, if* $\mathbf{SEM}_m = \{\mathbf{S}\}$ *then* $cs(m) \subseteq \mathbf{L}_{\mathbf{S}}(AF_m)$.

In a sense, top-down adherence requires the application of the combination schema to be a complete procedure w.r.t. the semantics involved, i.e. the obtained labellings include those prescribed by the semantics, while bottom-up adherence corresponds to correctness of the procedure, i.e. all obtained labellings are also prescribed by the semantics.

As a sort of counterpart of dependence properties, one may require the result of the combination schema to sometimes depart from the original semantics.

Definition 23. *A combination schema cs is* non symbiotic *if* $\exists m \in MIX$ *and* $\exists P \in \mathcal{P}_m$ *such that* $cs(m)\downarrow_P \nsubseteq (\mathbf{L}_{S_m(P)}(AF_m))\downarrow_P$.

3.2 Existence properties

Another significant requirement for a combination schema is being actually able to identify a non empty set of labellings for the mixing in input. Of course, this cannot be required in general for a *reasonable* argumentation schema if one of the argumentation semantics of the input mixing **S** is not universally defined. For instance, the schema returns the (potentially empty) set of labellings prescribed by **S** when applied w.r.t. a partition of $\mathcal{F}_{\mathtt{WH}}$ with the unique semantics **S** involved. As a consequence, we introduce the universal definition requirement under the assumption that all the semantics are universally defined.

Definition 24. *A combination schema cs is universally defined if* $\forall m \in MIX$ *such that* $\forall \mathbf{S} \in \mathbf{SEM}_m$ **S** *is universally defined, it holds that* $cs(m) \neq \emptyset$.

3.3 Properties that warrant basic properties of labellings

Whenever the labellings prescribed by all the involved semantics satisfy a desirable property, it would be nice this property to be satisfied also by the labellings returned by the combination schema. The basic properties we consider for labellings are conflict-freeness, admissibility, and completeness.

Definition 25. *A semantics* **S** *satisfies the CF (admissibility, completeness) criterion iff for any argumentation framework AF, all the labellings in* $\mathbf{L}_{\mathbf{S}}(AF)$ *are conflict-free (admissible, complete) labellings.*

Definition 26. *A combination schema cs is* conflict-freeness (admissibility) (completeness) *preserver if* $\forall m \in MIX$*, if* $\forall \mathbf{S} \in \mathbf{SEM}_m$ **S** *satisfies the CF (admissibility, completeness) criterion, then all the labellings in* $cs(m)$ *are conflict-free (admissible, complete) labellings.*

3.4 Skepticism-related properties

These properties concern the characterization of the labellings obtained by mixing semantics w.r.t. the skepticism relations between semantics.

We first introduce a criterion of skepticism monotony, basically stating that the level of skepticism of the labellings obtained by a combination schema monotonically depends on that of individual semantics. This can be formalized by requiring that replacing a semantics associated to a subframework with a more commited one should correspondingly result in a more committed set of labellings.

Definition 27. *A combination schema cs is* monotonic *w.r.t.* $\preceq^S$*, with* $\preceq^S \in \{\preceq^S_{SK}, \preceq^S_{CR}\}$*, iff* $\forall m_1, m_2 \in MIX$ *such that* $AF_{m_1} = AF_{m_2}$ *and* $\mathcal{P}_{m_1} = \mathcal{P}_{m_2}$*, if* $\forall P_i \in \mathcal{P}_{m_1}\ S_{m_1}(P_i) \preceq^S S_{m_2}(P_i)$ *then* $cs(m_1) \preceq^S cs(m_2)$.

We then consider a criterion of *boundedness* requiring for any mixing that if the set of involved semantics are lower (upper) bounded by a semantics w.r.t. a skepticism relation, then the resulting set of labellings is also bounded.

Definition 28. *A combination schema cs is* lower-bounded *w.r.t.* $\preceq^S$*, with* $\preceq^S \in \{\preceq^S_{SK}, \preceq^S_{CR}\}$*, iff* $\forall m \in MIX$*, if there is a semantics* $\mathbf{S}'$ *such that* $\forall \mathbf{S} \in \mathbf{SEM}_m\ \mathbf{S}' \preceq^S \mathbf{S}$ *then* $\mathbf{L}_{\mathbf{S}'}(AF_m) \preceq^S cs(m)$*. A combination schema cs is* upper-bounded *w.r.t.* $\preceq^S$*, with* $\preceq^S \in \{\preceq^S_{SK}, \preceq^S_{CR}\}$*, iff* $\forall m \in MIX$*, if there is a semantics* $\mathbf{S}'$ *such that* $\forall \mathbf{S} \in \mathbf{SEM}_m\ \mathbf{S} \preceq^S \mathbf{S}'$ *then* $cs(m) \preceq^S \mathbf{L}_{\mathbf{S}'}(AF_m)$.

It should be noted that, while seemengly related, monotonicity and (lower or upper) boundedness of combination schemas are independent properties. Monotonicity may hold even if the labellings arising from the combination schema are not bounded by a maximal (or minimal) semantics w.r.t. skepticism.

4 A decomposability-based approach

An approach to combine different argumentation semantics generalizes to multiple semantics the model introduced in [1] to analyze the decomposability properties of individual semantics, i.e. concerning the correspondences between semantics outcome at global and local level. This approach, first proposed in [10] and described in [8], works as follows. Given a mixing $m = (AF, \mathcal{P}, \mathbf{SEM}, S)$, for each element P_i of the partition $\mathcal{P}$ a function which models the application of the semantics $S(P_i)$ at the local level is applied, to determine the set of the labellings restricted to P_i. This local computation depends on the topology of the subframework $AF\downarrow_{P_i}$, on the attacks received by P_i from the external arguments and on the labels assigned to these attacking arguments by the other local functions. The following definition introduces the corresponding combination schema cs_{db}.

Definition 29. *Given a mixing* $m = (AF, \mathcal{P}, \mathbf{SEM}, S)$*,* $cs_{db}(m) \triangleq \{\bigcup_{P_i \in \mathcal{P}_m} Lab_{P_i} \mid Lab_{P_i} \in F_{S(P_i)}(AF\downarrow_{P_i}, {P_i}^{inp}, (\bigcup_{j=1\ldots n, j \neq i} Lab_{P_j})\downarrow_{{P_i}^{inp}}, {P_i}^R)\}$*, where* $F_{S(P_i)}$ *a* local function *which assigns to the arguments in* P_i *a set of labellings on the basis of* $AF{\downarrow}_{P_i}$, *of the set* ${P_i}^{inp} = \{A \notin P_i \mid \exists B \in P_i : A \to B\}$ *(including the external arguments attacking* P_i *and playing the role of* input *arguments for the set* P_i*), of the labels externally assigned to them, i.e.* $(\bigcup_{j=1\ldots n, j \neq i} Lab_{P_j}){\downarrow}_{{P_i}^{inp}}$, *and of the attack relation* ${P_i}^R \equiv att \cap ({P_i}^{inp} \times P_i)$ *from the input arguments of* ${P_i}^{inp}$ *to* P_i.

The question is then how to identify the local function $F_{\mathbf{S}}$ of a semantics **S** to be used in Definition 29. The local function can be univocally identified if the semantics **S** is *complete-compatible*.

Definition 30. *A semantics* **S** *is complete-compatible if it satisfies the following constraints:*

1. *For any argumentation framework* $AF = (Ar, att)$, *every labelling* $Lab \in \mathbf{L_S}(AF)$ *satisfies the following conditions:*
   - *if* $A \in Ar$ *is initial, then* $Lab(A) = \texttt{in}$
   - *if* $B \in Ar$ *and there is an initial argument* A *such that* $A \to B$, *then* $Lab(B) = \texttt{out}$
   - *if* $C \in Ar$ *is self-attacking, and there are no attackers of* C *besides* C *itself, then* $Lab(C) = \texttt{undec}$
2. *for any set of arguments* $\mathcal{I}$ *and any labelling* $Lab_{\mathcal{I}} \in \mathfrak{L}_{\mathcal{I}}$, *the argumentation framework* $AF' = (\mathcal{I}', att')$, *where* $\mathcal{I}' = \mathcal{I} \cup \{A' \mid A \in \texttt{out}(Lab_{\mathcal{I}})\}$ *and* $att' = \{(A', A) \mid A \in \texttt{out}(Lab_{\mathcal{I}})\} \cup \{(A, A) \mid A \in \texttt{undec}(Lab_{\mathcal{I}})\}$ *admits a labelling*[4], *i.e.* $|\mathbf{L_S}(AF')| > 0$.

In this case, the local function of **S** can be univocally determined by considering each input tuple $(AF_L, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})$, and constructing a corresponding *standard argumentation framework* [1] where the input arguments $\mathcal{I}$ as well as the attacks of $att_{\mathcal{I}}$ are added to AF_L, and the input labelling $Lab_{\mathcal{I}}$ is enforced through the addition of initial arguments attacking `out`-labelled arguments of $\mathcal{I}$ and self-attacks for all `undec`-labelled arguments of $\mathcal{I}$ (the input labelling is enforced due to the fact that **S** is complete-compatible). The output of the local function for the input $(AF_L, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})$ then includes the set of labellings obtained by applying the semantics to the standard argumentation framework, restricted to the set of original arguments of AF_L. This choice for the local function, called *canonical local function* in [1], is necessary to guarantee the combination schema cs_{db} to be adherent to a unique semantics at least for the simple cases represented by the standard argumentation frameworks identified by the various input tuples.

It is easy to verify that **CO**, **GR**, **PR** and **ID** are complete-compatible. The corresponding local functions can be explicitly expressed as in the following proposition.

Proposition 1. *The canonical local functions of complete, grounded, preferred and ideal semantics are as follows:*

[4] Due to the first point, this labelling is necessary unique, i.e. $|\mathbf{L_S}(AF')| = 1$.

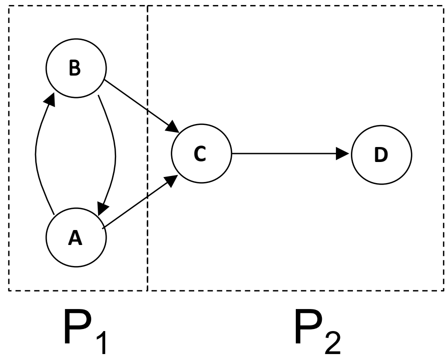

Fig. 2. A partitioned argumentation framework.

- $F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \triangleq \{Lab \in \mathfrak{L}(AF) \mid$
  $Lab(A) = \texttt{in} \rightarrow ((\forall B \in (Ar \cup att_{\mathcal{I}}) : (B, A) \in att, Lab(B) = \texttt{out})$
  $Lab(A) = \texttt{out} \rightarrow ((\exists B \in (Ar \cup \mathcal{I}) : (B, A) \in att \wedge Lab(B) = \texttt{in})$
  $Lab(A) = \texttt{undec} \rightarrow (((\forall B \in (Ar \cup \mathcal{I}) : (B, A) \in att, Lab(B) \neq \texttt{in}) \wedge ((\exists B \in (Ar \cup \mathcal{I}) : (B, A) \in att \wedge Lab(B) = \texttt{undec})\}$
- $F_{\mathbf{GR}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \triangleq \{Lab \mid Lab \textit{ is minimal w.r.t. } \sqsubseteq \textit{ in } F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})\}$
- $F_{\mathbf{PR}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \triangleq \{Lab \mid Lab \textit{ is maximal w.r.t. } \sqsubseteq \textit{ in } F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})\}$
- $F_{\mathbf{ID}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \triangleq \{Lab \mid Lab \textit{ is maximal w.r.t. } \sqsubseteq \textit{ in } F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})^*\}$, *where* $F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}})^* = \{Lab \in F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \mid \forall Lab_P \in F_{\mathbf{PR}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) Lab \sqsubseteq Lab_P\}$

Stable semantics is not complete-compatible, since self-attacking arguments otherwise unattacked prevent any stable labelling to exist (see the second item of Definition 30). It turns out that several local functions guarantee the combination schema cs_{db} to be adherent to a unique semantics. In particular, for all input labellings with at least one `undec`-labelled argument, the output of the local function can be choosen arbitrarily, as long as it includes no labellings with `undec`-labelled arguments. To maximize the set of labellings, the following local function can be adopted for **ST**:

$$F_{\mathbf{ST}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \triangleq \{Lab \in F_{\mathbf{CO}}(AF, \mathcal{I}, Lab_{\mathcal{I}}, att_{\mathcal{I}}) \mid \texttt{undec}(Lab) = \emptyset\}$$

As a simple example of application of cs_{db}, consider the partitioned argumentation as in Figure 2.

Considering a corresponding mixing m such that $S(P_1) = \mathbf{PR}$ and $S(P_2) = \mathbf{GR}$, the application of $F_{\mathbf{PR}}$ returns for $AF{\downarrow}_{P_1}$ the labellings $\{(A, \texttt{in}), (B, \texttt{out})\}$ and $\{(A, \texttt{out}), (B, \texttt{in})\}$. Taking into account that A and B are the input arguments of P_2, it is easy to see that $F_{\mathbf{GR}}$ returns for $AF{\downarrow}_{P_2}$ the labelling $\{(C, \texttt{out}), (D, \texttt{in})\}$ both with the input labelling $\{(A, \texttt{in}), (B, \texttt{out})\}$ and with the input labelling $\{(A, \texttt{out}), (B, \texttt{in})\}$. Summing up, it turns out that $cs_{db}(m) =$

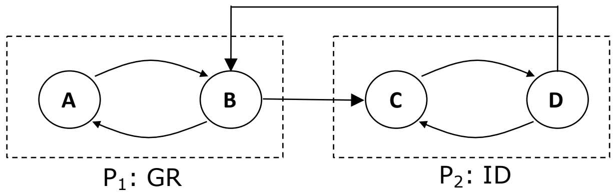

Fig. 3. A mixing showing that cs_{db} is not universally defined.

$\{\{(A,\texttt{in}),(B,\texttt{out}),(C,\texttt{out}),(D,\texttt{in})\},\{(A,\texttt{out}),(B,\texttt{in}),(C,\texttt{out}),(D,\texttt{in})\}\}$. This example shows in particular that cs_{db} is non symbiotic.

Considering instead a mixing m such that $S(P_1) = \mathbf{GR}$ and $S(P_2) = \mathbf{PR}$, the application of $F_{\mathbf{GR}}$ returns for $AF{\downarrow}_{P_1}$ the labelling $\{(A,\texttt{undec}),(B,\texttt{undec})\}$, and the application of $F_{\mathbf{PR}}$ with such an input returns for $AF{\downarrow}_{P_2}$ a singleton including the labelling $\{(C,\texttt{undec}),(D,\texttt{undec})\}$. Summing up, $cs_{db}(m) = \{\{(A,\texttt{undec}),(B,\texttt{undec}),(C,\texttt{undec}),(D,\texttt{undec}\}\}$.

In general, cs_{db} is not universally defined, since in particular examples where the partition elements are related by a cyclic relation induced by attacks the resulting set of labellings is empty even if the involved semantics are universally defined. As an example, consider the mixing depicted in Figure 3. First, note that P_2 has B as its only input argument, and $F_{\mathbf{ID}}$ returns for $AF{\downarrow}_{P_2}$ the labelling $\{(C,\texttt{out}),(D,\texttt{in})\}$ with the input labelling $(B,\texttt{undec})$, the labelling $\{(C,\texttt{undec}),(D,\texttt{undec})\}$ with the input labelling $(B,\texttt{out})$, and the labelling $\{(C,\texttt{out}),(D,\texttt{in})\}$ with the input labelling $(B,\texttt{in})$. As to P_1, $F_{\mathbf{GR}}$ returns for $AF{\downarrow}_{P_1}$ the labelling $\{(A,\texttt{undec}),(B,\texttt{undec})\}$ with the input labelling $(D,\texttt{undec})$, the labelling $\{(A,\texttt{undec}),(B,\texttt{undec})\}$ with the input labelling $(D,\texttt{out})$, and the labelling $\{(A,\texttt{in}),(B,\texttt{out})\}$ with the input labelling $(D,\texttt{in})$. It can be seen that there is no labelling satisfying Defnition 29. In particular, if B is undec-labelled then D is in-labelled according to **ID**, but in this case **GR** labels B as out. If instead B is out-labelled then D is undec-labelled according to **ID**, but in this case **GR** labels B as undec. Finally, if B is in-labelled then D is in-labelled according to **ID**, but in this case **GR** labels B as out.

The outcome of the previous example is due to the peculiar behavior of $F_{\mathbf{ID}}$ which is not monotonic, i.e. the outcome labellings does not respect the $\sqsubseteq$ relations between input labellings. In particular, considering the input labellings $Lab_{\mathcal{I}}{}^1 = (B,\texttt{undec})$ and $Lab_{\mathcal{I}}{}^2 = (B,\texttt{out})$ for which $Lab_{\mathcal{I}}{}^1 \sqsubseteq Lab_{\mathcal{I}}{}^2$, it turns out that $F_{\mathbf{ID}}(AF{\downarrow}_{P_2},\{B\},Lab_{\mathcal{I}}{}^1,\{(B,C)\}) = \{(C,\texttt{out}),(D,\texttt{in})\}$, and $F_{\mathbf{ID}}(AF{\downarrow}_{P_2},\{B\},Lab_{\mathcal{I}}{}^2,\{(B,C)\}) = \{(C,\texttt{undec}),(D,\texttt{undec})\}$. Thus it does not hold that $F_{\mathbf{ID}}(AF{\downarrow}_{P_2},\{B\},Lab_{\mathcal{I}}{}^1,\{(B,C)\}) \sqsubseteq F_{\mathbf{ID}}(AF{\downarrow}_{P_2},\{B\},Lab_{\mathcal{I}}{}^2,\{(B,C)\})$.

On the other hand, cs_{db} is universally defined under semantics with monotonic canonical local functions.

Definition 31. *Given a semantics* $\mathbf{S}$*, its canonical local function* $F_{\mathbf{S}}$ *is monotonic if for every* AF*,* $\mathcal{I}$*,* $att_{\mathcal{I}}$ *and for every pair of labellings* $Lab_{\mathcal{I}}{}^1$ *and* $Lab_{\mathcal{I}}{}^2$

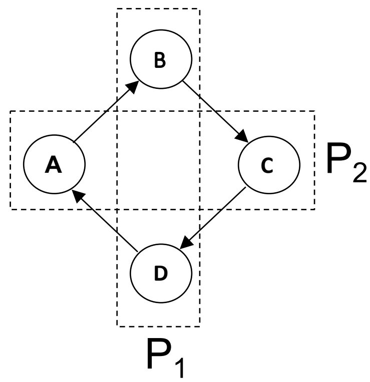

Fig. 4. cs_{db} is not adherent to a unique semantics under $\{\mathbf{GR}, \mathbf{PR}\}$.

such that ${Lab_{\mathcal{I}}}^1 \sqsubseteq {Lab_{\mathcal{I}}}^2$*, it holds that for every* $Lab_1 \in F_{\mathbf{S}}(AF, \mathcal{I}, {Lab_{\mathcal{I}}}^1, att_{\mathcal{I}})$ *there is a labelling* $Lab_2 \in F_{\mathbf{S}}(AF, \mathcal{I}, {Lab_{\mathcal{I}}}^2, att_{\mathcal{I}})$ *such that* $Lab_1 \sqsubseteq Lab_2$*.*

Proposition 2. *The combination schema* cs_{db} *is universally defined under any semantics* $\mathbf{S}$ *such that its canonical local function* $F_{\mathbf{S}}$ *is monotonic.*

In particular, since both $F_{\mathbf{GR}}$ and $F_{\mathbf{PR}}$ are monotonic, cs_{db} is universally defined under $\{\mathbf{GR}, \mathbf{PR}\}$.

Taking into account the relationship between cs_{db} and the decomposability schema proposed in [1], it is possible to prove that under some mild conditions cs_{db} is conflict-freeness, admissibility and completeness preserver.

Proposition 3. cs_{db} *is conflict-freeness, admissibility and completeness preserver under any set of semantics* $\mathbf{SSEM} \cup \{\mathbf{ST}\}$ *where all the semantics in* $\mathbf{SSEM}$ *are complete-compatible.*

As to semantics dependence, it is immediate to see that cs_{db} is reasonable. On the other hand, cs_{db} is not in general adherent to a unique semantics, while the following results can be obtained on the basis of [1]:

- cs_{db} is adherent to a unique semantics under $\{\mathbf{ST}\}$
- cs_{db} is top-down adherent to a unique semantics under $\{\mathbf{GR}, \mathbf{PR}\}$
- cs_{db} is adherent to a unique semantics under $\{\mathbf{GR}, \mathbf{PR}\}$ w.r.t. $\mathcal{F}_{\cup \mathsf{SCC}}$ (and thus also w.r.t. $\mathcal{F}_{\mathsf{SCC}}(AF)$), but not under $\{\mathbf{ID}\}$.

As a simple example showing that cs_{db} is not adherent to a unique semantics under $\{\mathbf{GR}, \mathbf{PR}\}$, consider the partitioned argumentation framework depicted in Figure 4. In a corresponding mixing m_1 such that $S_{m_1}(P_1) = S_{m_1}(P_2) = \mathbf{PR}$ as well as in a mixing m_2 such that $S_{m_2}(P_1) = S_{m_2}(P_2) = \mathbf{GR}$, cs_{db} returns $\{(A, \mathtt{in}), (B, \mathtt{out}), (C, \mathtt{in}), (D, \mathtt{out})\}$, $\{(A, \mathtt{out}), (B, \mathtt{in}), (C, \mathtt{out}), (D, \mathtt{in})\}$, and $\{(A, \mathtt{undec}), (B, \mathtt{undec}), (C, \mathtt{undec}), (D, \mathtt{undec})\}$ as output labellings. On

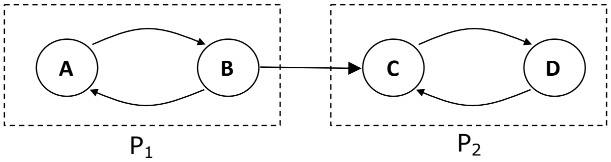

Fig. 5. Skepticism-related properties are poorly satisfied.

the other hand, $\mathbf{L_{GR}}(AF) = \{\{(A, \texttt{undec}), (B, \texttt{undec}), (C, \texttt{undec}), (D, \texttt{undec})\}\}$, and $\mathbf{L_{PR}}(AF)$ includes the labellings $\{(A, \texttt{in}), (B, \texttt{out}), (C, \texttt{in}), (D, \texttt{out})\}$ and $\{(A, \texttt{out}), (B, \texttt{in}), (C, \texttt{out}), (D, \texttt{in})\}$.

Skepticism-related properties turn out to be poorly satisfied, in particular according to the skeptical perspective. As to monotonicity, consider the partitioned argumentation framework of Figure 5, and two corresponding mixings m_1 and m_2 such that $S_{m_1}(P_2) = S_{m_2}(P_2) = \mathbf{PR}$, $S_{m_1}(P_1) = \mathbf{GR}$ and $S_{m_2}(P_1) = \mathbf{PR}$. Clearly, $S_{m_1}(P_i) \preceq^S_{SK} S_{m_2}(P_i)$ for $i \in \{1, 2\}$. However, it is not the case that $cs_{db}(m_1) \preceq^S_{SK} cs_{db}(m_2)$, thus cs_{db} is not monotonic w.r.t. $\preceq^S_{SK}$. In fact, it turns out that $cs_{db}(m_1) = \{\{(A, \texttt{undec}), (B, \texttt{undec}), (C, \texttt{out}), (D, \texttt{in})\}\}$, while $cs_{db}(m_2) = \{\{(A, \texttt{in}), (B, \texttt{out}), (C, \texttt{in}), (D, \texttt{out})\}, \{(A, \texttt{in}), (B, \texttt{out}), (C, \texttt{out}), (D, \texttt{in})\}, \{(A, \texttt{out}), (B, \texttt{in}), (C, \texttt{out}), (D, \texttt{in})\}\}$. cs_{db} is not upper-bounded, since $\forall \mathbf{S} \in \mathbf{SEM}_{m_1}$ $\mathbf{S} \preceq^S_{SK} \mathbf{PR}$, and it is not the case that $cs_{db}(m_1) \preceq^S_{SK} \mathbf{L_{PR}}(AF)$. The example of Figure 4 shows that cs_{db} is not lower-bounded either, since for m_1 it does not hold that $\mathbf{L_{PR}}(AF) \preceq^S_{SK} cs_{db}(m_1)$.

As to the credulous perspective, the following result has been obtained.

Proposition 4. *cs_{db} is lower-bounded w.r.t. $\preceq^S_{CR}$ under $\{\mathbf{GR}, \mathbf{PR}\}$.*

5 Discussion and perspectives for further research

We believe the considerations and results presented in this paper open the way to several interesting investigations, both at a conceptual and a technical level.

From a technical level, some other semantics can be considered, including e.g. semi-stable and CF2 semantics.

At a more general level, one might wonder whether the set of identified principles should be enlarged or, conversely, whether some principles should be weakened or given up. On the one hand, a significant set of properties has been identified for argumentation semantics [3, 11] and one may consider the relationship with combination schema. On the other hand, some principles seem rather difficult to achieve, in particular those related to skepticism. For instance, the example of Figure 5 suggests that the reason why cs_{db} is not monotonic w.r.t. $\preceq^S_{SK}$ concerns the semantics rather than cs_{db}, in particular the beaviour of $\mathbf{PR}$ that prescribes for $AF{\downarrow_{P_2}}$ with an undecided input a more committed outcome than the one that would be obtained with a more decided input. To put it in other terms, while $\mathbf{PR}$ leaves all arguments undecided in the argumentation framework of Figure 5, it yields argument D as justified if the two-lenght cycle in P_1 is replaced with a self attacking argument which attacks C. According to these considerations, skepticism-related properties can be considered not appropriate, or at least they should be refined to take into account the behavior of the semantics involved.

A relevant question concerns the analysis of the relationship holding between principles. For instance, under a set of semantics prescribing labellings for which $\sqsubseteq$ is coreflexive, if a combination schema satisfies both lower-boundedness and upper-boundedness then it is adherent to a unique semantics. This again confirms that skepticism-related properties in the current form are difficult to satisfy.

More generally, an interesting question is whether the proposed principles can actually be satisfied altogether. This in turn relates to the identification of the space of combination schemas that are worth considering. For instance, a combination schema which takes into account a greater set of input arguments w.r.t. cs_{db} may be able to be adherent to a unique semantics.

Another interesting issue to consider is the case when the same argument is interpreted according to different semantics with respect to different attackers, which may require an extension of the notion of mixing.

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