Methods of nulling numbers in the system of residual classes

Victor Krasnobayev $^{1[0000-0001-5192-9918]},$ Alina Yanko $^{2[0000-0003-2876-9316]},$ Oleksii Smirnov $^{3[0000-0001-9543-874X]}$ and Tetiana Kuznetsova $^{1[0000-0001-6154-7139]}$

¹ V. N. Karazin Kharkiv National University, Svobody sq., 4, Kharkiv, 61022, Ukraine v.a.krasnobaev@gmail.com, kuznetsova.tatiana17@gmail.com ² Poltava National Technical Yuri Kondratyuk University, Poltava, Ukraine

al9_yanko@ukr.net

³ Central Ukrainian National Technical University, avenue University, 8, Kropivnitskiy, 25006, Ukraine, dr.smirnovoa@gmail.com

Abstract. The article presents the nullification of numbers in the system of residual classes (SRC). This method is widely used in the non-positional number system in the SRC with the need to determine positional characteristics. Two methods of nullification are presented in the article: the method of successive subtractions and the method of parallel subtractions. Based on these methods, algorithms are developed for their implementation. The essence of the method of successive subtractions is that the nullification procedure is carried out consequently from the junior foundation to the oldest. The essence of the parallel subtraction method is that the nullification procedure is carried out parallel in time for two reasons. It is advisable to use these methods in the implementation of operations of comparing numbers in the SRC, and in monitoring data presented in the SRC. The estimation of these methods by the number of equipment and by the time of implementation of the nullification procedure is made. In terms of the efficiency of the nullification procedure, a parallel subtraction method was proposed.

Keywords: methods of nulling numbers; method of successive subtractions; nullification constants; nullification block; nullification procedure; parallel nullification method; sequential nullification; system of residual classes.

1 Methods of data control in the system of residual classes based on the principle of nulling

One of the methods for determining the correctness of a number is the nulling (nullification) method, which consists in the transition from the initial number.

$$A = (a_1, a_2, ..., a_n, a_{n+1})$$
 (1)

to the number:

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$$A^{Z} = (0, 0, ..., 0, \gamma_{n+1})$$
 (2)

with the help of such a sequence of transformations in which there is no output outside the working range of the system of residual classes (SRC) [1-8].

The process of nulling a number consists in successively subtracting from this number the nulling constants of the form:

$$(m_{1.1}, m_{1.2}, ..., m_{n+1})$$
, where $m_{1.1} = (1, 2, ..., m_1 - 1)$;

$$(0, m_{2,2}, ..., m_{n+1,2})$$
, where $m_{2,2} = (1, 2, ..., m_2 - 1)$.

In this case, the number $A = (a_1, a_2, ..., a_n, a_{n+1})$ is sequentially converted to a form $A' = (0, a'_2, ..., a'_n, a'_{n+1})$, then to a number $A'' = (0, 0, ..., a''_n, a''_{n+1})$ etc. By repeating the process n times, we get:

$$A^{(Z)} = (0, 0, ..., 0, \gamma_{n+1}).$$

If $\gamma_{n+1} = 0$, then the original number is correct and lies in the range [0, M), if $\gamma_{n+1} \neq 0$, then the number is incorrect and lies in the range [jM, (j+1)M), for $j = (1, 2, ..., m_{n+1} - 1)$, where (j+1) is the value of the number of the interval in which the operand \tilde{A} falls.

We show that an error ΔA can transfer the correct number A, lying in the interval [0, M), only in one of the two intervals.

We write $\tilde{A} = A + \Delta A$, i.e.

$$\tilde{A} = (a_1, a_2, ..., a_n, a_{n+1}) + (0, 0, ..., \Delta a_1, ..., 1)$$
 (3)

Obviously, ΔA is not in the first (working) interval [0, M), since the first interval contains the correct number:

$$A_0 = (0, 0, ..., 0, 0)$$
.

Let ΔA be in the k-th interval:

$$(k-1)M \le \Delta A \le kM$$
.

We write the system of two inequalities:

$$\begin{cases} 0 \le A < M, \\ (k-1)M \le \Delta A < kM. \end{cases}$$

Add up the two inequalities:

$$(k-1)M \le A + \Delta A \le (k+1)M \ .$$

Let j = k - 1, then we can write:

$$jM \leq \tilde{A} < (j+2)M$$
,

i.e. an error may cause the correct operand to be incorrect, lying only in one of two intervals [jM, (j+1)M) or [(j+1)M, (j+2)M).

2 Method of successive subtractions

Consider the nulling procedure in terms of the time of its implementation. The essence of the first (ZI), the basic in the theory of SRC, nullification procedure (the procedure of sequential nullification (SN)) consists of a sequence of operations for subtracting in form:

$$A^{(i+1)} = A^{(i)} - NC^{(i)}, (4)$$

by means of a set of NC of the form (5):

$$NC^{(0)} = [t_{1}^{(0)} \parallel t_{2}^{(0)} \parallel t_{3}^{(0)} \parallel \dots \parallel t_{i-1}^{(0)} \parallel t_{i}^{(0)} \parallel t_{i+1}^{(0)} \parallel \dots \parallel t_{n-3}^{(0)} \parallel t_{n-1}^{(0)} \parallel t_{n-1}^{(0)} \parallel t_{n-1}^{(0)} \parallel t_{n+1}^{(0)} \parallel$$

from the corresponding numbers:

$$\begin{split} A^{(0)} &= [a_1^{(0)} \parallel a_2^{(0)} \parallel a_3^{(0)} \parallel \dots \parallel a_{i-1}^{(0)} \parallel a_i^{(0)} \parallel a_{i+1}^{(0)} \parallel \dots \parallel a_{n-3}^{(0)} \parallel a_{n-2}^{(0)} \parallel a_{n-1}^{(0)} \parallel a_n^{(0)} \parallel a_{n+1}^{(0)} \parallel a_{n+1}^{($$

For example: performing the first subtraction operation:

$$\begin{split} A^{(1)} &= A^{(0)} - NC^{(0)} = [a_1^{(0)} \parallel a_2^{(0)} \parallel a_3^{(0)} \parallel \dots \parallel a_{i-1}^{(0)} \parallel a_i^{(0)} \parallel a_{i+1}^{(0)} \parallel \dots \\ \dots \parallel a_{n-3}^{(0)} \parallel a_{n-2}^{(0)} \parallel a_{n-1}^{(0)} \parallel a_n^{(0)} \parallel a_{n+1}^{(0)}] - [t_1^{(0)} \parallel t_2^{(0)} \parallel t_3^{(0)} \parallel \dots \parallel t_{i-1}^{(0)} \parallel t_i^{(0)} \parallel t_{i+1}^{(0)} \parallel \dots \\ \end{split}$$

$$\begin{split} \dots &\| t_{n-3}^{(0)} \| t_{n-2}^{(0)} \| t_{n-1}^{(0)} \| t_{n}^{(0)} \| t_{n+1}^{(0)}] = \left\{ [a_{1}^{(0)} - t_{1}^{(0)}] \operatorname{mod} m_{1} \| [a_{2}^{(0)} - t_{2}^{(0)}] \operatorname{mod} m_{2} \| \\ &\| [a_{3}^{(0)} - t_{3}^{(0)}] \operatorname{mod} m_{3} \| \dots \| [a_{i-1}^{(0)} - t_{i-1}^{(0)}] \operatorname{mod} m_{i-1} \| [a_{i}^{(0)} - t_{i}^{(0)}] \operatorname{mod} m_{i} \| \\ &\| [a_{i+1}^{(0)} - t_{i+1}^{(0)}] \operatorname{mod} m_{i+1} \| \dots \| [a_{n-3}^{(0)} - t_{n-3}^{(0)}] \operatorname{mod} m_{n-3} \| [a_{n-2}^{(0)} - t_{n-2}^{(0)}] \operatorname{mod} m_{n-2} \| \\ &\| [a_{n-1}^{(0)} - t_{n-1}^{(0)}] \operatorname{mod} m_{n-1} \| [a_{n}^{(0)} - t_{n}^{(0)}] \operatorname{mod} m_{n} \| [a_{n+1}^{(0)} - t_{n+1}^{(0)}] \operatorname{mod} m_{n+1} \right\} = \\ &= [0 \| a_{2}^{(1)} \| a_{3}^{(1)} \| \dots \| a_{i-1}^{(1)} \| a_{i}^{(1)} \| a_{i+1}^{(1)} \| \dots \| a_{n-3}^{(1)} \| a_{n-2}^{(1)} \| a_{n-1}^{(1)} \| a_{n}^{(1)} \| a_{n+1}^{(1)}]; \end{split}$$

performing the second subtraction:

$$A^{(2)} = A^{(1)} - NC^{(1)} =$$

$$= [0 \parallel a_2^{(1)} \parallel a_3^{(1)} \parallel ... \parallel a_{i-1}^{(1)} \parallel a_i^{(1)} \parallel a_{i+1}^{(1)} \parallel ... \parallel a_{n-3}^{(1)} \parallel a_{n-2}^{(1)} \parallel a_n^{(1)} \parallel a_n^{(1)} \parallel a_{n+1}^{(1)} \rfloor -$$

$$-[0 \parallel t_2^{(1)} \parallel t_3^{(1)} \parallel ... \parallel t_{i-1}^{(1)} \parallel t_i^{(1)} \parallel t_{i+1}^{(1)} \parallel ... \parallel t_{n-3}^{(1)} \parallel t_{n-2}^{(1)} \parallel t_n^{(1)} \parallel t_n^{(1)} \parallel t_{n+1}^{(1)} \rfloor =$$

$$= \left\{ 0 \parallel [a_2^{(1)} - t_2^{(1)}] \mod m_2 \parallel [a_3^{(1)} - t_3^{(1)}] \mod m_3 \parallel ... \parallel [a_{i-1}^{(1)} - t_{i-1}^{(1)}] \mod m_{i-1} \parallel \\ \parallel [a_i^{(1)} - t_i^{(1)}] \mod m_i \parallel [a_{i+1}^{(1)} - t_{i+1}^{(1)}] \mod m_{i+1} \parallel ... \parallel [a_{n-3}^{(1)} - t_n^{(1)}] \mod m_{n-3} \parallel \\ \parallel [a_{n-2}^{(1)} - t_{n-2}^{(1)}] \mod m_{n-2} \parallel [a_{n-1}^{(1)} - t_{n-1}^{(1)}] \mod m_{n-1} \parallel [a_n^{(1)} - t_n^{(1)}] \mod m_n \parallel \\ \parallel [a_{n+1}^{(1)} - t_{n+1}^{(1)}] \mod m_{n+1} \right\} =$$

 $= [0 \, \| \, 0 \, \| \, a_3^{(2)} \, \| \, \dots \| \, a_{i-1}^{(2)} \, \| \, a_i^{(2)} \, \| \, a_{i+1}^{(2)} \, \| \, \dots \| \, a_{n-3}^{(2)} \, \| \, a_{n-2}^{(2)} \, \| \, a_{n-1}^{(2)} \, \| \, a_n^{(2)} \, \| \, a_{n+1}^{(2)} \,] \, ;$ performing the third subtraction:

$$A^{(3)} = A^{(2)} - NC^{(2)} =$$

$$= [0 || 0 || a_{3}^{(2)} || \dots || a_{i-1}^{(2)} || a_{i}^{(2)} || a_{i+1}^{(2)} || \dots || a_{n-3}^{(2)} || a_{n-2}^{(2)} || a_{n-1}^{(2)} || a_{n+1}^{(2)} || -$$

$$-[0 || 0 || t_{3}^{(2)} || \dots || t_{i-1}^{(2)} || t_{i}^{(2)} || t_{i+1}^{(2)} || \dots || t_{n-3}^{(2)} || t_{n-2}^{(2)} || t_{n-1}^{(2)} || t_{n+1}^{(2)} || -$$

$$= \{0 || 0 || [a_{3}^{(2)} - t_{3}^{(2)}] \mod m_{3} || \dots || [a_{i-1}^{(2)} - t_{i-1}^{(2)}] \mod m_{i-1} || [a_{i}^{(2)} - t_{i}^{(2)}] \mod m_{i} ||$$

$$|| [a_{i+1}^{(2)} - t_{i+1}^{(2)}] \mod m_{i+1} || \dots || [a_{n-3}^{(2)} - t_{n-3}^{(2)}] \mod m_{n-3} || [a_{n-2}^{(2)} - t_{n-2}^{(2)}] \mod m_{n-2} ||$$

$$|| [a_{n-1}^{(2)} - t_{n-1}^{(2)}] \mod m_{n-1} || [a_{n}^{(2)} - t_{n}^{(2)}] \mod m_{n} || [a_{n+1}^{(2)} - t_{n+1}^{(2)}] \mod m_{n+1} \} =$$

$$= [0 || 0 || 0 || a_{4}^{(3)} || a_{5}^{(3)} || \dots || a_{i-1}^{(3)} || a_{i}^{(3)} || a_{i+1}^{(3)} || \dots || a_{n-3}^{(3)} || a_{n-2}^{(3)} || a_{n}^{(3)} || a_{n+1}^{(3)} || 1,$$

etc. The algorithm for performing the SN procedure is presented in Table. 1. In accordance with this algorithm, the initial number $A = A^{(0)} = (a_1^{(0)} \parallel a_2^{(0)} \parallel ... \parallel a_i^{(0)} \parallel a_{n+1}^{(0)})$ according to the formula (4) is sequentially converted to the form $A^{(Z)} = (0 \parallel 0 \parallel ... \parallel 0 \parallel \gamma_{n+1})$ with the help of a sequence of operations that does not result in the output of the numerical value of the number $A^{(0)}$ over the working range [0,M) of the SRC. In this case, the initial number $A = A^{(0)} = (a_1^{(0)} \parallel a_2^{(0)} \parallel ... \parallel a_i^{(0)} \parallel a_{i+1}^{(0)} \parallel ... \parallel a_n^{(0)} \parallel a_{n+1}^{(0)})$ is sequentially reduced to the form $A^{(H)}$, i.e.

$$A = A^{(0)} = (a_1^{(0)}, a_2^{(0)}, ..., a_i^{(0)}, a_{i+1}^{(0)}, ..., a_n^{(0)}, a_{n+1}^{(0)}),$$

$$A^{(1)} = (0, a_2^{(1)}, a_3^{(1)}, ..., a_n^{(1)}, a_{n+1}^{(1)}),$$

$$A^{(2)} = (0, 0, a_3^{(2)}, ..., a_n^{(2)}, a_{n+1}^{(2)}),$$

$$A^{(3)} = (0, 0, 0, a_4^{(3)}, ..., a_n^{(3)}, a_{n+1}^{(3)}) \text{ and so on.}$$

Table 1. SN Algorithm

Operation number	Contents of operation
1	Appeal by value $a_1^{(0)}$ and number $A^{(0)}$ in BNC_0 for the $NC^{(0)}$.
2	Performing a subtraction operation $A^{(1)} = A^{(0)} - NC^{(0)}$.
3	Appeal by value $a_2^{(1)}$ and number $A^{(1)}$ in BNC_1 for the $NC^{(1)}$.
4	Performing a subtraction operation $A^{(2)} = A^{(1)} - NC^{(1)}$.
5	Appeal by value $a_3^{(2)}$ and number $A^{(2)}$ in BNC_2 for the $NC^{(2)}$.
6	Performing a subtraction operation $A^{(3)} = A^{(2)} - NC^{(2)}$.
7	Appeal by value $a_4^{(3)}$ and number $A^{(3)}$ in BNC_3 for the $NC^{(3)}$.
8	Performing a subtraction operation $A^{(4)} = A^{(3)} - NC^{(3)}$.
•	
2n-3	Appeal by value $a_{n-1}^{(n-2)}$ and number $A^{(n-2)}$ in BNC_{n-2} for the $NC^{(n-2)}$.
2n-2	Performing a subtraction operation $A^{(n-1)} = A^{(n-2)} - NC^{(n-2)}$.
2n-1	Appeal by value $a_n^{(n-1)}$ and number $A^{(n-1)}$ in BNC_{n-1} for the $NC^{(n-1)}$.
2 <i>n</i>	Performing a subtraction operation $A^{(n)} = A^{(n-1)} - NC^{(n-1)}$. Getting a nullified number $A^{(Z)} = A^{(n)} = [0 \ 0 \ \dots \ 0 \ 0 \ \gamma_{n+1} = a_{n+1}^{(n)}] .$

By repeating the subtraction n times, we get the value $A^{(Z)} = (0 \| 0 \| ... \| 0 \| a_{n+1}^{(n)})$, or $A^{(Z)} = (0 \| 0 \| ... \| 0 \| \gamma_{n+1})$, where $\gamma_{n+1} = a_{n+1}^{(n)}$. The SN procedure is shown in Fig. 1.

$(\circ \circ \dots \circ \gamma_{n+1}), \dots (\circ \circ \gamma_{n+1}), \dots (\circ \circ \circ \circ \circ \circ \circ \circ$		
Operation (cycle) number	Contents of operation	
1	Appeal by the value of the remainder $a_1^{(0)}$ of a number $A = A^{(0)} = [a_1^{(0)} \parallel a_2^{(0)} \parallel$	

2	Performing a subtraction operation $A^{(1)} = A^{(0)} - NC^{(0)} = [a_1^{(0)} a_2^{(0)} $
	$\ a_{3}^{(0)} \ \dots \ a_{i-1}^{(0)} \ a_{i}^{(0)} \ a_{i+1}^{(0)} \ \dots \ a_{n-3}^{(0)} \ a_{n-2}^{(0)} \ a_{n-1}^{(0)} \ a_{n}^{(0)} \ a_{n+1}^{(0)}] -$
	$= \{ [a_1^{(0)} - t_1^{(0)}] \bmod m_1 \parallel [a_2^{(0)} - t_2^{(0)}] \bmod m_2 \parallel [a_3^{(0)} - t_3^{(0)}] \bmod m_3 \parallel \dots$

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...\|[a_{i-1}^{(0)}-t_{i-1}^{(0)}]\mod m_{i-1}\|[a_i^{(0)}-t_i^{(0)}]\mod m_i\|[a_{i+1}^{(0)}-t_{i+1}^{(0)}]\mod m_{i+1}\|...
            ... \| [a_{n-3}^{(0)} - t_{n-3}^{(0)}] \bmod m_{n-3} \| [a_{n-2}^{(0)} - t_{n-2}^{(0)}] \bmod m_{n-2} \| [a_{n-1}^{(0)} - t_{n-1}^{(0)}] \bmod m_{n-1} \|
            \|[a_n^{(0)} - t_n^{(0)}] \bmod m_n \|[a_{n+1}^{(0)} - t_{n+1}^{(0)}] \bmod m_{n+1} \Big\} = [0 \| a_2^{(1)} \| a_3^{(1)} \| \dots \| a_{i-1}^{(1)} \| a_i^{(1)} \|
            \parallel a_{i+1}^{(1)}\parallel ... \parallel a_{n-3}^{(1)}\parallel a_{n-2}^{(1)}\parallel a_{n-1}^{(1)}\parallel a_{n}^{(1)}\parallel a_{n+1}^{(1)} \rfloor\,.
            Appeal by the value of the remainder a_2^{(1)}
             A^{(1)} = [0 || a_2^{(1)} || a_3^{(1)} || \dots
            ... \|a_{i-1}^{(1)}\|a_i^{(1)}\|a_{i+1}^{(1)}\|...\|a_{n-3}^{(1)}\|a_{n-2}^{(1)}\|a_{n-1}^{(1)}\|a_n^{(1)}\|a_{n+1}^{(1)}\| in BNC_1 for the
           NC NC^{(1)} = [0 \parallel t_2^{(1)} \parallel t_3^{(1)} \parallel \dots \parallel t_{i-1}^{(1)} \parallel t_i^{(1)} \parallel t_{i+1}^{(1)} \parallel \dots \parallel t_{n-3}^{(1)} \parallel t_{n-2}^{(1)} \parallel
            ||t_{n-1}^{(1)}||t_n^{(1)}||t_{n+1}^{(1)}|; t_2^{(1)}=a_2^{(1)}; t_2^{(1)}=\overline{0, m_2-1}.
            Performing a subtraction operation
                A^{(2)} = A^{(1)} - NC^{(1)} = [0 | a_2^{(1)} | a_3^{(1)} | \dots
             ... \| a_{i-1}^{(1)} \| a_i^{(1)} \| a_{i+1}^{(1)} \| ... \| a_{n-3}^{(1)} \| a_{n-2}^{(1)} \| a_{n-1}^{(1)} \| a_n^{(1)} \| a_{n+1}^{(1)} \Big] - \left[ 0 \| t_2^{(1)} \| t_3^{(1)} \| ... \| t_{i-1}^{(1)} \| a_{n-1}^{(1)} \|
            t_{i}^{(1)} || t_{i+1}^{(1)} || \dots || t_{n-3}^{(1)} || t_{n-2}^{(1)} || t_{n-1}^{(1)} || t_{n}^{(1)} || t_{n+1}^{(1)} || = \{ 0 || [a_{2}^{(1)} - t_{2}^{(1)}] \mod m_{2} || \}
            ||[a_3^{(1)} - t_3^{(1)}] \mod m_3|| ... ||[a_{i-1}^{(1)} - t_{i-1}^{(1)}] \mod m_{i-1}||[a_i^{(1)} - t_i^{(1)}] \mod m_i||
           ||[a_{i+1}^{(1)} - t_{i+1}^{(1)}] \mod m_{i+1}|| ... ||[a_{n-3}^{(1)} - t_{n-3}^{(1)}] \mod m_{n-3}||[a_{n-2}^{(1)} - t_{n-2}^{(1)}] \mod m_{n-2}||
            ||[a_{n-1}^{(1)}-t_{n-1}^{(1)}] \mod m_{n-1}||[a_n^{(1)}-t_n^{(1)}] \mod m_n||[a_{n+1}^{(1)}-t_{n+1}^{(1)}] \mod m_{n+1}|
             = [0 \| 0 \| a_3^{(2)} \| \dots \| a_{i-1}^{(2)} \| a_i^{(2)} \| a_{i+1}^{(2)} \| \dots \| a_{n-3}^{(2)} \| a_{n-2}^{(2)} \| a_{n-1}^{(2)} \| a_n^{(2)} \| a_{n+1}^{(2)} \|...
 Appeal by the value of the remainder
                                                                                                                                                                                                                                                                                 of a number
  A^{(2)} = [0 || 0 || a_3^{(2)} || \dots
 ... \|a_{i-1}^{(2)}\|a_i^{(2)}\|a_{i+1}^{(2)}\|...\|a_{n-3}^{(2)}\|a_{n-2}^{(2)}\|a_{n-1}^{(2)}\|a_n^{(2)}\|a_{n+1}^{(2)}\| in BNC_2 for the NC
 NC^{(2)} = [0 \, \| \, 0 \, \| \, t_3^{(2)} \, \| \, \dots \, \| \, t_{i-1}^{(2)} \, \| \, t_i^{(2)} \, \| \, t_{i+1}^{(2)} \, \| \dots \, \| \, t_{n-3}^{(2)} \, \| \, t_{n-1}^{(2)} \, \| \, t_{n-1}^{(
  ||t_n^{(2)}||t_{n+1}^{(2)}|; t_3^{(2)} = a_3^{(2)}; t_3^{(2)} = \overline{0, m_3 - 1}
Performing a subtraction operation A^{(3)} = A^{(2)} - NC^{(2)} = [0 || 0 || a_2^{(2)} || \dots
 ... \| a_{i-1}^{(2)} \| a_i^{(2)} \| a_{i+1}^{(2)} \| ... \| a_{n-3}^{(2)} \| a_{n-2}^{(2)} \| a_{n-1}^{(2)} \| a_n^{(2)} \| a_{n+1}^{(2)} \| -[0 \| 0 \| t_3^{(2)} \| ... \| t_{i-1}^{(2)} \|
||t_{i}^{(2)}||t_{i+1}^{(2)}||\dots||t_{n-3}^{(2)}||t_{n-2}^{(2)}||t_{n-1}^{(2)}||t_{n}^{(2)}||t_{n+1}^{(2)}||=\{0\,||\,0\,||[a_{3}^{(2)}-t_{3}^{(2)}]\bmod m_{3}\,||\dots
...\|[a_{i-1}^{(2)}-t_{i-1}^{(2)}]\bmod m_{i-1}\|[a_i^{(2)}-t_i^{(2)}]\bmod m_i\|[a_{i+1}^{(2)}-t_{i+1}^{(2)}]\bmod m_{i+1}\|...
...\|[a_{n-3}^{(2)}-t_{n-3}^{(2)}]\bmod m_{n-3}\|[a_{n-2}^{(2)}-t_{n-2}^{(2)}]\bmod m_{n-2}\|[a_{n-1}^{(2)}-t_{n-1}^{(2)}]\bmod m_{n-1}\|
  ||[a_n^{(2)} - t_n^{(2)}] \mod m_n ||[a_{n+1}^{(2)} - t_{n+1}^{(2)}] \mod m_{n+1}||
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= [0 \parallel 0 \parallel 0 \parallel a_4^{(3)} \parallel a_5^{(3)} \parallel \dots \parallel a_{i-1}^{(3)} \parallel a_i^{(3)} \parallel a_{i+1}^{(3)} \parallel \dots \parallel a_{n-3}^{(3)} \parallel a_{n-2}^{(3)} \parallel a_{n-1}^{(3)} \parallel a_n^{(3)} \parallel a_{n+1}^{(3)} \parallel a_{n+
                                    Appeal by the value
                                                                                                                                                       of the remainder
                                     A^{(3)} = [0 || 0 || 0 || a_4^{(3)} ||
                                    ||a_5^{(3)}||...||a_{i-1}^{(3)}||a_i^{(3)}||a_{i+1}^{(3)}||...||a_{n-3}^{(3)}||a_{n-2}^{(3)}||a_{n-1}^{(3)}||a_n^{(3)}||a_{n+1}^{(3)}| in BNC_3 for
                                     the NC NC^{(3)} = [0 || 0 || 0 || t_4^{(3)} || t_5^{(3)} || \dots || t_{i-1}^{(3)} || t_i^{(3)} || t_{i+1}^{(3)} || \dots
                                    \parallel t_{n-3}^{(3)}\parallel t_{n-2}^{(3)}\parallel t_{n-1}^{(3)}\parallel t_{n}^{(3)}\parallel t_{n}^{(3)}\parallel t_{n+1}^{(3)}]\;;\;t_{4}^{(3)}=a_{4}^{(3)}\;;\;t_{4}^{(3)}=\overline{0,m_{4}-1}\;.
                                     Performing a subtraction operation A^{(4)} = A^{(3)} - NC^{(3)} = [0 || 0 || 0 || a_4^{(3)} ||
                                    \|a_{5}^{(3)}\|...\|a_{i-1}^{(3)}\|a_{i}^{(3)}\|a_{i+1}^{(3)}\|...\|a_{n-3}^{(3)}\|a_{n-2}^{(3)}\|a_{n-1}^{(3)}\|a_{n}^{(3)}\|a_{n+1}^{(3)}] - [0\|0\|0\|t_{4}^{(3)}\|
                                    ||t_{5}^{(3)}|| \dots ||t_{i-1}^{(3)}||t_{i}^{(3)}||t_{i+1}^{(3)}|| \dots ||t_{n-3}^{(3)}||t_{n-2}^{(3)}||t_{n-1}^{(3)}||t_{n}^{(3)}||t_{n+1}^{(3)}|] =
                                     = \{0 \mid \mid 0 \mid \mid \mid 0 \mid \mid [a_4^{(3)} - t_4^{(3)}] \mod m_4 \mid \mid [a_5^{(3)} - t_5^{(3)}] \mod m_5 \mid \mid \dots \mid \mid [a_{i-1}^{(3)} - t_{i-1}^{(3)}] \mod m_{i-1}
                                     ||[a_i^{(3)} - t_i^{(3)}] \mod m_i ||[a_{i+1}^{(3)} - t_{i+1}^{(3)}] \mod m_{i+1} || ... ||[a_{n-3}^{(3)} - t_{n-3}^{(3)}] \mod m_{n-3} || | | | | | | | | | | | | |
                                    ||[a_{n-2}^{(3)} - t_{n-2}^{(3)}] \mod m_{n-2} ||[a_{n-1}^{(3)} - t_{n-1}^{(3)}] \mod m_{n-1} ||[a_n^{(3)} - t_n^{(3)}] \mod m_n ||
                                     ||[a_{n+1}^{(3)} - t_{n+1}^{(3)}] \mod m_{n+1}| = |[0||0||0||0||a_5^{(4)}||...||a_{i-1}^{(4)}||a_i^{(4)}||a_{i+1}^{(4)}||...|
                                     ... || a_{n-3}^{(4)} || a_{n-2}^{(4)} || a_{n-1}^{(4)} || a_n^{(4)} || a_{n+1}^{(4)} ||...
                                     Appeal by the value of the remainder a_i^{(i-1)}
                                     A^{(i-1)} = [0 || 0 || 0 || \dots || 0 ||
                                    \parallel a_i^{(i-1)} \parallel a_{i+1}^{(i-1)} \parallel \dots \parallel a_{n-3}^{(i-1)} \parallel a_{n-2}^{(i-1)} \parallel a_{n-1}^{(i-1)} \parallel a_n^{(i-1)} \parallel a_{n+1}^{(i-1)} \rceil  in BNC_{i-1} for the NC
                                     NC^{(i-1)} = [0 \parallel 0 \parallel 0 \parallel \dots \parallel 0 \parallel t_i^{(i-1)} \parallel t_{i+1}^{(i-1)} \parallel \dots \parallel t_{n-3}^{(i-1)} \parallel t_{n-2}^{(i-1)} \parallel t_{n-1}^{(i-1)} \parallel
                                    ||t_n^{(i-1)}||t_{n+1}^{(i-1)}];\ t_i^{(i-1)}=a_i^{(i-1)};\ t_i^{(i-1)}=\overline{0,m_i-1}\ .
                                    Performing a subtraction operation A^{(i)} = A^{(i-1)} - NC^{(i-1)} = [0 \parallel 0 \parallel 0 \parallel \dots]
                                    ... \parallel 0 \parallel a_{i}^{(i-1)} \parallel a_{i+1}^{(i-1)} \parallel ... \parallel a_{n-3}^{(i-1)} \parallel a_{n-2}^{(i-1)} \parallel a_{n-1}^{(i-1)} \parallel a_{n}^{(i-1)} \parallel a_{n+1}^{(i-1)} \rfloor - [0 \parallel 0 \parallel 0 \parallel ... \parallel 0 \parallel
value
                                      \|t_{i}^{(i-1)} \|t_{i+1}^{(i-1)} \| \dots \|t_{n-3}^{(i-1)} \|t_{n-2}^{(i-1)} \|t_{n-1}^{(i-1)} \|t_{n}^{(i-1)} \|t_{n+1}^{(i-1)} \| = \{0 \|0 \|0 \| \dots \|0 \| 
A^{(i)}
                                     ||[a_i^{(i-1)} - t_i^{(i-1)}] \mod m_i ||[a_{i+1}^{(i-1)} - t_{i+1}^{(i-1)}] \mod m_{i+1} || \dots ||[a_{n-3}^{(i-1)} - t_{n-3}^{(i-1)}] \mod m_{n-3} ||
                                    \|[a_{n-2}^{(i-1)}-t_{n-2}^{(i-1)}]\bmod m_{n-2}\|[a_{n-1}^{(i-1)}-t_{n-1}^{(i-1)}]\bmod m_{n-1}\|[a_n^{(i-1)}-t_n^{(i-1)}]\bmod m_n\|
                                    ||[a_{n+1}^{(i-1)} - t_{n+1}^{(i-1)}] \mod m_{n+1}| = [0 || 0 || 0 || \dots || 0 || 0 || a_{i+1}^{(i)} || \dots || a_{n-3}^{(i)} || a_{n-2}^{(i)} || a_{n-1}^{(i)} ||
                                     ||a_n^{(i)}||a_{n+1}^{(i)}|.
```

```
Appeal by the value of the remainder a_{n-1}^{(n-2)}
                                                                                                       A^{(n-2)} = [0 || 0 || 0 || \dots
                                                                                                     ... \parallel 0 \parallel 0 \parallel 0 \parallel ... \parallel 0 \parallel 0 \parallel a_{n-1}^{(n-2)} \parallel a_{n}^{(n-2)} \parallel a_{n+1}^{(n-2)} ] \quad \text{in} \quad \mathit{BNC}_{n-2} \quad \text{for the NC}
                                                                                                     NC^{(n-2)} = [0 \, || \, 0 \, || \, 0 \, || \, \dots \, || \, 0 \, || \, 0 \, || \, 0 \, || \, \dots \, || \, 0 \, || \, 0 \, || \, t_{n-1}^{(n-2)} \, || \, t_n^{(n-2)} \, ||
                                                                                                  ||t_{n+1}^{(n-2)}|; t_{n-1}^{(n-2)} = a_{n-1}^{(n-2)}; t_{n-1}^{(n-2)} = \overline{0, m_{n-1} - 1}.
                                                                                                  Performing a subtraction operation A^{(n-1)} = A^{(n-2)} - NC^{(n-2)} = [0 \parallel 0 \parallel 0 \mid \dots]
                                                                                                     ... \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, a_{n-1}^{(n-2)} \, \| \, a_{n}^{(n-2)} \, \| \, a_{n+1}^{(n-2)} \, ] \, - [ \, 0 \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, ... \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 
                                                                                                \dots \| \, 0 \, \| \, 0 \, \| \, t_{n-1}^{(n-2)} \, \| \, t_n^{(n-2)} \, \| \, t_{n+1}^{(n-2)} \, ] = \left\{ 0 \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \|
                                                                                                     \|[a_{n-1}^{(n-2)}-t_{n-1}^{(n-2)}] \mod m_{n-1}\|[a_n^{(n-2)}-t_n^{(n-2)}] \mod m_n\|
                                                                                                  ||a_{n+1}^{(n-1)}||.
                                                                                                  Appeal by the value of the remainder a_n^{(n-1)} of a number
                                                                                                A^{(n-1)} = [0 \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \dots \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, a_n^{(n-1)} \, \| \, a_{n+1}^{(n-1)} \, ] \quad \text{in} \quad BNC_{n-1} for the NC NC^{(n-1)} = [0 \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, 0 \, \| \, \| \, t_n^{(n-1)} \, \| \, t_{n+1}^{(n-1)} \, ];
2n-1
                                                                                                     t_n^{(n-1)} = a_n^{(n-1)}; t_n^{(n-1)} = \overline{0, m_n - 1}.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    A^{(Z)} = A^{(n)} = A^{(n-1)} - NC^{(n-1)} =
                                                                                                                                                                                                                                                                                                                                                                                                                          number
                                                                      Getting
                                                                                                                                                                       a nullified
                                                                      = [0 || 0 || 0 || \dots || 0 || 0 || 0 || \dots || 0 || 0 || 0 || 0 || a_n^{(n-1)} || a_{n+1}^{(n-1)}] - [0 || 0 || 0 || \dots || 0 || 0 || 0 || \dots
                                                                      ...\,||\,0\,||\,0\,||\,0\,||\,t_{n}^{(n-1)}\,||\,t_{n+1}^{(n-1)}\,] = \Big\{0\,||\,0\,||\,0\,||\,...\,||\,0\,||\,0\,||\,0\,||\,...\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||\,0\,||
2n
                                                                      ||[a_n^{(n-1)} - t_n^{(n-1)}] \mod m_n ||[a_{n+1}^{(n-1)} - t_{n+1}^{(n-1)}] \mod m_{n+1}| = [0 || 0 || 0 || \dots || 0 || 0 || 0 || \dots || 0
                                                                      ... \parallel 0 \parallel 0 \parallel 0 \parallel 0 \parallel a_{n+1}^{(n)}], где a_{n+1}^{(n)} = \gamma_{n+1}.
                                                                                                                                                                                                                                                                                                                                                                                                   T_{z1} = 2 \cdot n \cdot \tau_{add}
```

Fig. 1. Procedure of SN of numbers in the SRC

Denoting the sampling time of the NC from the corresponding nullification block (NB) of the CS functioning in the SRC as t_1 , and the time of subtracting from the number $A^{(i-1)}$ the constant $NC^{(i-1)}$, i.e. performing the operation $A^{(i)} = A^{(i-1)} - NC^{(i-1)}$ – as t_2 , we get the total time T_{Z1} of the nullification procedure for the first Z1 method:

$$T_{Z1} = n (t_1 + t_2). (6)$$

3 Parallel subtraction method

Consider the following method (*Z*3) of operational control of data in the SRC (parallel nullification method (PNM)). The essence of the proposed control method is that the nullification procedure is carried out parallel in time for two reasons. For n-even numbers, we have $a_i^{(i-1)}$, $a_{n-i+1}^{(i-1)}$ ($i=\overline{1,n/2}$), namely $a_1^{(0)}$, $a_n^{(0)}$; $a_2^{(1)}$, $a_{n-1}^{(1)}$; $a_3^{(2)}$, $a_{n-2}^{(2)}$;... $a_{n/2}^{(n/2)}$, $a_{n/2+1}^{(n/2)}$ (see Fig. 2). For n-odd numbers, we have $a_1^{(0)}$, $a_n^{(0)}$; $a_2^{(1)}$, $a_{n-1}^{(1)}$; $a_3^{(2)}$, $a_{n-2}^{(2)}$; ... $a_{(n+1)/2}^{((n+1)/2-1)}$ (see Fig. 3). In this case, for an arbitrary value i of NC for

the corresponding number, have the following form: $A^{(i)} = [0 \parallel 0 \parallel \dots \parallel 0 \parallel a_{i+1}^{(i)} \parallel a_{i+1}^{(i)} \parallel a_{i+2}^{(i)} \parallel \dots \parallel a_{i+1}^{(i)} \parallel a_{i+1}^{($

Fig. 2. Sampling scheme for nullification constants for PNM method (*n* –even number)

For an arbitrary value i we have that:

$$\begin{split} A^{(i+1)} &= A^{(i)} - NC^{(i)} = \\ &= [0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, a^{(i)}_{i+1} \, \| \, a^{(i)}_{i+2} \, \| \, a^{(i)}_{i+3} \, \| \, \dots \| \, a^{(i)}_{n-i-2} \, \| \, a^{(i)}_{n-i-1} \, \| \, a^{(i)}_{n-i} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, a^{(i)}_{n+1} \,] - \\ &- [0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t^{(i)}_{i+1} \, \| \, t^{(i)}_{i+2} \, \| \, t^{(i)}_{i+3} \, \| \, \dots \| \, t^{(i)}_{n-i-2} \, \| \, t^{(i)}_{n-i-1} \, \| \, t^{(i)}_{n-i} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t^{(i)}_{n+1} \,] = \end{split}$$

$$\begin{split} = & \left\{ 0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| [a_{i+1}^{(i)} - t_{i+1}^{(i)}] \, \text{mod} \, m_{i+1} \, \| [a_{i+2}^{(i)} - t_{i+2}^{(i)}] \, \text{mod} \, m_{i+2} \, \| [a_{i+3}^{(i)} - t_{i+3}^{(i)}] \, \text{mod} \, m_{i+3} \, \| \, \dots \right. \\ & \quad \dots \| [a_{n-i-2}^{(i)} - t_{n-i-2}^{(i)}] \, \text{mod} \, m_{n-i-2} \, \| [a_{n-i-1}^{(i)} - t_{n-i-1}^{(i)}] \, \text{mod} \, m_{n-i-1} \, \| \\ & \quad \quad \| [a_{n-i}^{(i)} - t_{n-i}^{(i)}] \, \text{mod} \, m_{n-i} \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| [a_{n+1}^{(i)} - t_{n+1}^{(i)}] \, \text{mod} \, m_{n+1} \right\} = \\ & = & [0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, a_{i+2}^{(i+1)} \, \| \, a_{i+3}^{(i+1)} \, \| \, \dots \, \| \, a_{n-i-1}^{(i+1)} \, \| \, 0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \| \, 0 \, \| \, a_{n+1}^{(i+1)} \,] \, . \end{split}$$

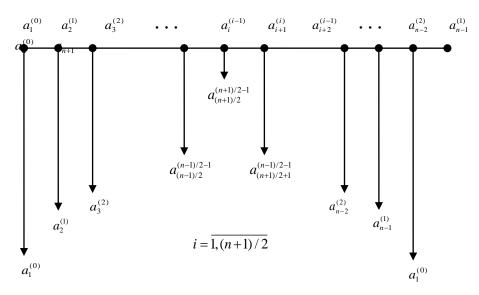


Fig. 3. Sampling scheme for nullification constants for PNM method (n - odd number)

The algorithm for performing the PNM procedure is presented in Table. 2. Before getting the value $\gamma_{n+1} = a_{n+1}^{(n/2)}$ for n - even number, we have that:

Before getting the value $\gamma_{n+1} = a_{n+1}^{(n/2)}$ for n -odd number, we have that

$$A^{((n+1)/2-1)} = \underbrace{[0 \, \| \, 0 \, \| \, \dots \, \| \, 0 \, \|}^{\frac{n+1}{2}-1 \text{ zeroes}} \underbrace{a_{(n+1)/2}^{((n+1)/2-1)}}_{(n+1)/2} \underbrace{\| \, 0 \, \| \, \dots \, \| \, 0 \, \|}_{n+1} a_{n+1}^{((n+1)/2-1)}].$$

Table 2. PNM algorithm

Operation number	Contents of operation
number	
	Appeal by the value of the remainders $a_1^{(0)}$ and $a_n^{(0)}$ of a number $A^{(0)}$ in
1	BNC_0 for the $NC^{(0)}$.
2	Performing a subtraction operation $A^{(1)} = A^{(0)} - NC^{(0)}$.
	Appeal by the value of the remainders $a_2^{(1)}$ and $a_{n-1}^{(1)}$ of a number $A^{(1)}$ in BNC_1
3	for the $NC^{(1)}$.
4	Performing a subtraction operation $A^{(2)} = A^{(1)} - NC^{(1)}$.
	Appeal by the value of the remainders $a_2^{(2)}$ and $a_{n-2}^{(2)}$ of a number $A^{(2)}$ in
5	BNC_2 for the $NC^{(2)}$.
6	Performing a subtraction operation $A^{(3)} = A^{(2)} - NC^{(2)}$.
i	Performing a subtraction operation $A^{(i)} = A^{(i-1)} - NC^{(i-1)}$.
	Appeal by the value of the remainders $a_{i+1}^{(i)}$ and $a_{n-i}^{(i)}$ of a number $A^{(i)}$ in
<i>i</i> +1	BNC_i for the $NC^{(i)}$.
<i>i</i> + 2	Performing a subtraction operation $A^{(i+1)} = A^{(i)} - NC^{(i)}$.
	Appeal by the value of the remainders $a_{n/2-1}^{(n/2-2)}$ and $a_{n/2+2}^{(n/2-2)}$ of a number
n-3	$A^{(n/2-2)}$ in $BCN_{n/2-2}$ for the $NC^{(n/2-2)}$.
n-2	Performing a subtraction operation $A^{(n/2-1)} = A^{(n/2-2)} - NC^{(n/2-2)}$.
	Appeal by the value of the remainders $a_{n/2}^{(n/2-1)}$ and $a_{n/2+1}^{(n/2-1)}$ of a number $A^{(n/2-1)}$
n-1	in $BNC_{n/2-1}$ for the $NC^{(n/2-1)}$.
	Performing a subtraction operation $A^{(n/2)} = A^{(n/2-1)} - NC^{(n/2-1)}$.
10	Getting nullified number $A^{(Z)}$
n	$A^{(Z)} = A^{(n/2)} = [0 0 \dots 0 \dots 0 0 \gamma_{n+1} = a_{n+1}^{(n/2)}].$

$$\begin{split} NC^{((n+1)/2-1)} = & \left[0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t_{(n+1)/2}^{((n+1)/2-1)} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t_{n+1}^{((n+1)/2-1)} \, \right], \\ t_{(n+1)/2}^{((n+1)/2-1)} = & \overline{0, m_{(n+1)/2}} \; ; \; t_{(n+1)/2}^{((n+1)/2-1)} = a_{(n+1)/2}^{((n+1)/2-1)} \; . \\ A^{(Z)} = & A^{(n+1)/2} = A^{((n+1)/2-1)} - NC^{((n+1)/2-1)} = \\ = & \left\{ 0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \left[a_{(n+1)/2}^{((n+1)/2-1)} - t_{(n+1)/2}^{((n+1)/2-1)} \right] \bmod m_{(n+1)/2} \, \| \, 0 \, \| \dots \| \, 0 \, \| \, 0 \, \| \dots \| \, 0 \, \| \, 0 \, \| \\ \| \left[a_{n+1}^{((n+1)/2-1)} - t_{n+1}^{((n+1)/2-1)} \right] \bmod m_{n+1} \right\} = & \left[0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \dots \| \, 0 \, \| \, 0 \, \| \, a_{n+1}^{(n+1)/2} \right], \; \text{where} \\ \gamma_{n+1} = & a_{n+1}^{(n+1)/2} \; . \end{split}$$

PNM method in the SRC presented in Fig. 4.

Opera-	
tion	Contents of operation
number	
(cycle)	
1	Appeal by the value of the remainders $a_1^{(0)}$ and $a_n^{(0)}$ of a number
	$A = A^{(0)} = [a_1^{(0)} \mid $
	$ \ a_2^{(0)} \ a_3^{(0)} \ \dots \ a_{i-1}^{(0)} \ a_i^{(0)} \ a_{i+1}^{(0)} \ \dots \ a_{n-3}^{(0)} \ a_{n-2}^{(0)} \ a_{n-1}^{(0)} \ a_n^{(0)} \ a_{n+1}^{(0)}] $
	BNC_0 for the $NC^{(0)} = [t_1^{(0)} \parallel t_2^{(0)} \parallel t_3^{(0)} \parallel \dots \parallel t_{i-1}^{(0)} \parallel t_i^{(0)} \parallel t_{i+1}^{(0)} \parallel t_{i+1}^{(0)} \parallel \dots$
	$t_n^{(0)} = \overline{0, m_n - 1}$

2	Performing a subtraction operation $A^{(1)} = A^{(0)} - NC^{(0)} = [a_1^{(0)} a_2^{(0)} $
	$\parallel a_{3}^{(0)} \parallel \ldots \parallel a_{i-1}^{(0)} \parallel a_{i}^{(0)} \parallel a_{i+1}^{(0)} \parallel \ldots \parallel a_{n-3}^{(0)} \parallel a_{n-2}^{(0)} \parallel a_{n-1}^{(0)} \parallel a_{n}^{(0)} \parallel a_{n+1}^{(0)} \rfloor - [t_{1}^{(0)} \parallel t_{2}^{(0)} \parallel$
	$= \{ [a_1^{(0)} - t_1^{(0)}] \mod m_1 \parallel [a_2^{(0)} - t_2^{(0)}] \mod m_2 \parallel [a_3^{(0)} - t_3^{(0)}] \mod m_3 \parallel \dots$
	$\ [a_{i-1}^{(0)}-t_{i-1}^{(0)}]\bmod m_{i-1}\ [a_{i}^{(0)}-t_{i}^{(0)}]\bmod m_{i}\ [a_{i+1}^{(0)}-t_{i+1}^{(0)}]\bmod m_{i+1}\ $
	$ \ a_{i-1}^{(1)} \ a_i^{(1)} \ a_{i+1}^{(1)} \ \ a_{n-3}^{(1)} \ a_{n-2}^{(1)} \ a_{n-1}^{(1)} \ 0 \ a_{n+1}^{(1)}].$

 $A ppeal by the value of the remainders $a_2^{(1)}$ and $a_{n-1}^{(1)}$ of a number $A^{(1)} = [0 \, \| \, a_2^{(1)} \, \| \ \ \| \, a_3^{(1)} \, \| \, \ldots \| \, a_{i-1}^{(1)} \, \| \, a_{i+1}^{(1)} \, \| \, \ldots \| \, a_{n-3}^{(1)} \, \| \, a_{n-2}^{(1)} \, \| \, a_{n-1}^{(1)} \, \| \, 0 \, \| \, a_{n+1}^{(1)} \,] \ \ in BNC_1 for the $NC^{(1)} = [0 \, \| \, t_2^{(1)} \, \| \, t_3^{(1)} \, \| \, \ldots \| \, t_{i-1}^{(1)} \, \| \, t_i^{(1)} \, \| \, t_{i+1}^{(1)} \, \| \, \ldots \| \, t_{n-3}^{(1)} \, \| \ \ \| \, t_{n-2}^{(1)} \, \| \, t_{n-1}^{(1)} \, \| \, 0 \, \| \, t_{n+1}^{(1)} \,] \, ; \qquad t_2^{(1)} = a_2^{(1)} \, , \qquad t_{n-1}^{(1)} = a_{n-1}^{(1)} \, ; \qquad t_2^{(1)} = \overline{0, m_2 - 1} \, , \\ t_{n-1}^{(1)} = \overline{0, m_{n-1} - 1} \, .$

```
Performing a subtraction operation A^{(2)} = A^{(1)} - NC^{(1)} = [0 || a_2^{(1)} || a_3^{(1)} || \dots
                     ... \| a_{i-1}^{(1)} \| a_i^{(1)} \| a_{i+1}^{(1)} \| ... \| a_{n-3}^{(1)} \| a_{n-2}^{(1)} \| a_{n-1}^{(1)} \| 0 \| a_{n+1}^{(1)} ] - [0 \| t_2^{(1)} \| t_3^{(1)} \| ... 
                     ... \|t_{i-1}^{(1)} \|t_{i}^{(1)} \|t_{i+1}^{(1)} \|... \|t_{n-3}^{(1)} \|t_{n-2}^{(1)} \|t_{n-1}^{(1)} \|0\|t_{n+1}^{(1)}] = \{0 \|[a_{2}^{(1)} - t_{2}^{(1)}] \mod m_{2} \|
                    ||[a_3^{(1)} - t_3^{(1)}] \mod m_3||[a_4^{(1)} - t_4^{(1)}] \mod m_4||...||[a_{i-1}^{(1)} - t_{i-1}^{(1)}] \mod m_{i-1}||
                    ||[a_i^{(1)} - t_i^{(1)}] \mod m_i ||[a_{i+1}^{(1)} - t_{i+1}^{(1)}] \mod m_{i+1} || \dots ||[a_{n-3}^{(1)} - t_{n-3}^{(1)}] \mod m_{n-3} ||
                    ||[a_{n-2}^{(1)}-t_{n-2}^{(1)}] \mod m_{n-2}||[a_{n-1}^{(1)}-t_{n-1}^{(1)}] \mod m_{n-1}||0||[a_{n+1}^{(1)}-t_{n+1}^{(1)}] \mod m_{n+1}|
                    = [0 \| 0 \| a_3^{(2)} \| a_4^{(2)} \| \dots \| a_{i-1}^{(2)} \| a_i^{(2)} \| a_{i+1}^{(2)} \| \dots \| a_{n-3}^{(2)} \| a_{n-2}^{(2)} \| 0 \| 0 \| a_{n+1}^{(2)} ].
                   Appeal by the value of the remainders a_3^{(2)} and a_{n-2}^{(2)} of a number
                    A^{(2)} = [0 || 0 ||
                    ||a_3^{(2)}|| \dots ||a_{i-1}^{(2)}||a_i^{(2)}||a_{i+1}^{(2)}|| \dots ||a_{n-3}^{(2)}||a_{n-2}^{(2)}||0||0||a_{n+1}^{(2)}| in BNC, for the
                    NC^{(2)} = [0 \parallel 0 \parallel t_3^{(2)} \parallel \dots \parallel t_{i-1}^{(2)} \parallel t_i^{(2)} \parallel t_{i+1}^{(2)} \parallel \dots \parallel t_{n-3}^{(2)} \parallel t_{n-2}^{(2)} \parallel
                    \parallel 0 \parallel 0 \parallel t_{n+1}^{(2)} \rfloor \; , \; t_{3}^{(2)} = a_{3}^{(2)} \; , \; t_{n-2}^{(2)} = a_{n-2}^{(2)} \; ; \; t_{3}^{(2)} = \overline{0, m_{3} - 1} \; , \; t_{n-2}^{(2)} = \overline{0, m_{n-2} - 1} \; .
                   Performing a subtraction operation A^{(3)} = A^{(2)} - NC^{(2)} = [0 || 0 || a_3^{(2)} || \dots
                    ... \| a_{i-1}^{(2)} \| a_i^{(2)} \| a_{i+1}^{(2)} \| ... \| a_{n-3}^{(2)} \| a_{n-2}^{(2)} \| 0 \| 0 \| a_{n+1}^{(2)} ] - [0 \| 0 \| t_3^{(2)} \| ... \| t_{i-1}^{(2)} \| t_i^{(2)} \|
                    ||t_{i+1}^{(2)}|| \dots ||t_{n-3}^{(2)}||t_{n-2}^{(2)}||0||0||t_{n+1}^{(2)}| = \{0||0||[a_3^{(2)} - t_3^{(2)}] \mod m_3 ||
                    \|[a_4^{(2)}-t_4^{(2)}] \bmod m_4\|...\|[a_{i-1}^{(2)}-t_{i-1}^{(2)}] \bmod m_{i-1}\|[a_i^{(2)}-t_i^{(2)}] \bmod m_i\|
                    ||[a_{i+1}^{(2)} - t_{i+1}^{(2)}] \mod m_{i+1}|| \dots ||[a_{n-3}^{(2)} - t_{n-3}^{(2)}] \mod m_{n-3}||[a_{n-2}^{(2)} - t_{n-2}^{(2)}] \mod m_{n-2}||
                    ||0||0||[a_{n+1}^{(2)}-t_{n+1}^{(2)}]\mod m_{n+1} = [0||0||0||a_4^{(3)}||a_5^{(2)}||...||a_{i-1}^{(3)}||a_i^{(3)}||a_{i+1}^{(3)}||...|
                    ... || a_{n-4}^{(3)} || a_{n-3}^{(3)} || 0 || 0 || 0 || a_{n+1}^{(3)} ].
                   Appeal by the value of the remainders a_i^{(i-1)} and a_{n-i+1}^{(i-1)} of a number
                    A^{(i-1)} = [0 || 0 || \dots
                    ... \parallel 0 \parallel a_{i}^{(i-1)} \parallel a_{i+1}^{(i-1)} \parallel a_{i+2}^{(i-1)} \parallel ... \parallel a_{n-i-3}^{(i-1)} \parallel a_{n-i}^{(i-1)} \parallel a_{n-i+1}^{(i-1)} \parallel 0 \parallel 0 \parallel ... \parallel 0 \parallel a_{n+1}^{(i-1)} ] \quad \text{in}
For
value
                    BNC_{i-1} for the NC^{(i-1)} = [0 || 0 || ... || 0 ||
                    \parallel t_{i}^{(i-1)} \parallel t_{i+1}^{(i-1)} \parallel t_{i+2}^{(i-1)} \parallel \dots \parallel t_{n-i-1}^{(i-1)} \parallel t_{n-i}^{(i-1)} \parallel t_{n-i+1}^{(i-1)} \parallel 0 \parallel 0 \parallel \dots \parallel 0 \parallel t_{n+1}^{(i-1)} \rceil; \ t_{i}^{(i-1)} = a_{i}^{(i-1)}
                    ,\ t_{n-i+1}^{(i-1)}=a_{n-i+1}^{(i-1)}\ ;\ t_i^{(i-1)}=\overline{0,\,m_i-1}\ ,\ t_{n-i+1}^{(i-1)}=\overline{0,\,m_{n-i+1}-1}\ .
```

```
 \begin{array}{c} \text{Performing a subtraction operation} \ \ A^{(i)} = A^{(i-1)} - NC^{(i-1)} = [0 \, \| \, 0 \, \| \, 0 \, \| \, \dots \\ \dots \, \| \, 0 \, \| \, a_i^{(i-1)} \, \| \, a_{i+1}^{(i-1)} \, \| \dots \, \| \, 0 \, \| \, 0 \, \| \, a_{n+1}^{(i-1)} \, ] - [0 \, \| \, 0 \, \| \, 0 \, \| \dots \, \| \, 0 \, \| \, t_i^{(i-1)} \, \| \, t_{i+1}^{(i-1)} \, \| \dots \, \| \, 0 \, \| \\ \| \, 0 \, \| \, t_{n+1}^{(i-1)} \, ] = \left\{ 0 \, \| \, 0 \, \| \dots \, \| \, 0 \, \| \, [a_i^{(i-1)} - t_i^{(i-1)}] \, \text{mod} \, m_i \, \| \, [a_{i+1}^{(i-1)} - t_{i+1}^{(i-1)}] \, \text{mod} \, m_{i+1} \, \| \\ \| \, [a_{i+2}^{(i-1)} - t_{i+2}^{(i-1)}] \, \text{mod} \, m_{i+2} \, \| \dots \, \| \, [a_{n-i-1}^{(i-1)} - t_{n-i-1}^{(i-1)}] \, \text{mod} \, m_{n-i-1} \, \| \\ \| \, [a_{n-i}^{(i-1)} - t_{n-i}^{(i-1)}] \, \text{mod} \, m_{n-i} \, \| \, [a_{n-i+1}^{(i-1)} - t_{n-i+1}^{(i-1)}] \, \text{mod} \, m_{n-i+1} \, \| \, 0 \, \| \, 0 \, \| \dots \, \| \, 0 \, \| \\ \| \, [a_{n+1}^{(i-1)} - t_{n+1}^{(i-1)}] \, \text{mod} \, m_{n+1} \, \right\} = [0 \, \| \, 0 \, \| \dots \, \| \, 0 \, \| \, 0 \, \| \, a_{i+1}^{(i)} \, \| \, a_{i+2}^{(i)} \, \| \, a_{i+3}^{(i)} \, \| \, \dots \\ \dots \, \| \, a_{n-i-1}^{(i)} \, \| \, a_{n-i}^{(i)} \, \| \, 0 \, \| \, 0 \, \| \dots \, \| \, 0 \, \| \, a_{n+1}^{(i)} \, \right]. \end{array}
```

Appeal by the value of the remainders $a_{i+1}^{(i)}$ and $a_{n-i}^{(i)}$ of a number $A^{(i)} = [0 \parallel 0 \parallel 0 \parallel ...$

$$\begin{split} \dots &\|\ 0\ \|\ 0\ \|\ a_{i+1}^{(i)}\ \|\ \dots \|\ a_{n-i-1}^{(i)}\ \|\ a_{n-i}^{(i)}\ \|\ 0\ \|\ 0\ \|\ a_{n+1}^{(i)}\] \quad \text{in} \quad BNC_i \qquad \text{for} \quad \text{the} \\ NC^{(i)} &= & [0\ \|\ 0\ \|\ 0\ \|\ \dots \|\ 0\ \|\ 0\ \|\ t_{i+1}^{(i)}\ \|\ \dots \|\ t_{n-i-1}^{(i)}\ \|\ t_{n-i}^{(i)}\ \|\ 0\ \|\ 0\ \|\ t_{n+1}^{(i)}\]\ ; \qquad t_{i+1}^{(i)} &= a_{i+1}^{(i)}\ , \\ t_{n-i}^{(i)} &= & a_{n-i}^{(i)}\ ;\ t_{i+1}^{(i)} &= & \overline{0,m_{i+1}-1}\ ,\ t_{n-i}^{(i)} &= & \overline{0,m_{n-i}-1}\ . \end{split}$$

For value $A^{(i+1)}$

 $\begin{aligned} & \text{Performing a subtraction operation} \ \ A^{(i+1)} = A^{(i)} - NC^{(i)} = [0 \, \| \, 0 \, \| \, \dots \\ & \dots \| \, 0 \, \| \, a^{(i)}_{i+1} \, \| \, a^{(i)}_{i+2} \, \| \, a^{(i)}_{i+3} \, \| \, \dots \| \, a^{(i)}_{n-i-1} \, \| \, a^{(i)}_{n-i-1} \, \| \, a^{(i)}_{n-i} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, a^{(i)}_{n+1} \,] - \\ & - [0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t^{(i)}_{i+1} \, \| \, t^{(i)}_{i+2} \, \| \, t^{(i)}_{i+3} \, \| \, \dots \| \, t^{(i)}_{n-i-1} \, \| \, t^{(i)}_{n-i-1} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, t^{(i)}_{n+1} \,] = \\ & = \left\{ 0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, [a^{(i)}_{i+1} - t^{(i)}_{i+1}] \, \text{mod} \, m_{i+1} \, \| \, [a^{(i)}_{i+2} - t^{(i)}_{i+2}] \, \text{mod} \, m_{i+2} \, \| \\ & \| \, [a^{(i)}_{n+3} - t^{(i)}_{n+3}] \, \text{mod} \, m_{i+3} \, \| \, \dots \| \, [a^{(i)}_{n-i-2} - t^{(i)}_{n-i-2}] \, \text{mod} \, m_{n-i-2} \, \| \\ & \| \, [a^{(i)}_{n-i-1} - t^{(i)}_{n-i-1}] \, \text{mod} \, m_{n-i-1} \, \| \, [a^{(i)}_{n-i} - t^{(i)}_{n-i}] \, \text{mod} \, m_{n-i} \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \\ & \| \, [a^{(i)}_{n+1} - t^{(i)}_{n+1}] \, \text{mod} \, m_{n+1} \right\} = [0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, 0 \, \| \, a^{(i+1)}_{i+2} \, \| \, a^{(i+1)}_{i+3} \, \| \, \dots \| \, a^{(i+1)}_{n-i-1} \, \| \, a^{(i+1)}_{n-i-1} \, \| \\ & \| \, 0 \, \| \, 0 \, \| \, \dots \| \, 0 \, \| \, 0 \, \| \, a^{(i+1)}_{n+1} \,] \, . \end{aligned}$

Further for n even n odd numbers we get: for n even number. Appeal by the value of the remainders $a_{n/2}^{(n/2-1)}$ and $a_{n/2+1}^{(n/2-1)}$ of a number $A^{(n/2-1)} = [0 || 0 || \dots || 0 || a_{n/2}^{(n/2-1)} || a_{n/2+1}^{(n/2-1)} || 0 || \dots || 0 || 0 || a_{n+1}^{(n/2-1)}]$ in $BNC_{n/2-1}$ for the $NC^{(n/2-1)} = [0 || 0 || \dots || 0 || t_{n/2}^{(n/2-1)} ||$

 $n-1 \qquad \begin{aligned} \|t_{n/2-1}^{(n/2-1)} \|0\| \dots \|0\|0\|t_{n+1}^{(n/2-1)}\}; & t_{n/2}^{(n/2-1)} = a_{n/2}^{(n/2-1)}, & t_{n/2+1}^{(n/2-1)} = a_{n/2+1}^{(n/2-1)}; \\ t_{n/2}^{(n/2-1)} = \overline{0, m_{n/2} - 1}, & t_{n/2+1}^{(n/2-1)} = \overline{0, m_{n/2+1} - 1}. \end{aligned}$

For n odd number.

Appeal by the value of the remainder $a_{(n+1)/2}^{((n+1)/2-1)}$ of a number $A^{((n+1)/2-1)}=[0\,\|\,0\,\|\dots$

 $... \|\, 0\, \|\, a_{\scriptscriptstyle (n+1)/2}^{\scriptscriptstyle ((n+1)/2-1)}\, \|\, 0\, \|\, ... \|\, 0\, \|\, a_{\scriptscriptstyle n+1}^{\scriptscriptstyle ((n+1)/2-1)}\,] \qquad \text{in} \qquad \mathit{BNC}_{\scriptscriptstyle (n+1)/2-1} \qquad \text{for} \qquad \text{the}$

$$NC^{((n+1)/2-1)} = [0 || 0 || \dots || 0 || t_{(n+1)/2}^{((n+1)/2-1)} || 0 || \dots || 0 || t_{n+1}^{((n+1)/2-1)}],$$

$$t_{(n+1)/2}^{((n+1)/2-1)} = a_{(n+1)/2}^{((n+1)/2-1)}; t_{(n+1)/2}^{((n+1)/2-1)} = 0, m_{(n+1)/2} - 1.$$
For a even and a odd numbers, we obtain the following values of the

For n even and n odd numbers, we obtain the following values of the number $A^{(Z)}$ to be nullified. For n even number. Getting a nullified $A^{(Z)}$ number: $A^{(Z)} = A^{(n/2)} = A^{(n/2-1)}$ $-NC^{(n/2-1)} = [0 \parallel 0 \parallel \dots \parallel 0 \parallel a_{n/2}^{(n/2-1)} \parallel a_{n/2+1}^{(n/2-1)} \parallel 0 \parallel \dots \parallel 0 \parallel 0 \parallel a_{n+1}^{(n/2-1)}] -[0\,||\,0\,||\,\dots||\,0\,||\,t_{n/2}^{(n/2-1)}\,||\,t_{n/2+1}^{(n/2-1)}\,||\,0\,||\,\dots||\,0\,||\,0\,||\,t_{n+1}^{(n/2-1)}\,] = \big\{0\,||\,0\,||\,\dots||\,0\,||\,$ $\|[a_{n/2}^{(n/2-1)}-t_{n/2}^{(n/2-1)}]\operatorname{mod} m_{n/2}\,\|[a_{n/2+1}^{(n/2-1)}-t_{n/2+1}^{(n/2-1)}]\operatorname{mod} m_{n/2+1}\,\|\,0\,\|\,...\,\|\,0\,\|\,0\,\|$ $\|[a_{n+1}^{(n/2-1)}-t_{n+1}^{(n/2-1)}]\bmod m_{n+1}\} = \left[0\|0\|...\|0\|0\|0\|...\|0\|0\|a_{n+1}^{(n/2)}\right], \text{ where }$ $\gamma_{n+1} = a_{n+1}^{(n/2)} \, .$ For n odd number. Getting a nullified $A^{(Z)}$ number: $A^{(Z)} = A^{(n+1/2)} = A^{((n+1/2-1))}$ $NC^{((n+1)/2-1)} = [0 \parallel 0 \parallel \dots \parallel 0 \parallel a_{(n+1)/2}^{((n+1)/2-1)} \parallel 0 \parallel \dots \parallel 0 \parallel a_{n+1}^{((n+1)/2-1)}] - [0 \parallel 0 \parallel \dots$ $\dots \| 0 \| t_{(n+1)/2}^{((n+1)/2-1)} \| 0 \| \dots \| 0 \| t_{n+1}^{((n+1)/2-1)}] = \{ 0 \| 0 \| \dots \| 0 \|$ $\|[a_{(n+1)/2}^{((n+1)/2-1)}-t_{(n+1)/2}^{((n+1)/2-1)}] \bmod m_{(n+1)/2}\,\|\,0\,|\|\ldots\|\,0\,\|$ $\| [a_{n+1}^{((n+1)/2-1)} - t_{n+1}^{((n+1)/2-1)}] \bmod m_{n+1} \Big\} = [0 \| 0 \| \dots \| 0 \| \dots \| 0 \| 0 \| a_{n+1}^{((n+1)/2)}] ,$ where $\gamma_{n+1} = a_{n+1}^{((n+1)/2)}$. $T_{Z3} = n \cdot \tau$

Fig. 4. Procedure of SN of numbers in the SRC

The time T_{Z3} for performing the zeroing procedure for the first (Z3) method of the PNM is defined as:

$$T_{Z3}=n\cdot \tau_{add}$$
 (7)

When implementing the nullification procedure for the second (Z3) method in the block of nullification constants (NB) of the calculator in the SRC, it is necessary to

have $K_{Z3} = \sum_{i=1}^{\left[\frac{n}{2}\right]} (m_i \cdot m_{n-i+1} - 1)$ nullification constants. In this case, the number of N_{Z3} double digits of the NB nullification constants is determined by the expression $\left[\frac{n}{2}\right]$

$$K_{Z3} = \sum_{i=1}^{\left\lfloor \frac{n}{2} \right\rfloor} (m_i \cdot m_{n-i+1} - 1) \cdot (n-2i+1) .$$

4 Conclusions

The article presents the nullification of numbers in the system of residual classes (SRC). This method is widely used in the non-positional number system in the SRC with the need to determine positional characteristics. Two methods of nullification are presented in the article: the method of successive subtractions and the method of parallel subtractions. Based on these methods, algorithms are developed for their implementation. The essence of the method of successive subtractions is that the nullification procedure is carried out consequently from the junior foundation to the oldest. The essence of the parallel subtraction method is that the nullification procedure is carried out parallel in time for two reasons.

It is advisable to use these methods in the implementation of operations of comparing numbers in the SRC, and in monitoring data presented in the SRC [9-13]. The estimation of these methods by the number of equipment and by the time of implementation of the nullification procedure is made. In terms of the efficiency of the nullification procedure, a parallel subtraction method was proposed.

The obtained research results can be useful for various methods of increasing the reliability and fault tolerance of computer systems [14-18].

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