Linear Quadratic Gaussian Control for Robotic Excavator

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Abstract. When developing a robotic excavator, one of the main issues is tracking the given trajectories using its manipulator. This task is complicated by the fact that the system is subject to disturbances and measurement noise, which can most naturally be modelled as stochastic white noise processes. This paper describes an LQG regulator aimed at improving tracking performance at the levelling operation made by the robotic excavator. Simulation is performed on the attached backhoe equipment of the Boreks 2201excavator. The results show that the proposed control algorithm is effective for improving the trajectory tracking accuracy.

Keywords: robotic excavator, trajectory tracking, LQG, Kalman filter

1 Introduction

Hydraulic excavators are widely used in many areas, such as construction, mining and agriculture, as well as in dangerous areas for rescue and recovery operations. Strict requirements for improving the quality of digging operations and reducing energy and time costs for the entire workflow lead to a change in the look of traditional hydraulic excavators. For instance, the number of degrees of freedom for working equipment has increased; articulated booms and sticks have appeared; multi-handed excavators have become available on the market, etc. However, growing complexity of the excavator requires great skill to control it, and now even an experienced operator cannot realize the full capabilities of the machine. Thus, automation of excavators is considered as a challenge in modern construction machinery. Moreover, as excavators are often used to accomplish dangerous tasks, an unmanned robotic excavator can greatly improve the safety.

Nevertheless, the lack of information about the environment and unpredictable mechanical parameters such as flexibility, friction, various nonlinearities in hydraulic actuators prevent the extensive use of unmanned excavators. To mitigate the above problems varieties of studies have been made recently concerning adaptive and robust control of an excavator arm.

In particular, Kim [1] has proposed a nonlinear proportional-derivative (PD) controller coupled with the μ-controller to compensate parametric and unstructured uncertainties. Wang [2] has introduced the nonlinear proportional-integral (PI) controller combined with a cross-coupled pre-compensation algorithm to improve tracking accuracy by an unmanned excavator bucket edge. PI-based adaptive velocity controller for each cylinder of a mini excavator has been presented by Wind in [3].

Some researchers have utilized Genetic Algorithms [4], artificial neural networks and fuzzy logic [5–9], ant colony optimization [10, 11] in order to adjust the parameters of the traditional proportional-integral-derivative (PID) and PD controllers. Sliding-mode and Time delay controls have been applied to compensate for the nonlinearity of complex dynamics of hydraulic manipulators in [12, 13, 14]. Besides, in some studies, other intelligence algorithms are used, such as guaranteed cost control [15, 16], model predictive control [17], model reference adaptive control [18], a robust stochastic control, based on the method of analytical design of aggregated regulators (ADAR) [19], that were applied to control the trajectory of the excavator.

All the mentioned and other related works have contributed significantly to robotic excavator development. However, these controllers are quite complex; their use requires significant re-equipment of excavators, in particular, replacement of the hydraulic system. Therefore, no unmanned excavator is mass-produced today. Thus, automation of the excavators remains a topical issue.

Since in the process of digging the values of external disturbances and measurement noise are unknown, it is expedient to assume them as stochastic white noise processes. In such a case, a well-known in modern control theory Linear Quadratic Gaussian regulator (LQG), which is relatively easy to design and implement [20], can be used to control the excavator manipulator.

Thus, ***this paper aims*** at designing and applying the LQG in a hydraulic robotic excavator to control the manipulator trajectory.

The remainder of this paper is organized as follows. The formal statement of the problem is given in Section 2. Section 3 is dedicated to the excavator modelling; nonlinear as well as linearized models of the excavator manipulator are described in this section. Section 4 presents the design procedure of the controller. The simulation results are provided in Section 5. Finally, Section 6 presents concluding remarks and future directions for this work.

2 Formal Problem Statement

It is assumed that the excavator manipulator model is linearized and the state equations of its motion look like:

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Bu(t)+w(t), \\ y(t) &= Cx(t)+v(t), \end{aligned} \tag{1}$$

where $y(t)$ is the ($r \times 1$) observation vector of the true state ($n \times 1$) vector $x(t)$ at time t; $u(t)$ is the control input ($m \times 1$) vector, $w(t)$ and $v(t)$ are stochastic processes associated with the working process and the manipulator state measurement, respectively.

The state matrix A $(n \times n)$, control input gain matrix B $(n \times m)$ and measured state matrix C $(r \times n)$ are all linear time invariants, $n = 6$, $m = 3$, $r = 3$.

The process noise $w(t)$ and measurement noise $v(t)$ are white Gaussian random sequences with zero mean value, and $V(t)$ and $W(t)$ are the $v(t)$ and $w(t)$ covariance matrices.

It is necessary to obtain a control law in the form of static linear feedback:

$$u(t) = -Kx(t)\,, \tag{2}$$

where the gain matrix K is meant to minimize a quadratic cost function formulated as follows:

$$J = \int_0^T \left(x(t)^T Qx(t) + u(t)^T Ru(t) \right) dt\,, \tag{3}$$

where Q and R are symmetric positive definite matrices of compatible dimensions.

3 Excavator manipulator modelling

In this paper, the attached working equipment of the Boreks 2201 excavator is considered as an example. To study this excavator, its 3D model was originally built in Autodesk Inventor 2016, and afterwards it was imported into MATLAB Simscape Multibody [21]. The general view of the model is given in Fig. 1. The adequacy of the model was proved by the comparison with the results of a real excavator testing. The inputs of the model are the desired rods displacements of the boom, the stick and the bucket hydro cylinders, calculated on the base of the desired rotations of the corresponding joints [21].

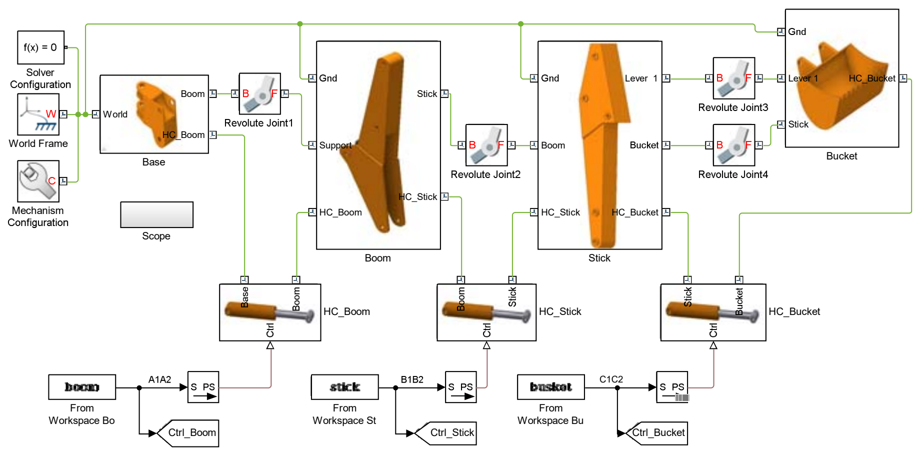


Fig. 1. General view of the excavator manipulator model

The model in Fig. 1 is a nonlinear one; however, the LQG theory is developed for linear systems described by (1). Therefore, in this section, the linear model of the excavator manipulator is under consideration. In robotics the feedback linearization [22] in widely known, though the obtained model is quite complex and requires knowledge of all the dynamic parameters of the manipulator. On the other part, a model of the manipulator link can be simplistically described by the second order system [14, 22]:

$$G_i(s) = \frac{1}{m_i s^2 + b_i s + c}, \tag{4}$$

where m_i, b_i, c_i are positive parameters to be determined, $G_i(s)$ is a transfer function from the desired θ_{di} to the actual θ_i rotation angle of the excavator manipulator links, and $i = 1,2,3$.

The transfer function (4) can be rewritten in the state-space form by the following relation:

$$\begin{aligned} \dot{x}_i(t) &= \begin{bmatrix} 0 & 1 \\ -c_i/m_i & -b_i/m_i \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ 1/m_i \end{bmatrix} u_i(t), \\ y_i(t) &= [1 \quad 0] x_i(t), \end{aligned} \tag{5}$$

where $x_i(t) = \theta_i(t)$, $i = 1,2,3$.

In order to obtain the linear dynamic model of the excavator manipulator, step responses of the system in Fig. 1 were obtained for each joint (Fig. 2).

The values of m_i, b_i and c_i in (4) can be estimated using any of the numerical optimization methods that minimize the error between the outputs of the real system and its model based on the known information about the inputs and outputs of the real system. Here, to automate this procedure, the MATLAB System Identification Toolbox was used. The estimated values are given in Table 1, and Fig. 2(d) presents the errors between the actual outputs of each link and the outputs of the corresponding linear models.

Table 1. Estimated values of the linearized model for each joint

	m	b	c
Boom	$1.1285 \cdot 10^{-4}$	0.020057	1.000
Stick	$1.0065 \cdot 10^{-4}$	0.020011	1.000
Bucket	$1.0001 \cdot 10^{-4}$	0.020000	1.000

Fig. 2(d) shows that the results of the linear model coincide with the nonlinear one. The response of the linear model is somewhat slower but the agreement is satisfactory with respect to the high degree of simplification in the linear model. Therefore, it can be concluded, that the established linear model is accurate and credible.

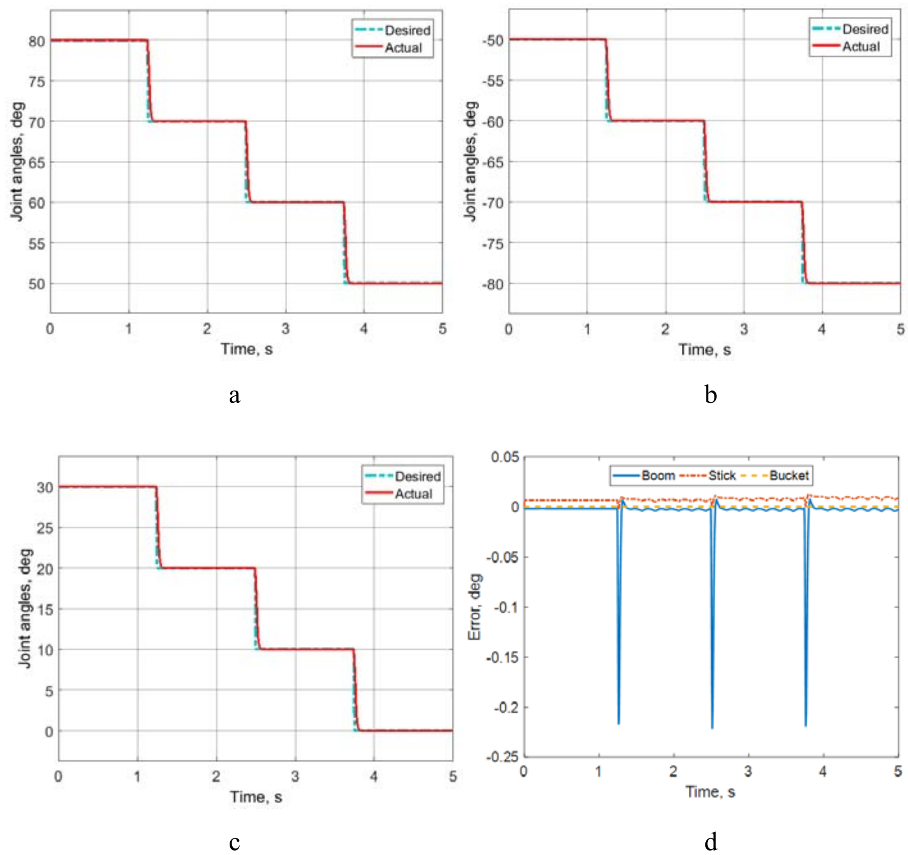


Fig. 2. Desired and actual joint angles of the boom (a), the stick (b) and the bucket (c), and the errors of the linearized model (d)

4 Controller design

4.1 LQG general form

LQG control is a modern state-space technique for designing optimal dynamic regulators [20]. It is rooted in optimal stochastic control theory and combines both the concepts of Linear Quadratic Regulators (LQR) for full state feedback and Kalman filters for state estimation. The plant is described by the state and output equations (1) and the controller design methodology enables a controller to be synthesized which is optimal with respect to the quadratic cost function (3), where Q and R are weighting matrices that define the trade off between regulation performance and control efforts. Taking into consideration the second term in (3) allows getting control signals of limited amplitude at its minimization, which is especially important when designing control systems of such unmanned vehicles as excavators.

The gain matrix K in the control law (2) has the following form

$$K = R^{-1} B^T P \,, \tag{6}$$

and is obtained by solving the algebraic Riccati equation (CARE):

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \,, \tag{7}$$

where P is the unknown ($n \times n$) symmetric matrix.

Due to uncertainties, the Kalman filter is used to obtain the estimate of the state vector $\hat{x}$ (Fig. 3):

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\big(y(t) - C\hat{x}(t)\big) \,. \tag{8}$$

The Kalman filter gain L in (8) equals:

$$L = V^{-1} C^T S \,, \tag{9}$$

where S is the solution of the following matrix Riccati equation:

$$A^T S + SA - SCV^{-1} C^T S + W = 0 \,. \tag{10}$$

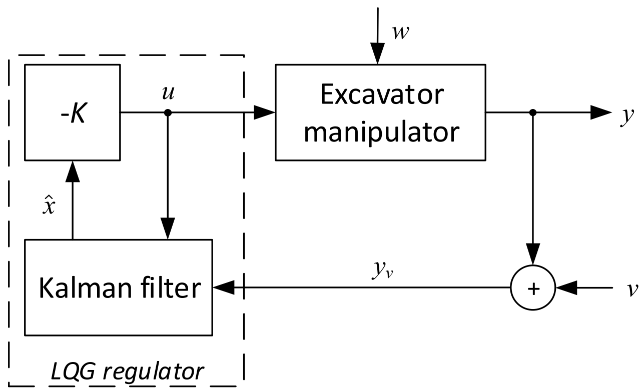


Fig. 3. Block diagram of the excavator control system with LQG regulator

4.2 Controller design

The controller design was accomplished in MATLAB. According to the common approach [20], the LQG design was split into two independent steps: first, the LQR was designed, using the combined state space model of the manipulator as (1):

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -c_{bo}/m_{bo} & -b_{bo}/m_{bo} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -c_{st}/m_{st} & -b_{st}/m_{st} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -c_{bu}/m_{bu} & -b_{bu}/m_{bu} \end{bmatrix},$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 1/m_{bo} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/m_{st} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/m_{bu} \end{bmatrix},\ C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

where x_1, x_3, x_5 are joint angles of the boom, the stick and the bucket, x_2, x_4, x_6 are their angular velocities, respectively.

The initial values of the weighting matrices in (3) were selected by using Bryson's rule, relating the reciprocal of the maximum squared values of the states with Q and the reciprocal of the maximum squared values of the control inputs with R:

$$Q=\frac{1}{\max(x^2)}, \tag{11}$$

$$R=\frac{1}{\max(u^2)}, \tag{12}$$

and then were refined experimentally. Satisfactory closed-loop responses were obtained using weighting matrices

$$Q=\begin{bmatrix} 1.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5\cdot 10^{-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5\cdot 10^{-4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\cdot 10^{-4} \end{bmatrix},\ R=\begin{bmatrix} 1\cdot 10^{-5} & 0 & 0 \\ 0 & 1\cdot 10^{-5} & 0 \\ 0 & 0 & 1\cdot 10^{-5} \end{bmatrix}.$$

The obtained gain matrix K is:

$$K=\begin{bmatrix} 386.30 & 7.06 & 0 & 0 & 0 & 0 \\ 0 & 0 & 386.30 & 7.06 & 0 & 0 \\ 0 & 0 & 0 & 0 & 446.21 & 4.46 \end{bmatrix}.$$

4.3 Kalman filter design

At the second step, the Kalman filter was designed in MATLAB using noise covariances $W = \text{diag}\{0.1\}$, $V = \text{diag}\{0.02\}$. The Kalman gain matrix is:

$$L = \begin{bmatrix} 0.0651 & 0 & 0 \\ -2.4979 & 0 & 0 \\ 0 & 0.0626 & 0 \\ 0 & -2.4980 & 0 \\ 0 & 0 & 0.0625 \\ 0 & 0 & -2.4980 \end{bmatrix}.$$

5 Simulation results

The effectiveness of the proposed controller was verified by simulation, which was performed for levelling works using the nonlinear model in Fig.1. The desired bucket edge path and the initial and final configurations of the manipulator are given in Fig. 4.

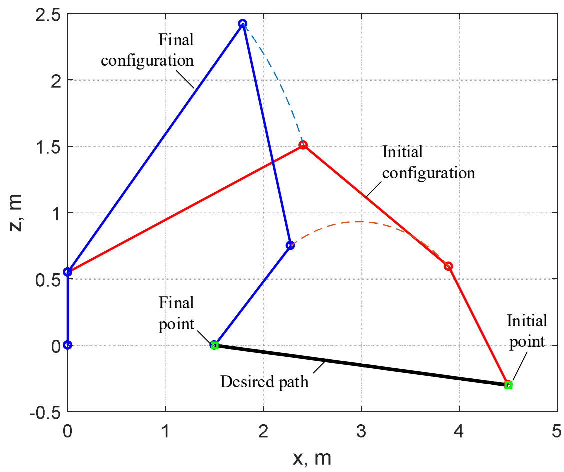


Fig. 4. The desired bucket edge path and extreme positions of the manipulator

During digging, resistance force F_r acts at the cutting edge of the bucket teeth, this force is a resultant reaction force of the tangential, F_t and the normal F_n forces. At the simulation, the tangential force was simplistically determined as

$$F_t = k_c b \sum h \Delta x \,, \tag{13}$$

where k_c is the specific cutting force, that takes into account soil resistance to cutting, frictional resistance of the bucket with the soil, resistance to the movement of the prism of soil and all other forces; h and b are the thickness and width of the cut slice of soil; Δx is an increment of the bucket teeth path along the x-axis [15].

The normal component F_n was calculated as:

$$F_n = \psi F_t \,, \tag{14}$$

where ψ is a dimensionless factor depending on the digging angle, digging conditions and the cutting edge where $\psi = 0.1–0.45$. Higher values of ψ correspond to more blunting of the bucket teeth edge.

It was assumed that $k_c = 181.3$ kN/m^2, $b = 0.75$ m, $h = 0.1$ m, $\psi = 0.1$. The forces F_t and F_n were unknown for the controller during simulation.

Figs. 5(a)–(c) show the desired, actual and measured trajectories of the boom, the stick and the bucket respectively, and the tracking errors are depicted in Fig. 5(d). Fig. 6 describes the levelling error.

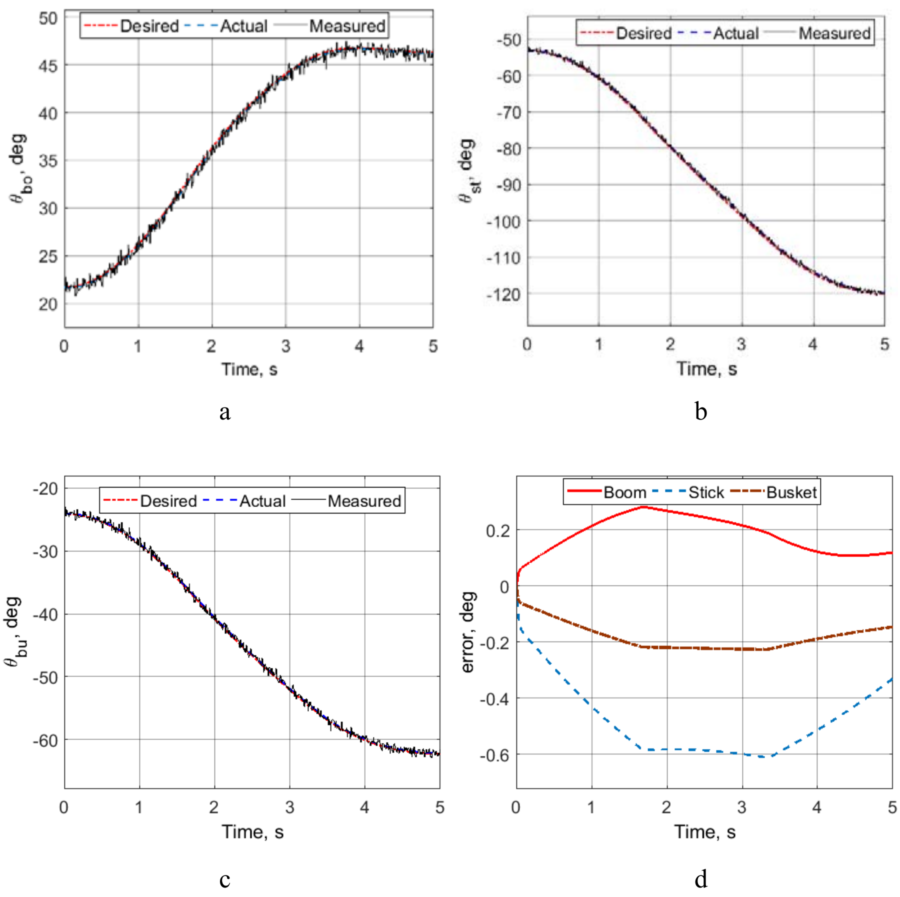
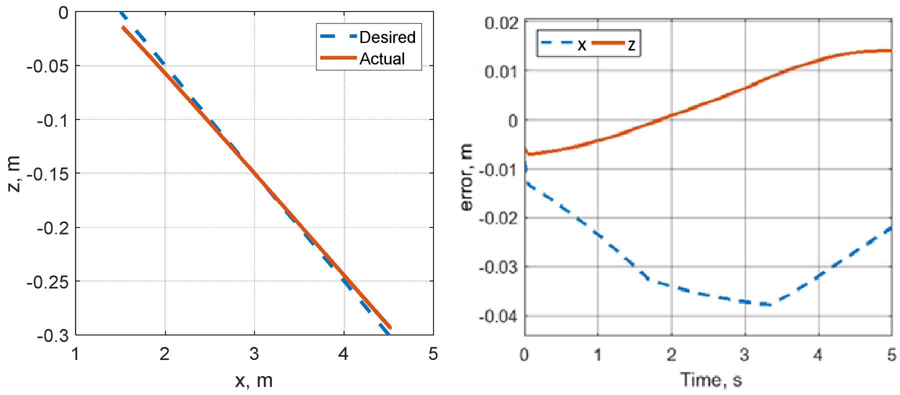


Fig. 5. Displacements of the manipulator links: a – for the boom, b – for the stick; c – for the bucket; d – links trajectories tracking errors



a b

Fig. 6. Levelling error: a – in *x*-*z* plane; b – on each of the coordinates

There is a good agreement between the desired and measured displacements of the manipulator links (Fig. 5), and the fit to the desired trajectories is near 99%. As can be seen from Fig. 6, the error of the levelling does not exceed 4 cm along the *x*-axis and 1.5 cm – along the *z*-axis. These values are far higher than those obtained using the PID controller [21] (1 cm and 0.1 cm, respectively), however, in [21] the simulation was performed under ideal conditions, without taking into account the disturbances. Hence, the results prove that the LQG control ensures good system performance. It should be noted, though, that backlashes, which have not taken into the consideration in the model in Fig. 1, as well as variability of soil properties can increase the tracking error.

6 Conclusion and future work

In this paper, we have shown how an LQG regulator can be used to control robotic excavator trajectory. For this purpose, a linear model of the excavator manipulator was obtained, and its parameters were identified by the comparison with the nonlinear model, described in the previous work.

The control system is designed in the way that it provides a deviation of the bucket edge from a given path no more than 4 cm along the *x*-axis and 1.4 cm along the *z*-axis at the levelling operation in the presence of random measurement noise and unknown forces appearing when the bucket touches the soil. Tracking accuracy can be improved by adding the integrator to the controller. However, this can cause difficulties associated with time delays [20], which may not be taken into account in the model in Fig. 1. Moreover, the simulation was carried out for uniform soil, so more significant tracking deviations can be expected in real conditions. Therefore, our future work will be focused on reducing tracking errors under unknown external disturbances, including the variability of soil properties.

References

1. Kim, S., Park, J., Kang, S. et al. A Robust Control Approach for Hydraulic Excavators Using μ-synthesis. Int. J. Control Autom. Syst. **16**, 1615–1628 (2018). https://doi.org/10.1007/s12555-017-0071-9
2. Wang, D., Zheng, L., Yu, H. et al. Robotic excavator motion control using a nonlinear proportional-integral controller and cross-coupled pre-compensation. Automation in Construction, **64**, 1–6 (2016). https://doi.org/10.1016/j.autcon.2015.12.024.
3. Wind H., Jenisch L., Renner A. et al. Design of an Adaptive Velocity Controller for a Hydraulic Mini Excavator using a Moving-Horizon-Estimator. In: IEEE Conference on Control Technology and Applications, pp. 685–690 (2019). https://doi.org/10.1109/CCTA.2019.8920496
4. Feng, H., Yin, C.-B., Weng, W. et al. Robotic excavator trajectory control using an improved GA based PID controller. Mechanical Systems and Signal Processing. **105**, 153–168 (2018). https://doi.org/10.1016/j.ymssp.2017.12.014
5. Le Hanh, D., Ahn, K.K., Kha, N.B. et al. Trajectory control of electro-hydraulic excavator using fuzzy self tuning algorithm with neural network. J Mech Sci Technol **23**, 149–160 (2009). https://doi.org/10.1007/s12206-008-0817-7
6. Yan, J., Li, B., Guo, G. et al. Adaptive control for bucket position of excavator based on single neuron PID. Journal of PLA University of Science and Technology (Natural Science Edition) **3**, 316–319 (2012).
7. Park, J., Cho, D., Kim, S. et al. Utilizing online learning based on echo-state networks for the control of a hydraulic excavator. Mechatronics **24**(**8**), 986–1000 (2014). https://doi.org/10.1016/j.mechatronics.2014.10.004
8. Li, Y., Wang, Q. Adaptive neural finite-time trajectory tracking control of hydraulic excavators. Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering **232**(**7**), 909–925 (2018). https://doi.org/10.1177/0959651818767770
9. Bo, L., Jun, Y., Gang. G., et al. High performance control of hydraulic excavator based on fuzzy-pi soft-switch controller. In: IEEE international conference on Computer science and automation engineering, pp. 676–679 (2011). https://doi.org/10.1109/CSAE.2011.5952558
10. Hassan, M. Y., Sugban, A. F. Ant Colony Optimization Based Type-2 Fuzzy Force-Position Control for Backhoe Excavator Robot. Al-Nahrain Journal for Engineering Sciences **21**(**1**), 1-11, (2018). https://doi.org/10.29194/NJES21010001
11. Feng, H., Yin, C., Ma, W., et al. Parameters identification and trajectory control for a hydraulic system. ISA Transactions **92**, 228–240. (2019). https://doi.org/10.1016/j.isatra.2019.02.022.
12. Xu, J., Yoon, H. Sliding Mode Control of Hydraulic Excavator for Automated Grading Operation. SAE International Journal of Commercial Vehicles. **11(2)**, 113–124 (2018). https://doi.org/10.4271/02-11-02-0010
13. Wang, J., Lee, M. C., Kallu, et al. Trajectory tracking control of a hydraulic system using TSMCSPO based on sliding perturbation observer. Applied Sciences, **9**(**7**), 1455 (2019). https://doi.org/10.3390/app9071455
14. Kim, J., Jin, M., Choi, W., et al. Discrete time delay control for hydraulic excavator motion control with terminal sliding mode control. Mechatronics **60**, 15–25 (2019). https://doi.org/10.1016/j.mechatronics.2019.04.008.
15. Gurko, A. G., Sergiyenko, O. Yu., Hipólito, J. I. N., et al. Guaranteed Control of a Robotic Excavator During Digging Process. In: 12th International Conference on Informatics in Control, Automation and Robotics, **2,** pp. 52–59. SciTePress (2015).

16. Gurko, A. G., Sergiyenko, O. Yu., Hipólito, J.I.N., et al. Trajectory Tracking Control of an Excavator Arm Using Guaranteed Cost Control. In: Lecture Notes in Electrical Engineering, **383**, 177–196 (2016) Springer, Cham https://doi.org/10.1007/978-3-319-31898-1_10
17. Tomatsu, T., Nonaka, K., Sekiguchi, K., et al. Model predictive trajectory tracking control for hydraulic excavator on digging operation. In: IEEE Conference on Control Applications, pp. 1136–1141 (2015). https://doi.org/10.1109/CCA.2015.7320765
18. Chiang, M., Huang, C. Experimental implementation of complex path tracking control for large robotic hydraulic excavators. Int J Adv Manuf Technol **23**, 126–132 (2004). https://doi.org/10.1007/s00170-003-1636-8.
19. Kolesnikova, S., Tsvetnitskaya S., Pobegailo, P. An Extension of the Method of Aggregated Regulators to a Discrete Stochastic Object. In: International Siberian Conference on Control and Communications, pp. 1–6 (2019). https://doi.org/10.1109/SIBCON.2019.8729595
20. Kwon W.H., Park P. LQG Optimal Controls. In: Stabilizing and Optimizing Control for Time-Delay Systems. Communications and Control Engineering. Springer, Cham (2019). https://doi.org/10.1007/978-3-319-92704-6_7
21. Gurko, A., Kyrychenko, I., Yaryzhko, A. Trajectories Planning and Simulation of a Backhoe Manipulator Movement. In: CEUR Workshop Proceedings, pp. 771–785 (2019). Online http://ceur-ws.org/Vol-2353/paper61.pdf.
22. Siciliano B., Khatib O. (Eds.): Springer Handbook of Robotics. Springer (2016).