Irregular layout problem for additive production

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Abstract. One of the interesting applications of optimization layout problems is additive production. The problem of layout of 3D objects (parts) inside a container (a working chamber of a 3D printer) to minimize the container height is studied. It is aimed to reduce printing costs by minimizing the number of 3D-printing layers while reducing the number of the printer starts. A mathematical model of the layout problem is provided in the form of nonlinear programming problem using the phi-function technique. A solution algorithm to search for optimized layouts is proposed. Computational results demonstrate the efficiency of our approach.

Keywords: additive production, packing, mathematical modeling, phi-function, quasi phi-function, nonlinear optimization.

1 Introduction

Optimization 3D layout problems have a wide spectrum of real-word applications, including transportation, logistics, chemical and aerospace engineering, shipbuilding, robotics, additive manufacturing, materials science. In this paper the smart technique to optimize the 3D-printing process for selective laser sintering (SLS) additive manufacturing [1] is developed. The SLS technology uses high power laser sintering for small particles of plastic, ceramic, glass or metal flour in three-dimensional structure.

This technology empowers the fast, flexible, cost-efficient, and easy manufacture of prototypes for various application of required shape and size by using powder based material. A physical prototype is an important for design confirmation and operational examination by creating the prototype unswervingly from CAD data.
The main feature of this technology is the use of powder, consisting of particles of metal coated polymer. After the sintering process piece is placed in a high temperature kiln to burn plastic and fusible took the bronze. The advantages of the technology include no need for material support. Parts immersed into a powder, which works on as a support [2].

Recently 3D-prototyping technologies are evolving rapidly. The purpose of the research is development of smart technology to improve 3D-printing process for advanced additive production. We propose the approach for accelerating printing cycle due to the simultaneous printing of several parts providing dense filling the entire volume of the working chamber 3D printer using SLS technology.

One of the important problems arising in the process of creating new prototypes (final products) is reducing the time and cost production. For each start of SLS printer requires time and energy for heating and maintaining temperature. In [3] data on what savings can be achieved by optimizing the layout of objects to be created are provided.

Our approach allows optimizing the process of 3D printing for the following factors:

- printing of several prototypes (products) providing dense filling the volume of the 3D printer working chamber [4];
- minimizing the time and cost of 3D parts production by reducing printing cycle.

In this paper the optimization layout problem of irregular 3D objects into optimized cuboid is studied.

Our approach is based on the mathematical modelling of relations between irregular geometric objects by means of the phi-function technique. It allows us reducing the layout problem to nonlinear programming model.

2 Literature review

The list of publications related to the layout problem of irregular 3D objects, taking into account the minimum allowable distances is very scarce within the field of Packing and Cutting. Arbitrary shaped objects in most cases are approximated by sets of cuboids or spheres. To solve the layout problems heuristic and meta-heuristic algorithms are used that resulting in the loss of optimal solutions.

3D object layout problems is NP-hard. In order to find feasible solutions some researchers use different techniques, including heuristics (based on different approximation rules heuristics [5], genetic algorithms [6], simulated annealing algorithms [7], artificial bee colony algorithms [8]), extended pattern search [8], traditional optimization methods [9, 10], nonlinear mathematical programming [11].

In the majority of papers, either orientation of 3D objects is fixed or only discrete rotations (by 45 or 90 degrees) are allowed. In particular, paper [2] uses the parallel translation algorithm for packing convex polytopes. The authors of [12] propose the HAPE3D algorithm which can be applied to arbitrarily shaped polyhedra that can be rotated around each coordinate axis at eight different angles. In [13] the issue is discussed that for 3D packing problems making calculations of 0 to 360 degrees orientations of objects with respect to each axis is impossible. Analysis of irregular three-dimensional packing problems in additive manufacturing is provided in [14]. The paper [15 22] presents an intelligent layout planning for rapid prototyping.

Only few works consider continuous rotations of 3D objects (see, e.g. [16- 22].

3 Problem statement

In order to minimize the time of 3D parts production using SLS-technology the number of layers should be minimized. The problem of minimizing layers can be formulated as a problem of layout (packing) of parts in the container of minimum height (fig.1).

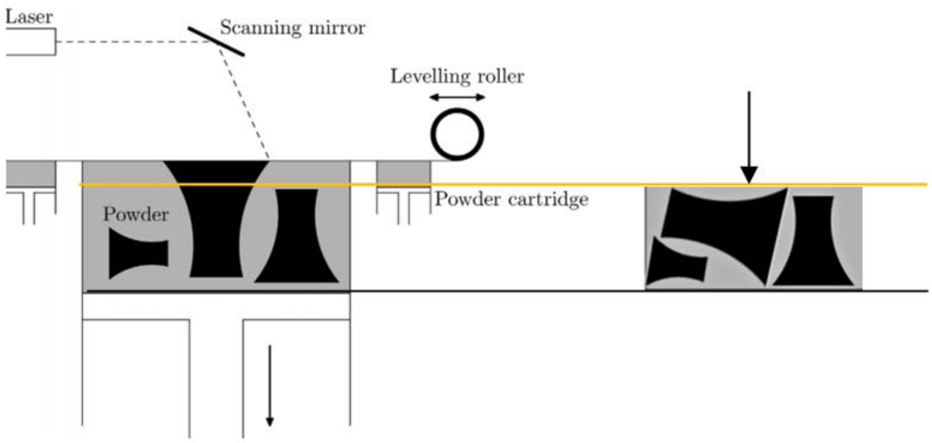


Fig. 1. Minimizing of the height of the occupied part of the 3D printer working chamber

Let the set of irregular 3D objects T_i, $i \in I_n = \{1,2,...,n\}$, and container $\Omega = \{(x,y,z) \in R^3, 0 \le w_1 \le x \le w_2, 0 \le l_1 \le y \le l_2, 0 \le h_1 \le z \le h_2\}$ be given. Here h_1 and h_2 are variable. Denote the container Ω of variable sizes by $\Omega(h_1,h_2)$.

Each object T_i is presented by a union of convex polyhedra

$$T_i = \bigcup_{k=1}^{n_i} T_{ik},\ i \in I_n,$$

where T_{ik} is defined by the collection of vertices $\{p_{ik}\}$.

Layout of T_i in R^3 determined by the translation vector $v_i = (x_i, y_i, z_i)$ and the vector of rotation angles $\theta_i = (\alpha_i, \beta_i, \gamma_i)$, $i \in I_n$ Thus, vector $u_i = (v_i, \theta_i)$ determines placement of P_i in the tree-dimensional space R^3.

Further object T_i, translated on the vector v_i and rotated by angles α_i, β_i, γ_i is denoted by $T_i(u_i)$.

Optimization layout problem. Find vector $u = (u_1,...,u_n)$ that provides layout of objects $T_i(u_i)$, $i \in I$, inside the container $\Omega(h_1, h_2)$ so that the height $H = h_2 - h_1$ will reach the minimum value.

4 Mathematical model and its properties

Using the phi-function technique [16-22] a mathematical model of the optimization layout problem can be presented as the following nonlinear programming problem:

$$\min_{X \in W} H \,, \tag{1}$$

$$W = \{X \in R^m : \Phi'_{ij}(u_i, u_j, u'_{ij}) \geq 0, i < j \in I_n, \Phi_i(u_i, h_1, h_2) \geq 0, i \in I_n, \; h_2 - h_1 \geq 0\} \,, \tag{2}$$

where $X = (h_1, h_2, u, u')$, $\Phi'_{ij}(u_i, u_j, u'_{ij})$ is the quasi phi-function for polyhedra T_i and T_j [18, 21], $u' = (u'_{ij}, i < j \in I_n)$, u'_{ij} is the vector auxiliary variables for the quasi phi-function $\Phi'_{ij}(u_i, u_j, u'_{ij})$, $\Phi_i(u_i, h_1, h_2)$ is the phi-function for objects T_i and $\Omega^* = R^3 \setminus int\Omega$.

The inequality $\Phi'_{ij}(u_i, u_j, u'_{ij}) \geq 0$ provides non-overlapping T_i and T_j and inequality $\Phi_i(u_i, h_1, h_2) \geq 0$ provides containment of T_i into Ω.

The problem (1)-(2) is an exact formulation of the optimization layout problem of 3D objects.

The feasible region *W* of the problem (1)-(2), in the general case, is a disconnected set, and each of its connected components is a multiply connected.

5 Solution approach

Our solution approach is addressed to the placement of non-convex continuously rotated objects. To construct feasible starting points the clustering algorithm is proposed. Local optimization is performed using the IPOPT code combined with the decomposition strategy. To search for local extrema, a multistart strategy is used.

Firstly we solve the problem of clustering of pairs of 3D objects into optimized containing spheres or cuboids. Then depending on the shape of clusters auxiliary subproblems of packing cuboids or spheres are solved, employing the clusters homothetic transformations. This allows constructing fast feasible starting points.

The reduction of computational costs is also facilitated by the fact that the process of finding a local extremum of the problem is divided into two stages: solving NLP subproblems by fixing the rotation angles and solving NLP subproblems allowing free object rotations. In addition, the strategy of finding an approximation to the global extremum is used.

As an approximation to the global minimum of the optimization layout problem (1)-(2) the best local minimum found by our approach is considered.

5.1 Generation of feasible starting points

In order to generate a feasible starting point for problem (1) - (2) we use the following algorithm. Firstly, pairs of non-overlapping objects are placed into containing regions (cuboids or spheres) of the minimum volume. Then we solve the problem of packing the set of the obtained clusters into the container (cuboid) of minimum height. This algorithm returns feasible placement parameters for each polyhedron. To compute rotation angles of each of polyhedra the following algorithm is proposed.

The set of objects T_i, $i \in I_n$, is divided into k groups. Each group involves l_k identical polyhedra.

Each object T_i is contained into the sphere S_i of minimum radius r_i^*, using the following NLP subproblem:

$$r_i^* = \min_{(v_i, r_i) \in D_i \subset R^4} r_i \ , \ \ i \in I_n,$$

$$D_i = \left\{ (v_i, r_i) \in R^4 : \Psi_{ij} = r_i^2 - (x'_{ij} - x_i)^2 - (y'_{ij} - y_i)^2 - (z'_{ij} - z_i)^2 \geq 0, j \in J_i \right\}.$$

Denote a local minimum point of the subproblem by (v_i^*, r_i^*). Then each object T_i is translated by the vector v_i^*.

Further $C_n^2 + n$ subproblems of packing the objects T_i, $i \in I_n$, into cuboid Ω_{ij} of minimum volume D_{ij}^C are solved:

$$r_i^* = \min_{(u_i, u_j, h_1, h_2) \in W_{ij} \subset R^{18}} D_{ij}^c(h_1, h_2), \qquad (3)$$

$$\begin{aligned} W_{ij} = \{ & (u_i, u_j, h_1, h_2) \in R^{18} : \Phi_{ij}(u_i, u_j) \geq 0, \ \Phi_i(u_i, h_1, h_2) \geq 0, \\ & \Phi_j(u_j, h_1, h_2) \geq 0, \ F(h_1, h_2) \geq 0 \} \end{aligned} \qquad (4)$$

where $i < j \in I_n$,

$$D_{ij}^c(h_1, h_2) = (h_2 - h_1)(w_2 - w_1)(l_2 - l_1), \ \ F(h_1, h_2) = \min\{h_2 - h_1, w_2 - w_1, l_2 - l_1\}.$$

The inequality $\Phi_{ij}(u_i, u_j) \geq 0$ implies that $\operatorname{int} T_i \cap \operatorname{int} T_j = \varnothing$, while the inequalities $\Phi_i(u_i, h_1, h_2) \geq 0$ and $\Phi_j(u_j, h_1, h_2) \geq 0$ guarantee the arrangement of T_i and T_j fully inside containing region Ω_{ij}.

Next we solve the layout problem of subset of clusters Q_i, $i \in M$, inside the cuboid Ω of minimum height.

Now the problem (1)-(2) is reduced to the following NLP model:

$$\min_{(\tilde{u},h_1,h_2)\in\tilde{W}\subset R^{6\mu+6}} H(h_1,h_2), \quad (5)$$

$$\tilde{W}=\{(\tilde{u},h_1,h_2)\in R^{6\mu+6}:\Phi_{ij}(\tilde{u}_i,\tilde{u}_j)\geq 0, i<j\in M, \Phi_i(\tilde{u}_i,h_1,h_2)\geq 0, \\ i\in M, h_2-h_1\geq 0\}, \quad (6)$$

where $H(h_1,h_2)=h_2-h_1$.

Let the point $(\tilde{u}^*,h_1^*,h_2^*)\in R^{6\mu+6}$ be an approximation to the global minimum point of the problem (5) - (6). The point corresponds to packing clusters $Q_i(u_i^*)$, $i\in M$ into cuboid $\Omega(h_1^*,h_2^*)$. Each cluster Q_i. contains the pair of polyhedra T_{k_i} and T_{t_i} with placement parameters $u_{k_i}^Q$ and $u_{t_i}^Q$ in the local coordinate system of the cluster Q_i.

In order to construct a feasible point $(u^0,h_1^0,h_2^0)\in W$ of the problem (1) - (2) regarding the arrangement of clusters Q_i, $i\in M$, we set the arrangement of object T_i using the equation $v_i^0=\tilde{v}_i^*+v_i^Q$ for $i\in I_n$.

To define the rotation angles θ_i^0 of polyhedra T_i, $i\in I_n$, we solve the sequence of n subproblems of the following form:

$$r_{13}^{i*}=\min_{R_i\in D_i\subset R^9} r_{13}^i, \quad (7)$$

$$D_i=\{R_i\in R^9: V_1^iR^i=\tilde{V}_1^i,\ V_2^iR^i=\tilde{V}_2^i, V_3^iR^i=\tilde{V}_3^i,\ \sum_i r_{ij}r_{ik}=\delta_{jk}, \\ \sum_i r_{ji}r_{ki}=\delta_{jk}, i=1,2,3\} \quad (8)$$

where V_1^i, V_2^i, V_3^i are vectors of initial coordinates of the first three vertices of the polyhedron P_i, $\tilde{V}_j^i=\tilde{R}_i^*(R_i^QV_j^i+v_i^Q)$, $j=1,2,3,$ R_i is the rotation matrix, $i\in I_n$. Let r_i^* be a solution of the problem (7) - (8). Then the angles of T_i can be derived in the form: $\beta_i=\arcsin r_{13}^{i*}$, $\alpha_i=\arcsin(-r_{23}^{i*}/\cos\beta_i)$, $\gamma_i=\arccos(-r_{12}^{i*}/\cos\beta_i)$.

5.2 Local optimization

To find a local extremum of the problem (1)-(2) the following algorithm is used. This algorithm allows reducing CPU.

The feasible region of the problem (1)-(2) can be always represented by a union of subregions (see e.g. [21]). It enables to search for a local minimum of the problem (1)-(2) by solving a collection of NLP subproblems with a considerably smaller number of inequalities.

The key idea of the proposed algorithm is based on the decomposition strategy (see, e.g. [23]). The large scale problem (1)-(2) is reduced to a sequence of subproblems of smaller dimension. The following stages are performed:

- generating feasible subregions of the feasible region (2) related to the appropriate starting points;
- forming the system of $\varepsilon-$active constraints;
- searching for local extrema of the subproblems generated at the first step, employing state-of-the-art NLP-solvers;
- replacing subregions.

Now we consider the algorithm in detail.

Let the point $X^{\bullet} \in W$ be a starting point. Then we select an appropriate subregion W_0, such that $X^{\bullet} \in W_0 \subset W$ and substitute the point $X^{\bullet}$ in the inequality system (2). Each quasi phi-function has the form

$$\Phi'_{ij}\left(u_i, u_j, u'\right) = \max\left\{\Psi^s_{ij}\left(u_i, u_j, u'\right), s = 1, \ldots, \wp_{ij}\right\}.$$

Then we select one of the functions $\Psi^{a_{ij}}_{ij}\left(u_i, u_j, u'\right)$, $a_{ij} \in \left\{1, \ldots, \wp_{ij}\right\}, i < j \in I$, such that

$$\Phi'_{ij}\left(u^{\bullet}_i, u^{\bullet}_j, u'\right) = \Psi^{a_{ij}}_{ij}\left(u^{\bullet}_i, u^{\bullet}_j, u'\right) = \chi^{\bullet}_{ij}.$$

Similarly we choose $\Phi_i\left(u_i, u_\Omega\right) \geq 0, i \in I$. It results in the system of inequalities $\Upsilon^0\left(X\right) \geq 0$ describing the subregion W_0. Then the subproblem

$$F\left(u^{0*}_\Omega\right) = \min_{X \in W_0 \subset R^{\vartheta}} F\left(u_\Omega\right)$$

is solved. The inequality system $\Upsilon^0\left(X^{0*}\right) \geq 0$ distinguishes the active inequality $\varsigma_{j0}\left(\xi^{0*}_j\right) \geq 0, \quad j \in \Gamma_0 = \{1, ..., \mu_0\} \subset \Gamma = \{1, ..., \mu\}$. Denote the subsystem by $\Psi^a_{ij}(u_i, u_j) \geq 0, \;\; i \in I_{0\eta_1} \subset I, \;\; j \in I_{0\eta_2} \subset I$. This allows choosing quasi phi-functions $\Phi'_{ij}\left(u_i, u_j, u'\right)$ that involve functions $\Psi^a_{ij}(u_i, u_j)$, for $i \in I_{0\eta_1}, \;\; j \in I_{0\eta_2}$.
Then we calculate the values of the functions at the point X^{0*}.
Let

$$\Phi'_{ij}\left(u^{0*}_i, u^{0*}_j\right) = \Psi^c_{ij}(u^{0*}_i, u^{0*}_j) = \chi^0_{ij}, \;\; i \in I_{0\eta_1}, \;\; j \in I_{0\eta_2}.$$

If $\chi^0_{ij} > 0, \;\; i \in I_{0\eta_1}, \;\; j \in I_{0\eta_2}$ then replace subsystems $\Psi^a_{ij}(u_i, u_j) \geq 0$ by systems $\Psi^c_{ij}(u_i, u_j) \geq 0, i \in I_{0\eta_1}, \;\; j \in I_{0\eta_2}$. Thus a new subsystem of inequalities defining a new subregion $W_1 \subset W$ is generated. Obviously, $X^{0*} \in W_1$.

Taking the starting point X^{0*}, we solve the problem

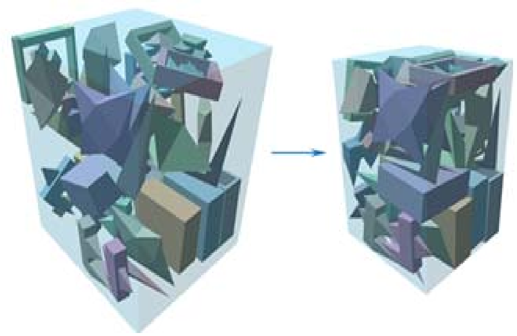
$$F\left(u_{\Omega}^{1*}\right) = \min_{X \in W_1 \subset R^m} F\left(u_{\Omega}\right),$$

and search for a local minimum point X^{1*}.

The computational process is repeated until $F\left(u_{\Omega}^{(\rho-1)*}\right) = F\left(u_{\Omega}^{\rho*}\right)$.

The search for a local minimum of the problem (1) - (2) can be divided into two stages: optimization of the system with linear constraints and nonlinear optimization. The first stage is realized by fixing the rotation angles θ_i^0 of objects T_i, $i \in I_n$ at the feasible starting point $(u^0, u_{\Omega}^0) \in W$. Fixing rotation angles significantly reduces the dimension of the problem (1) - (2) switching to the linear constraints to describe the feasible region.

Figure 3 depicts layout of irregular 3D objects that corresponds to a) a feasible starting point and b) the appropriate local minimum found by our algorithm.



a) **b)**

Fig. 1. Example of layouts of irregular objects corresponding: a) a starting point; b) a local minimum point

6 Computation experiments

We present some examples to demonstrate the efficiency of our methodology. We have run all experiments on an Intel I5 2320 computer, programming language C++, Windows 10 OS. To solve NLP problems IPOPT [24] is used, which is available at an open access software depository (https://projects.coin-or.org/Ipopt).

Figure 2 demonstrates some benchmark examples of irregular layouts obtained by our approach.

In order to show the efficiency of our approach a number of benchmarks instances given in [12] are tested. The results are shown in Table 1.

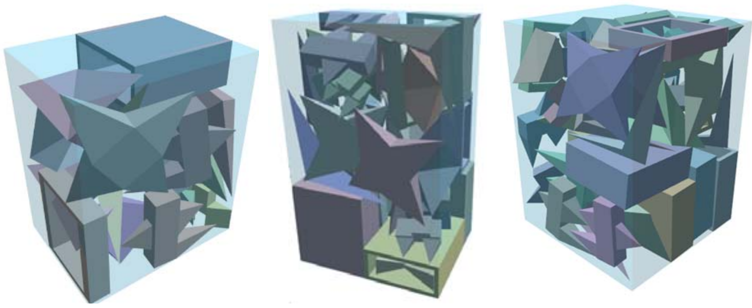


Fig. 2. Examples of 3D irregular object layouts

Table 1. Comparison of our results with those publised in [12]

Approach	*HAPE3D*	*Our algorithm*
The result of packing 20 irregular 3D objects		
Volume	32550	28500
Runtime (sec)	26202	6656
The result of packing 30 irregular 3D objects		
Volume	12480	10720
Runtime (sec)	9637	4789
The result of packing 36 irregular 3D objects		
Volume	48300	42450
Runtime (sec)	53741	9543
The result of packing 40 irregular 3D objects		
Volume	61950	56012
Runtime (sec)	99952	24543
The result of packing 50 irregular 3D objects		
Volume	77280	71800
Runtime (sec)	125210	36543

7 Conclusions

The 3D-printing procedure using SLS technology takes a long time (many hours or even days) and requires a great financial cost associated with: the printer running, the camera heating and the temperature stabilization. Development of the optimization techniques allowing saving time and material is of paramount importance.

The optimization problem of layout of irregular 3D objects into cuboid of minimum height is formulated. The mathematical model is constructed, using the phi-function technique. The solution strategy is proposed. To demonstrate the efficiency of our methodology some instances are provided. Obtainment of optimized layouts of 3D objects makes possible reducing the printing cost by minimizing the number of layers of 3D printing and therefore reducing the number of the printer starts.

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