

# Optimal Strategy for the Development of Insurance Business Structures in a Competitive Environment

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**Abstract.** This paper explores the application of the game theory to the problem of formation of optimal strategy for the development of insurance business structures in a competitive environment and demonstrates a class of optimization models that could be employed for insurance portfolio optimization. The designed game theory model aims to develop the optimal strategy was implemented at the insurance company, which offers various kinds of insurance. A model designed to analyze a set of insurance products will be organized to follow the costs spent by the insurance business on the provision of the insurance services and income received by the insurance company for the provision of the insurance services. The task included the calculation of the company's optimal insurance portfolio with the aim of receiving a maximum income from the developed insurance products realization. The game model enables to define the percentage ratio of the providing of the insurance services at the insurance business, considering the conditions of the insurance market and behavior of competitors. Performed calculations allow the insurance company's executives to determine the favorable insurance market conditions for certain types of insurance products and to enhance the decision-making process regarding the reduction or the increase of providing of the insurance services.

**Keywords:** game theory, optimal strategy, insurance business, insurance market, insurance portfolio, insurance services.

## Introduction

Latest political, social and economic developments nearing an unforeseen scenario motivate further study of game theory approaches for insurance portfolio

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optimization. Insurance portfolio optimization and cost savings - these are the challenges of insurance management of businesses. In order to secure the highest value for assets, insurance company's specialists support in the optimization of insurance portfolios, by structuring the attractiveness of insurance products. The insurance industry is facing a complicated position with regard to insurance portfolio optimization given the raising trend in different types of losses.

Optimization of insurance portfolio is important part of continual improvement of insurance processes. Continual improvement it means to improve management of insurance products. Continual improvement is a development for most management systems such as insurance portfolio management, insurance business management, risk management. Continual improvement is a type of modification that is focused on increasing the efficiency of insurance company to fulfil its policy and objectives.

Through optimal parameters of management of insurance products is possible to decrease costs of the insurance company and to economize business. It is the first big step to reduce waste of resources. Quantitative methods are significant tools for insurance management. Choice of appropriate alternative decision of insurance problem and their application in financial practice needs to know conditions of risks and utility of alternative for insurance process in the company.

The insurance market in Ukraine and in the world is a necessary condition for formation of demand for insurance products. In our country, one sees a creation of the national insurance system, decrease in demand for insurance services, reduction of insurance payments and bonuses, number of reinsurers and contracts.

The modern insurance market in Ukraine is currently experiencing not the best time, since it is negatively affected by external and internal factors. Given the instability of the domestic economy and the military conflict in the country, insurance, as an investment instrument, is not developing sufficiently but steadily and demonstrates a tendency for further growth [1].

We consider that in such circumstances the relevance of research requires the selecting those types of insurance that protect the interests of consumers, bring profits to insurance business structures. This is all extremely important for effective risk management, improvement of insurance activities.

A significant number of scientists and economists focus on the research of development of different types of insurance, a ratemaking perspective for a typical property and casualty insurance portfolio [2], estimation of interdependence of insurance loss coverage and demand elasticities [3].

These problem take place in the studying insurance management services, insurance market and are analyzed in the works of [1, 4] and others.

The game theory is used to solve real and current economic problems in different spheres of the economy. Solving economic problems using the practical application of the game theory is still popular in fields of science [5-8].

The new research of formation of insurance services portfolio allow us to enrich the methodological arsenal and to significantly supplement the databases for further A new researches conceptual and practical understanding of game theory in the field of insurance. This determined the choice of research topic.

The game theory in the field of insurance gives the opportunity to formulate optimal logical models of behavior of all participants.

Game theory provides a framework for analyzing a specific interaction of agents (insurance business structures) participating in the insurance services market and predicting the behavior of units within the economy.

The uncertainty of the result becomes an indispensable feature of every conflict situation. It allows to predict the future directions of company's development, taking into account external and internal factors (existing internal reserves of insurance companies, tendencies of development of the insurance sector, etc.).

Therefore, the psychology of actions of all participants is characterized by some complexity. They are governed by the following aspects: self-interest, goals, assessment of their status and chances to win, evaluation of the competitor's interests, realistic view of the competitor's goals and motivations, and overall results.

The reason for the implementation of game theory in the processes in the insurance industry is the complexity of formalizing tasks and presenting conflicts through the use of game forms. In this case, the formalization of conflicts has the following characteristics: players (insurance business structures (companies), consumers, insurance market); a set of strategies for all players; the matrix of wins, the result of the game, the degree of personal satisfaction of the participants.

In this regard, understanding of the development of insurance business, its products and services on the basis of economic and mathematical modeling in conditions of instability and uncertainty is becoming increasingly relevant.

However, today there is no published research on the possibility of implementation economic and mathematical models that attempts to optimize (maximize or minimize) the provision of insurance services. It is important for insurance companies to develop the economic and mathematical model of formation of insurance services portfolio.

The designed game theory model based on the finite antagonistic games theory aims to develop the optimal strategy was implemented at the insurance company, which offers various kinds of insurance. A model designed to analyze a set of insurance products will be organized to follow the costs spent by the insurance business on the provision of the insurance services and income received by the insurance company for the provision of the insurance services. The task included the calculation of the company's optimal insurance portfolio with the aim of receiving a maximum income from the developed insurance products realization.

## **Literature Review**

Especially interesting is the application of the game theory in the field of insurance. Concepts of the game theory have a growing importance in sustainable development economics and sciences related to effective decision-making processes. The application of the game theory in the field of insurance is fascinating. Research interest mostly concentrated on strategy and development insurance companies concerns, including the issues of formation of optimal insurance portfolio.

Many scientists are investigating optimization problems arising in the insurance market framework and creating mathematical models in insurance theory. Portfolio optimization is a central topic in financial mathematics.

So, Bäuerle, Rieder [9] prove that portfolio optimization with stochastic market data is more realistic than standard models with constant coefficients. The formulation of the market condition as a continuous-time Markov chain makes the analysis simpler as in the case of a driving diffusion. For the utility functions treated here, the maximal portfolio value can be computed as a solution of a simple linear differential equation. More complicated is the case of benchmark optimization.

In a portfolio optimization problem, Hillairet, Jiao [10] considered a finite family of investable assets whose prices are described by a stochastic process with a positive real number, which denotes the time horizon. And an investment strategy is described by a stochastic process.

In work of Lwin, Qu, MacCarthy [11], an efficient learning-guided hybrid multi-objective evolutionary algorithm (MODE-GL) is proposed to solve mean-VaR portfolio optimization problems with real-world constraints such as cardinality, quantity, pre-assignment, round-lot and class constraints. A learning-guided solution generation strategy is incorporated into the multi-objective optimization process to promote efficient convergence by guiding the evolutionary search towards promising regions of the search space. The proposed algorithm is compared with the Non-dominated Sorting Genetic Algorithm and the Strength Pareto Evolutionary Algorithm. Experimental results using historical daily financial market data from S & P 100 and S & P 500 indices are presented. The results show that MODE-GL outperforms two existing techniques for this important class of portfolio investment problems in terms of solution quality and computational time. The results highlight that the proposed algorithm is able to solve the complex portfolio optimization without simplifications while obtaining good solutions in reasonable time and has significant potential for use in practice.

The idea of another study [12] is to describe how modern insurance companies use catastrophe model results to optimize their selection of risks in an insurance or reinsurance portfolio. An optimization procedure is described based on both, simulated annealing that takes care of second-order minima as well as the computationally less expensive hill climbing method, that is likely to produce suboptimal results in case of complex portfolios with several second-order optima.

Zieling, Mahayni, Balder [13] use S&P 500 index return data for the time period 1985–2013 to evaluate the performance of portfolio insurance strategies. They shed light on the question if the performance of a constant proportion portfolio insurance (CPPI) strategy can be improved by means of a time-varying multiplier which depends on the estimated future volatility. It turns out that even time-varying multiple strategies based on a rolling window of historical volatility estimates give a significant improvement of CPPI strategies.

Guan, Liang [14] study the stochastic Nash equilibrium portfolio game between two pension funds under inflation risks. Dynamic programming method is employed to derive the Nash equilibrium strategies. A numerical analysis is presented to reveal the economic behaviors of the two DC pension funds.

The theory of games [15-17] for the formation of insurance contracts, the implementation of auction tenders and design of auctions, cooperative agreements have been successfully applied.

Scientists [18-19] believe that a guideline for development of the insurance industry should be an increase in the use value of insurance products. This greatly improving customer satisfaction by meeting the insurance protection requirements. Considering the development of insurance products, Farny [20] said that the systemic concept of insurance relations is important. Insurance is a special relationship between an insurer and an insured, and the usefulness of the insurance product is determined on the basis of trust to the customer experience

Concepts of games theory were aimed at determining the abilities of market participants to estimate the expected costs [21], taking into account the mathematical models used to measure the economic activity of insurance companies and auctions [22-23]. The winner's curse theory [22] suggests that an insurer who bids the lowest price and wins the business is likely to have underestimated the cost and therefore is likely to be cursed by less profit than expected.

Analyzing the impact of insurance cycles on an insurer's surplus, probability of occurrence of the event insured and potential losses, and using mathematical modeling Dagg [24], Boor [25], Zhang, Chen [26] have been explored the formation of strategies that allow saving market share and capital.

A noncooperative game to model a non-life insurance market have been formulated Dutanga, Albrecherb, Loisel [23]. The effects of competition between insurers through different indicators (the market premium, the solvency level, the market share and the underwriting results) has been analyzed.

Lemaire [27] emphasizes the role and meaning of the basic concepts of cooperative game theory, at an elementary level, including insurance applications for the insurance industry.

In the works of Eid, El-Adaway, Coatney [28] there was formed the recommendations for mitigating the financial impacts associated with natural disasters.

Backovic, Popovic [29] focuses attention on the importance of analyzing of game theoretic application to mutual insurance model, with hierarchical structure in agent positioning, in conditions of both full and partial information obtained by the player, with the goal of finding equilibrium strategies.

A group of scientists [30] are dealing with the set of Pareto optimal insurance contracts and the core of an insurance game. Closed-form and numerical solutions are found for various preferences that the insurance players might have. They showed how one may find robust and Pareto efficient contracts, which is a key decision-making problem under uncertainty.

The classic models of management for insurance companies include the pursuit of strategic goals of insurance companies, the profit maximization and cost minimization. To maximize the benefits and return on investment, managers of companies must also assist in the identification and development of new insurance products.

## **Methodology**

Improving the insurance process focused on establishing conditions of insurance company's activity. This context is defined by a set of organizational measures directly linked to the strategy of insurance business and the current dynamics of increasing instability and growing ambiguity of the environment. These processes can be quantified and simulated by a systematic approach to the analysis of business processes in insurance company.

The insurance price is an important means of selling insurance products. For insurance business structures, competition on the domestic insurance market is a major threat as the Pricing factors are extremely important in the insurance sector. This is partly the result of the high degree of sensitivity of the population to the established prices for insurance products.

The price of the insurance product, of course, depends on the insurance rate, which is established by laws of competition under market economy conditions.

The insurance rate is characterized by structure, with attributes such as wages for insurance employees, cost of doing business, costs of preventive measures (coverage of the expected claims in the course of the insurance period, establishment of insurance reserves), contribution to the investment returns. Elements of the insurance rate provide financing for all actions and functions of the insurance company.

For the correct calculation of the insurance rates company can fulfill all the obligations required, cover their own cost of insurance and receive a large profit. Excessive insurance rates in comparison with the probability of emerging risks will prevent from signing of the insurance contracts with customers and reduce the company's competitive ability in the insurance market.

One of the main goals of the development of insurance companies is to maximize profits and the end depends on rising prices for insurance products, beneficial use of the insurance market conditions, reducing costs for the development and marketing of insurance products.

The achievement of the established goals of the development of insurance companies leads to a search for a development strategy. The development strategy should take into account the complex of resource support, the system of factors of the external environment, the state of the economy of the country and the insurance market. Different insurance services provided by insurance companies have different profitability, which depends on the cost of providing these services and losses.

An important landmark of the development of insurance companies is a search for a development strategy for the achievement of the goals established. The development strategy should take into account the complex of resource support, the influence of external factors, the state of the country's economy and insurance market. Different insurance services provided by insurance companies have different profitability, which depends on the costs of providing these services and losses.

At the same time, various insurance services are in some sort of conflict in formation of an insurance portfolio. Such a situation would be covered by the modeling and optimization from the use of games theory tools. And the behavior of

any insurance company cannot be set forth in advance, as well as potential behavior of all participants in the insurance market.

In article, we will explore the task of the formulation of strategies for the development an insurance company that offers different insurance products for consumers. The sale of insurance products could provide an appropriate financial benefit to the insurance company in the insurance market. The profitability of the insurance organization as a result of the provision of insurance services depends on many factors: provision of resources (financial (insurance premiums, equity, insurance reserves, etc.), organizational, personnel, information, technological), types of services rendered, conditions for the execution of contracts, possible amount of financial losses, etc.

Insurance services are characterized by different levels of profitability. The balance between the objectives of the insurance company and resources is based on the choice of insurance services options that is shown in minimizing contradictions among price, quality of the insurance service, and profit for the organization. The formation of insurance services is based on an understanding of such economic processes as the lack of adequate funds to coverage of losses, underestimation of risks and profit.

Under such conditions, for each insurance company, the task is to create a set of insurance products (insurance portfolio), for which benefits and costs could be equal and balanced with the cost of providing insurance coverage.

Taking into account the criterion of maximizing benefits and minimizing costs from insurance activities, the formation of optimal strategies for an insurance company will be represented using a matrix game ( $T$ ) [31]:

$$T = \langle A, B, S \rangle, \quad (1)$$

$A$  is the set of possible measures (action) and change in demand for the insurance market;

$B$  is the set of possible measures (action) of insurance company for offering insurance;

$S$  is the utility function of the insurance company, which characterizes the multiplication  $A \times B$  of couples  $(a, b)$ ,  $a \in A, b \in B$ .

The solving of the matrix game is based on the idea that players (insurance companies) choose certain pure strategies  $A$  and  $B$  independently of each other. This allows us to formulate some equilibrium situations  $(A, B)$ . Since the number of possible measures (action) of each player (insurance company) is finished, one can assume that:

$$\begin{aligned} A &= \{1, \dots, m\}, \\ B &= \{1, \dots, n\}, \end{aligned} \quad (2)$$

$m, n$  are respectively, the number of pure strategies of the insurance company and insurance market.

We will assume that the number of possible strategies of the insurance company and insurance market are the same, that is:  $m = n$ .

In this case, the value of the function  $S$  must be represented as a matrix:

$$S = \|s_{ij}\|, 1 \leq i \leq m, 1 \leq j \leq n \quad (3)$$

On the basis of this matrix, we will find optimal strategies for the matrix game. In this matrix, an insurance company and an insurance market that has an impact on the amount of insurance company's expenses and income is considered as a conflict point.

We assume that  $c_i$  ( $1 \leq i \leq m$ ) is the income received by the insurance company from the provision of insurance services, and  $o_i$  is the costs incurred by the insurance company in the course of its business.

In this case, the matrix form of  $S$  wins will take the following form:

$$S = \begin{pmatrix} c_1 & -o_1 & \dots & -o_1 \\ -o_2 & c_2 & \dots & -o_2 \\ \dots & \dots & \dots & \dots \\ -o_m & -o_m & \dots & c_m \end{pmatrix} \quad (4)$$

Let's carry out the transformation of the matrix  $S$  (4) to a form that would make it possible to simplify the process of seeking strategies for the development of an insurance company.

To this end, the first row of the matrix  $S$  (4) is multiplied by the number  $d_1$ , the second row multiplied by the number  $d_2$ , the third row multiplied by the number  $d_3$ , etc. In this case, the following conditions must be fulfilled [31]

$$d_1 \cdot c_1 = d_2 \cdot c_2 = \dots = d_m \cdot c_m = c \quad (5)$$

Next we carry out the following actions in the modified matrix  $S$  (4): subtract the  $c$  value from all components. As a result, we get the matrix  $S^*$ :

$$S^* = \begin{pmatrix} 0 & -f_1 & \dots & -f_1 \\ -f_2 & 0 & \dots & -f_2 \\ \dots & \dots & \dots & \dots \\ -f_m & -f_m & \dots & 0 \end{pmatrix}, \quad (6)$$

where

$$f_i = d_i \cdot o_i + c. \quad (7)$$

Let's form the rows in the matrix  $S^*$  (6), taking into account the condition (8)

$$f_1 > f_2 > \dots > f_m > 0 \quad (8)$$

Two matrices  $S^*$  and  $S$  are equal to each other (equivalent). Thereby, all the actions performed by us for the transformation of the matrix  $S$  (4) do not change the entire set of optimal strategies for the market participants. Therefore, the value of the matrix game  $W^*$  and  $W$  are equivalent.

Now we will calculate the optimal strategies for forming the insurance portfolio of the company and the insurance market, which is given by the matrix  $S^*$ .

Let's consider the situation when  $m$  strategy of the insurance market is optimal. In this case, the value of the matrix game  $W$  (win) will be determined:

$$W = -f_m, \quad (9)$$

because

$$S^*(m, 1) = -f_m. \quad (10)$$

Let  $B = (\delta_1, \delta_2, \dots, \delta_m)$  be an insurance company's strategy. If we have the case of equilibrium, then the mathematical expectation of win of the insurance market  $S^*(m, B)$  should be equal to  $-f_m$ . So,

$$S^*(m, B) = -f_m \sum_{i=1}^{m-1} \delta_i = -f_m (1 - \delta_m) = -f_m, \quad (11)$$

we get  $\delta_m = 0$ .

As a result of optimality  $B$ , conditions must be met. These conditions will be equal to the following inequalities:

$$S^*(j, B) = -f_j (1 - \delta_j) = -f_m, \quad 1 \leq j \leq m-1, \quad (12)$$

$$\delta_i \leq 1 - \frac{f_m}{f_i}, \quad 1 \leq i \leq m-1 \quad (13)$$

In the case of adopting a strategy  $B$  for which the conditions (13) and (12) are given and fulfilled, equality (9) will be satisfied. And such a strategy is optimal for an insurance company that operates in an unstable market environment under uncertainty. Therefore, the conditions are necessary and sufficient to be fulfilled in order to achieve the goal

$$1 \leq m-1 - f_m \sum_{i=1}^{m-1} \frac{1}{f_i}, \quad (14)$$

or inequality that is equivalent to this condition

$$0 \leq m - 2 - r_m \sum_{i=1}^{m-1} \frac{1}{f_i}. \quad (15)$$

This statement is a consequence of the condition (11). The inverse statement will also be correct because

$$0 \leq \frac{1 - f_m}{f_i} \leq 1. \quad (16)$$

The fulfillment of the condition (14) must be verified at the beginning of the solution of the matrix game (6). If the condition is fulfilled, the value of the game will have the form (9), and the insurance market will have the optimal strategy expressed by the  $m$  row of the proposed matrix (6).

The strategy of an insurance company can be found as follows: to select all the values of  $\delta_i$  and satisfy the condition (14).

If the condition (15), which determines the optimal condition of the activity of an insurance company, is not fulfilled:

$$1 - m + f_m \sum_{i=1}^{m-1} \frac{1}{f_i} > 0, \quad (17)$$

then the insurance market and the insurance company will have optimal strategies in which pure strategies are applied with positive probability.

Optimum strategy  $B$  for an insurance company will satisfy the system of such equations [32]:

$$S^*(j, B) = -f_j(1 - \delta_j) = W, \quad 1 \leq j \leq m \quad (18)$$

Define the value of  $\delta_j$ , based on the system of the proposed equations (18):

$$\delta_j = 1 + \frac{W}{f_j}, \quad 1 \leq j \leq m \quad (19)$$

Summarizing the probability  $\delta_j$  of the strategy  $B$  with respect to  $i$  and taking into account that the sum must be equal to 1, we obtain the following equality:

$$W \cdot \sum_{i=1}^m \frac{1}{f_i} = 1 - m. \quad (20)$$

The final solution of the value  $W$  is obtained from equation (20):

$$W = \frac{1-m}{\sum_{i=1}^m \frac{1}{f_i}} \quad (21)$$

Formulas for calculating the optimal strategy of an insurance company  $\delta_j = (\delta_1, \delta_2, \dots, \delta_n)$  are obtained by solving the equation (16):

$$\delta_j = \frac{1-m + f_j \cdot \sum_{i=1}^m \frac{1}{f_i}}{f_j \cdot \sum_{i=1}^m \frac{1}{f_i}}, \quad 1 \leq j \leq m \quad (22)$$

It follows from (8) and (17) that all values of  $\delta_j$  are positive. The equation (22) defines a mixed strategy that corresponds to the system of the proposed equations (18). The final value of the game is calculated according to the formula (21).

A mixed strategy for the insurance market  $A = (a_1, a_2, \dots, a_m)$  is obtained by the formula:

$$a_i = \frac{1}{f_i \cdot \sum_{j=1}^m \frac{1}{f_j}}, \quad 1 \leq i \leq m, \quad (23)$$

and corresponds to the system of such equations:

$$S^*(A, i) = \sum_{j \neq i} f_j \cdot a_j = W \quad (24)$$

Consequently, the strategies of the insurance company and the insurance market will be optimal in the matrix game  $S(1)$ . In order to obtain the value of a matrix game  $W^*$ , it is necessary to take into account the relations (4), (5), (6), (7) and (21):

$$W^* = \frac{1-m + \sum_{i=1}^m \frac{c_i}{o_i + c_i}}{\sum_{i=1}^m \frac{c_i}{d_i \cdot (o_i + c_i)}} \quad (25)$$

The value  $\delta_j$  will be interpreted as the proportion of services in the insurance portfolio, which should be given greater advantage in the event that the insurance company is able to provide different types of insurance services in the course of its

activities. The probabilities of  $a_i$  should be presented as the shares of the influence of the insurance market on the insurance company activity. The sum of all variants of actions (probabilities) of the insurance company and the insurance market will be equal to units.

## Results of Investigation

Consider the process of strategic management by TAS Insurance Group case. TAS is the insurance company that offers the following services to the individuals: life insurance, health insurance, transport insurance, property insurance, disability insurance. The insurance company offers various kinds of insurance.

Table 1 shows data on insurance products of the insurance company and the income received by the insurance company from the provision of insurance services and the costs incurred by the insurance company in the course of its business.

**Table 1.** The data on the range of insurance services by the company for 2019

№	Type of insurance services	Costs spent by the insurance company on the provision of the insurance services unit ( $o_i$ ), UAH		Income received by the insurance company for the provision of the insurance services unit ( $c_i$ ), UAH	
		2015	2019	2015	2019
1	Life Insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 20 UAH	2270	7950	10000	35000
2	Life Insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 30 UAH	5675	11357	25000	50000
3	Travel Medical Insurance	14	27	60	120
4	Accident Insurance	16	34	70	150
5	Car Insurance	27	57	120	250

Source: the data of TAS Insurance Group

It is always necessary to evaluate the profitability of insurance services for an insurance company. It is necessary to be able to navigate in a market environment, to optimize the insurance portfolio, to identify the most relevant market insurance services.

According to the proposed model, the calculation and evaluation of the strategy of the insurance company development was carried out. The model used data on such

services as life insurance under the two program, travel medical insurance, accident insurance, car insurance.

The results of calculating the insurance company's strategy components  $\dot{a}_3$ , the strategy of insurance market influence  $\delta_3$  and the results of the matrix game (win), which the insurance company receives, are presented in table 2.

**Table 2.** Calculated optimal strategy of the insurance company and the insurance market

№	Type of insurance services	2015		2019	
		Optimal strategy of the insurance company $\dot{a}_3$	Optimal strategy of the insurance market $\delta_3$	Optimal strategy of the insurance company $\dot{a}_3$	Optimal strategy of the insurance market $\delta_3$
1	Life Insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 20 UAH	20,019	19,923	19,994	20,023
2	Life Insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 30 UAH	20,019	19,923	19,994	20,023
3	Travel Medical Insurance	19,916	20,335	20,029	19,883
4	Accident Insurance	19,994	20,026	20,002	19,992
5	Car Insurance	20,052	19,793	19,98	20,079
6	Win of the insurance company	2444,5		5013,6	

Source: own calculation

The game value for the insurance company TAS Insurance Group (income from providing insurance services unit) in 2015 equaled 2444,5 UAH and in 2019 increased to 5013,6 UAH.

In accordance with the values of vector  $\delta_3$ , the state of the insurance market was the most beneficial in 2015 for travel medical insurance (20,335%) and, on the contrary, adverse in case of car insurance sales (20%). In 2019, the most beneficial state of the insurance market concerned the development of car insurance (22%).

In 2015 and 2019, the insurance company had the opportunities to obtain optimal results from the development and providing of car insurance (20,052% in 2015), travel medical insurance (20,029% in 2019).

## Conclusion

Managers in the insurance field are seeking directions to profit growth amid quick financial, social and economic change. No longer can the insurance companies lean on cost cuts alone to boost income. The insurance companies, the chief executives must strike a balance that will promote profit growth, productivity and returns over the long term. Many insurance companies aren't getting the effective results they need from traditional approaches. That's because traditional approach for creating strategy weren't designed for the challenges (optimization of the insurance portfolio) confronting insurance companies today.

Customer demand for travel medical insurance and accident insurance in 2015, as well as for life insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 20 and 30 UAH, car insurance in 2019, was not taken into consideration. Therefore, insurance company got less income from the insurance products sales.

The insurance company executives in 2015 should have decided to increase of travel medical insurance and accident insurance. The performed estimates suggest that there was a deficiency of these insurance products in the insurance market.

This situation was defined in time by the chief executives the insurance company. In 2019, the contrary situation at the insurance company occurred regarding the provision of insurance services. In 2019, this perspective was somewhat improved in the insurance company and the costs spent on the development of travel medical insurance and accident insurance have increased.

In 2019, the wins of the insurance company were maximum, and the ratio of each of the offered types of insurance services was almost optimal: life insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 20 UAH – 19,99%, life insurance under the program «Insurance for permanent disability as a result of an accident» and an insurance premium of 30 UAH – 19,99%, travel medical insurance – 20,03%, accident insurance – 20,0%, car insurance – 19,98%.

Establishing an insurance company's strategy provides an opportunity to create a direction for sustainable development of the company, taking into account maximization of possible profits and optimization of various types of insurance.

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