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Investigation of the Influence of Errors on the Parameters of the Layers of Optical Filters on the Stability of Their Spectral Characteristics

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Abstract

Mathematical modeling is performed in order to analyze the effects of technological errors originated from partial inhomogeneity, variations of the refractive index and the differences in geometric thickness of layers with high refractive index on spectral characteristics of narrow- and wide-band filters. Using a Monte Carlo method the influence of technological errors on optical parameters of the layers and their influences on spectral characteristics of interference filters are investigated and discusses in details.

Keywords: Monte Carlo method, slightly inhomogeneous films, light transmission, multilayer interference coating, the narrowband interference filters, the wideband interference filters.

MSC: 8208; 82B80

1. Introduction

Investigation of physical properties of some new film-formation materials used for interference and high power optics indicate that the formation of optical coating on various substrates can create spartially inhomogeneous films with transitional areas [1-3]. Therefore, the influence of this effect on spectral characteristics of interference filters should be considered. Apart from partial inhomogeneity of layers, spectral characteristics are also influenced by technological errors. In this article the influence of technological errors occurred in layers with high refractive on optical parameters of filters is investigated using the Monte Carlo method [4]. This investigation builds on the assumption that error has a normal distribution. The purpose of this paper is to develop an appropriate mathematical model and to investigate the effects of different technological errors.

The objective of this article is to develop an appropriate mathematical model and to investigate the influence of error originated from partial inhomogeneity of layers with high refractive index and technological errors in layers' parameters on spectral characteristics of narrow-band and wide-band filters. The refraction index of inhomogeneous areas was selected with different step-functions having linear, quadratic, logarithmic, and exponential distributions.

2. Mathematical modeling

When the refractive index n, geometric thickness of a layer d and wavelength λ are selected as parameters, the characteristic matrix of one layer can be written in the following way [5]:

$$M_s(n,d,\lambda) = \begin{Vmatrix} \cos\delta(n,d,\lambda) & -\frac{i}{n}\sin\delta(n,d,\lambda) \\ -in\sin\delta(n,d,\lambda) & \cos\delta(n,d,\lambda) \end{Vmatrix}, \tag{2.1}$$

where $\delta(n,d,\lambda) = \frac{2\pi nd}{\lambda}$.

While identifying the characteristic matrix of partially inhomogeneous structure we will use the theoretical considerations described in [3].

Using the characteristic matrix of the whole layered structure the transmittance coefficient dependent on wavelength λ can be found by following equation:

$$T(\lambda) = \frac{4}{2 + \frac{n_0}{n_s}M_{11}^2(\lambda) + \frac{n_s}{n_0}M_{22}^2(\lambda) + n_0 n_s M_{12}^2(\lambda) + \frac{1}{n_0 n_s}M_{21}^2(\lambda)}, \tag{2.2}$$

where n_0, n_s – refractive indexes of the external medium and substrate, respectively.

The function of quality can be established as follows [6]:

$$F(\overline{n},\overline{d}) = \Big(\frac{1}{L}\sum_{i=1}^{L} T^2(\overline{n},\overline{d},\lambda_{(i)})\Big)^{\frac{1}{2}}, \tag{2.3}$$

where L is the number of points of the net of spectral interval from λ_1 to λ_2, $\lambda_{(i)}$ are the values of wavelengths on the given net.

Fot determination of stability of spectral characteristics of interference filters it is necessary to consider that the error in refraction index values measured for one layer is not higher than ± 0.05 and the deviation in geometric thickness is in range of ± 2 nm. The type of distribution of error values in the given boundaries was not taken into account in the previous study [3]. In this article the normal distribution of errors was considered.

The Monte Carlo method was used to solve the problem (2.1)–(2.3) within several steps. The first step is to input information necessary for calculations including data on the structure of coating, the planned number of experiments k, interval boundaries (a, b) from which parameter of the layer assumes a random value.

The second step is to define that the first layer is considered and that the number of experiment is 1.

The third step is to generate a random number.

The fourth step is to identify parameters of a layer though the obtained random number.

The fifth step is to calculate characteristics of the coating and compute the value of objective function $F_i(x)$ in the experiment No i.

The sixth step is to use calculated $F_i(x)$ value for formation of sums of the following form:

$$\sum_i F_i, \sum_i F_i^2.$$

The seventh step is to check the condition of conducting of the planned number of experiment. If this condition is met, then the numerical characteristics of distribution of the objective functions can be calculated:

$$M = \frac{1}{k} \sum_{i=1}^{k} F_i(x),$$

$$\sigma = \sqrt{\frac{1}{k-1} \Big(\sum_{i=1}^{k} F_i^2(x) - kM^2 \Big)},$$

where M and σ are mathematical expectation and dispersion of a random $F_i(x)$ value.

The eighth step is to move to the next layer.

The ninth step is to check the condition for ending the calculation process.

3. Computing experiment

Let us to consider the influence of partially inhomogeneous areas on spectral characteristics of a narrowband filter (Fig. 1).

As it can seen from Fig. 1, the types of distribution of refractive index influences the deviation of spectral characteristics of 17-layer narrow-band filteroperating at wavelength of 630 nm. This deviation from the ideal case is depending on value of average refractive index of spectral range.

For all types of distributions the increase of average refractive index of the spectral range lead to widening of waoking spectral region to red-shift of $\lambda_{\max}$ value, while transmission half-width $\Delta\lambda_{0.5}$ and bandwidth $\Delta\lambda_{0.1}$ decrease.

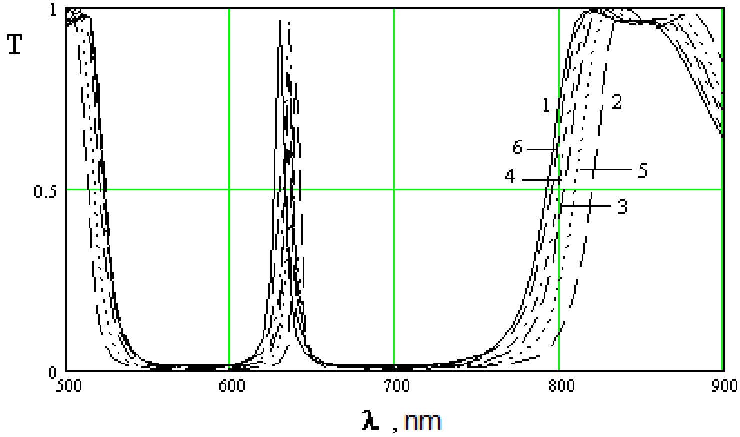

Figure 1: Spectral characteristics of 17-layer structure $S - HLH..2H..HLH$ ($\lambda_0 = 630nm$) in ideal case and in presence of near-surface and transitional areas with different refractive index distributions: 1 - ideal case, 2 - step-function distribution; 3 - linear distribution; 4 - quadratic distribution; 5 - logarithmic distribution; 6 - exponential distribution.

Except exponential distribution of refractive index with increasing the average value of refractive index the transmission coefficient, $T_{\max}$, is increasing in comparison with the ideal case. For exponential distribution of refractive index, the value of transmission coefficient $T_{\max}$ at working wavelengths of 480, 630, 760 and 1000 nm is lower than those found in ideal case.

The increasing of number of layers leads to decrease the deviation of $\lambda_{\max}$ value from ideal case for all types of refractive index distribution. It was found that the transmission coefficient $T_{\max}$ changes less than 10^{-3} during variation of number of layers. Also, the increasing of number of layers results in decreasing of both transmission half-width $\Delta\lambda_{0.5}$ and band-width $\Delta\lambda_{0.1}$. Once again, the higher is the average value of refractive index of spectral range, the lower are their values.

Let us model the stability of spectral characteristics of a $S - BHB \ldots BHB$ type narrow-band filter with regard to possible errors of layers' parameters using the Monte Carlo method. For a 9-layer narrow-band filter the sensitivity of spectral characteristics with regard to variations of refractive index of the second layer is much higher than other variations of layers' characteristics (Fig. 2). For a $(4k+1)$-layer narrow-band filter the spectral characteristics are several times more sensitive to the variations of refractive index of layers with low refractive index than to the variations of refractive index of layers with high refractive index (Fig. 2). The exception is $(2k+1)$- layer which is half-wave one and sensitivity to variations of which is significant too.

The sensitivity of spectral characteristics to errors of geometric thickness of low-

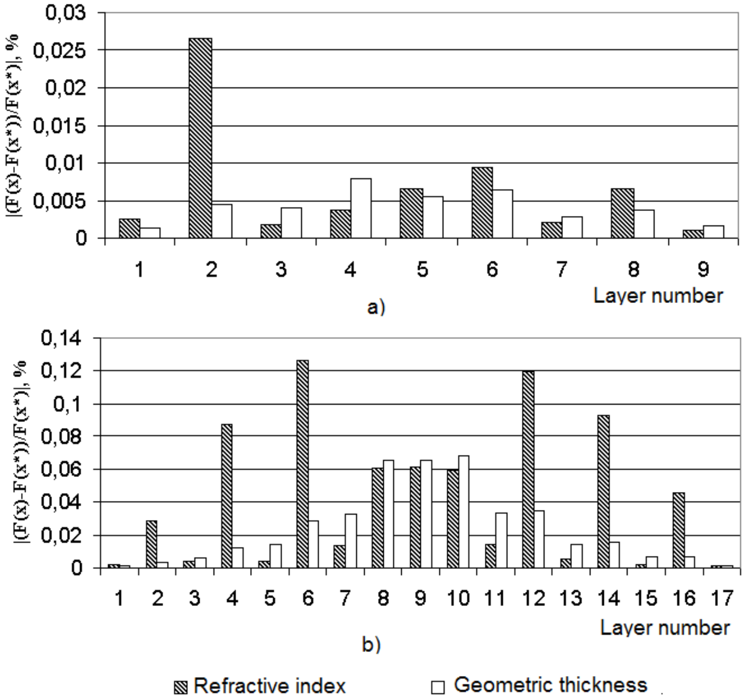

Figure 2: Fig. 2. Diagram of dispersion of the objective function for a $S-HLH..2H..HLH$ type narrowband filter with step-function distribution of refractive index of near-surface and transitional areas at the working wavelength $= 630nm$ based on the results of Monte Carlo simulation: a) 9-layer structure; b) 17-layer structure.

refracting layer is much lower than to errors induced by refractive index changes. The exceptions are $2k$ and $(2k+2)$ layers which are adjacent to the half-wave layer. The sensitivity of spectral characteristics of these layers to errors of the geometric thickness is higher than to the refraction index errors. This also applies to high refracting layers, except for the first and the last layers of multilayered structure of interference filter. For a narrow-band filter, unlike a cutting filter, an average value of refractive index of a spectral range does not influence the stability of spectral characteristics. The highest dispersion range has the step-function distribution of refractive index.

Let us now investigate the influence of partially inhomogeneous areas on spectral characteristics of a wide-band filter (Fig. 3). As can be seen from Fig. 3, for a 17-layer wide-band filter at working wavelength of 630 nm the left boundaries of transmittance range for different distributions of refractive index are not ranked (unlike for a cutting filter) in order of magnitude of average refractive index value of spectral range.

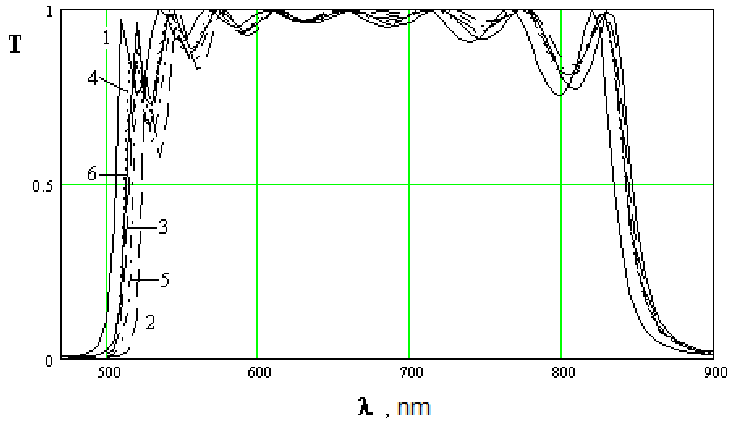

Figure 3: Fig. 3. Spectral characteristics of the 17-layer structure $S - 2BH2B..2BH2B$ ($\lambda_0 = 630nm$) in ideal case and in presence of near-surface and transitional areas with different distributions of refractive index: 1 – ideal case; 2 with step-function distribution of refractive index; 3- with linear distribution of refractive index; 4 – with quadratic distribution of refractive index; 5 – with logarithmic distribution of refractive index; 6 – with exponential distribution of refractive index.

Main characteristics of wide-band filters are bandwidth $\Delta\lambda_{0.5}$, bandwidth $\Delta\lambda_{0.1}$ and the middle of transmittance range λ_{md}. The middle of transmittance range is given by the formula:

$$\lambda_{md} = \frac{\lambda_{0.5}^{L} + \lambda_{0.5}^{R}}{2},$$

where $\lambda_{0.5}^{L}$, $\lambda_{0.5}^{R}$ are left and right boundaries of the transmittance range with the transmittance value of 0.5, respectively.

The middle of the transmittance range of most filters with different distributions of refractive index, λ_{md}, shifts to the long waves area in comparison with ideal case at the value which directly depends on average value of refractive index of spectral range. The exception is the linear distribution of refractive index at working wavelength $\lambda_0 = 480nm$. Except exponential distribution of refractive index, the bandwidths of $\Delta\lambda_{0.1}$ and $\Delta\lambda_{0.5}$ at working wavelengths (λ_0) of 480, 630, 750 and 1000 nm are decreasing when the average refractive index of spectral range increase. The values of main characteristics of filters with exponential distribution of refractive index are higher in comparison with the ideal case. At working wavelength $\lambda_0 = 3000nm$ the deviation from ideal case increase with increasing of the average value of refractive index of spectral range.

The values of main characteristics (middle of transmittance range λ_{md} and bandwidth $\Delta\lambda_{0.5}$) is increasing with increasing of number of layers. The bandwidth $\Delta\lambda_{0.1}$ for 17 layers is higher than for 25 layers. This is due to more rapid decreasing of transmittance coefficient at the boundaries of the transmittance range.

The role of small changes of layers' parameters with different distributions of refractive index on stability of spectral characteristic of the $S-2BH2B\cdots 2BH2B$ type wideband filter were studies using the Monte Carlo method.

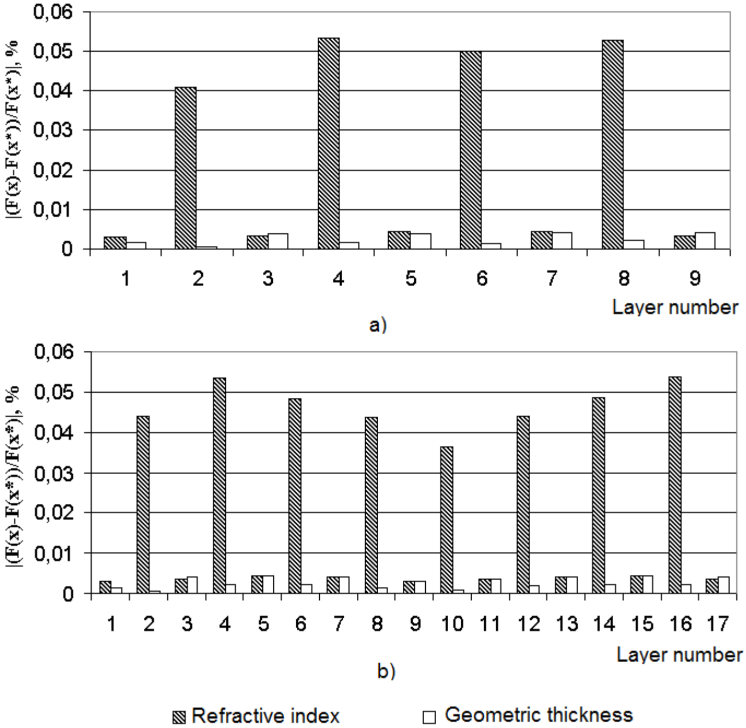

Figure 4: Fig. 4. Dispersion diagram of objective function for $2BH2B\cdots 2BH2B$ type wideband filter with the exponential distribution of refractive index of near-surface and transitional areas at the working wavelength $\lambda_0 = 630nm$ obtained using the results of Monte Carlo simulation: a) 9-layer structure; b) 17-layer structure.

As can be seen from Fig. 4, the sensitivity of spectral characteristics to errors of refractive index of even layers for both a wide-band and a narrow-band filters are much higher than to errors of other parameters.

The influence of error in geometric thickness of odd layers on stability of spectral characteristics of filters is approximately at the same level as those induced by error in refractive index but more than error in geometric thickness of even layers.

The dispersion range does not always increase with the increasing of number of layers. However, in general, the increasing trend is maintained: for the 9-layers structure the maximal value of dispersion range is 0.052, for the 17-layers – 0.054, for 25-layers – 0.053 and for 33-layers– 0.089. Generally, the spectral characteristics of a wide-band filter are more stable to errors in layers' parameters than spectral characteristics of a narrow-band filter.

4. Conclusion

The detailed analysis of results indicate that the stability to different technological errors should be taken into account during development of interference filter structures. Among few structures of interference filters which set roughly similar spectral characteristics the more stable to errors should be preferred for practical implementation. The practical implementation of Monte Carlo method faces two problems. The first problem regards the necessity to conduct the large number of experiments since the statistical error of M and σ values decreases very slowly (inversely proportional to square root of number of experiments). The second problem relate with the retrieving information on the law of distribution of errors in defining layers' parameters. This particularly applies to refractive index of layers, which value depends on numerous technological parameters including substrate and evaporation temperatures, evaporation and condensation rates, residual gas in coating chamber, presence or absence of oxidizing medium, cleanness of raw materials and other parameters.

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