Query Answering in Fuzzy DL-Lite with Graded Axioms (Extended Abstract)*

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Fuzzy description logics have been studied as extensions of classical description logics (DLs) [1] capable of dealing with imprecision and vagueness. In fuzzy logics, vague concepts are modelled by fuzzy predicates whose interpretation extends the usual truth degrees 0 (*false*) and 1 (*true*) to allow any value from the unit interval $[0, 1]$. To handle these multiple truth degrees, the interpretation of the logical operators needs to be generalised. The choices made in the definition of these interpretations give rise to different fuzzy semantics.

The study of fuzzy DLs can be traced back decades (see e.g. [11] for a survey from twelve years ago), but early work on the area focused exclusively on the so-called Zadeh semantics. Although this semantics is a very intuitive generalisation of the interpretation of the standard logical constructors, it does not satisfy important properties from the point of view of mathematical logic. The semantics which do satisfy these properties have been studied under the umbrella of *mathematical fuzzy logic* [9], and are defined by the choice of a triangular norm (or *t-norm* for short): a binary operator over $[0, 1]$ used to interpret the conjunction [10]. The choice of the t-norm implicitly determines also how the other constructors (e.g., disjunction, implication, and negation) are interpreted.

About ten years ago, fuzzy description logics started considering the mathematical fuzzy logic point of view as well [3, 7, 12]. This study focused first on expressive DLs but, after some initial negative results [2, 8], smaller members of the DL family were considered in an attempt to capture the limits of decidability of ontology consistency and concept subsumption in these logics [5]. Interestingly, during this active period in the area, fuzzy extensions of the lightweight DL-Lite logics were mainly ignored. Perhaps for related reasons, the task of query answering in this setting went also largely unexplored.

We note that there exists a very large corpus of work on query answering on fuzzy description logics (see [15, 17] and references therein). However, most of it considers the Zadeh semantics only. To our knowledge, only Mailis and Turhan [13] have considered query answering with a t-norm based semantics. Their approach, implemented in the system FLite [14], rewrites a degree query into a standard (crisp) query which can be answered by standard database management systems. However, the approach is limited to the idempotent Gödel t-norm and can only handle graded ABox axioms; that is, it requires that the TBox is crisp. A different approach which allows for non-idempotent conjunc-

tions and graded TBox axioms was proposed in [6]. Unfortunately, this latter approach is limited to a *finite* (and fixed) class of truth degrees.

We decided to tackle the existing gap in the literature by developing query answering methods which could handle also graded GCIs, with semantics based on t-norms. In our work [16], we developed a method for answering conjunctive (degree) queries over ontologies written in the fuzzy extension of DL-Lite$_R$ based on the Gödel semantics. Interestingly, despite covering a more general case (that is, allowing intermediate membership degrees in all the axioms of the ontology, instead of just the ABox) our approach is also conceptually and practically simpler than the algorithm underlying FLite. In fact, while FLite rewrites the query following the different degrees appearing in the ABox, we show that it suffices to consider only the d-cuts of the ontology (also called α-cuts): the (classical) sub-ontologies obtained by considering only those axioms holding to a degree at least d. We show that, under the Gödel semantics, the (Boolean) conjunctive query q holds with degree at least d in the fuzzy DL-Lite$_R$ ontology $\mathcal{O}$ iff the d-cut of $\mathcal{O}$ entails q. This result allows us to use any classical query answering method as a black-box to answer degree queries over fuzzy ontologies.

The drawback of our approach is that it does not work with other continuous t-norms. Specifically, the lack of idempotency of any t-norm different from Gödel may lead to erroneous results if an axiom is used more than once in answering a query. However, as already noted in [5], for any t-norm without zero-divisors ontology consistency remains reducible to the classical case. This in particular means that for the product t-norm, consistency is as costly, in terms of computational complexity, as for classical DL-Lite$_R$. For the Łukasiewicz t-norm, which is known to cause issues for standard reasoning [4], the problem of handling degree queries remains open.

For future work, we intend to study methods for answering CQs over ontologies based on arbitrary t-norms. We will also consider other variations of conjunctive queries which are relevant in the presence of degrees; specifically, threshold queries and relaxed queries.

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