Connections Between Answer Set Semantics and Information Terms Semantics in Constructive $\mathcal{EL}_\bot$

(Extended Abstract)

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Constructive description logics are interpretations of description logics (DLs) under appropriate constructive semantics. The application of such non-classical semantics to description logics is motivated by the interest in using the formal properties of constructive semantics in knowledge representation. Starting from different constructive semantics, several constructive description logics have been proposed, see e.g. [4,6,11].

Constructive description logics have been mostly studied from a theoretical viewpoint, and they have also been applied to tackle different representation and reasoning problems (see, e.g., [2,8,9,10]); however, the interaction between formal and practical aspects of constructive DLs has been scarcely investigated in the literature. To bridge this gap, in [1] we have proposed a simple constructive DL based on $\mathcal{EL}$ and we have discussed its relationship with *Answer Set Programming (ASP)*. Exploiting such a connection, we have presented a Datalog encoding managing a reasoning task over the constructive semantics (namely, the generation of "valid states" of a knowledge base) and we have provided a prototype based on the standard OWL-EL profile, together with "off the shelf" tools for manipulation of OWL 2 ontologies and ASP reasoning.

Our constructive interpretation of $\mathcal{EL}$ is based on *information terms (ITs) semantics* [3,6,12], a constructive semantics related to the *BHK (Brower-Heyting-Kolmogorov)* interpretation of logical connectives [13]. Intuitively, an information term η for a formula K is a syntactical object that constructively justifies the truth of K in a classical model $\mathcal{M}$. For instance, let us consider the $\mathcal{EL}$ concept $\exists R.C$ and the formula $\exists R.C(d)$, to be understood as "the individual d belongs to $\exists R.C$". The validity of $\exists R.C(d)$ in a model $\mathcal{M}$ can be explained by an information term (e, η) providing the filler e such that $(d^{\mathcal{M}}, e^{\mathcal{M}})$ (the pair of elements interpreting d and e in $\mathcal{M}$) belongs to $R^{\mathcal{M}}$ (the relation interpreting R in $\mathcal{M}$) and, inductively, an information term η justifying the validity of the formula $C(e)$. Information terms have been used to represent the *state* or *answer* of a formula (and, by extension, a notion of "snapshot" representing a valid state of a knowledge base). The relation between information terms and answer sets semantics has been firstly studied in [7], where propositional theories are considered. Along these lines, [1] extends this approach, for the first time, to the context of constructive description logics.

In this work, we continue the investigation started in [1] by "pushing the envelope" towards more expressive DLs: as a first step we introduce falsum in $\mathcal{EL}$, thus providing a constructive interpretation for the DL $\mathcal{EL}_\bot$. Similarly to [3,6], negated formulas of the kind $\neg C(d)$ are treated akin to atomic formulas with respect to their IT interpretation: intuitively, this corresponds to considering these formulas as constraints, i.e. we only require that $C(d)$ *does not hold*, without any constructive information about non-validity.

We can then show how the results linking IT semantics and answer sets semantics can be extended in presence of negative information. Following [7], negative information can be represented similarly to default negation in ASP: negative formulas are used as constraints and answer sets are formulated over a suitable positive *reduction* of the input formulas w.r.t. negative information. Intuitively, as a result, we can show that the answer sets of an $\mathcal{EL}_\bot$ formula K (and, by extension, of an $\mathcal{EL}_\bot$ KB) correspond to the "positive answers" encoded by the minimal ITs of K that respect such negative constraints.

We note that the constructive reading of formulas provided by information terms semantics can be related to the recent interest in Explainable AI (which is being discussed also in the field of symbolic Knowledge Representation and Reasoning). For example, as shown in [5], the generation of a "valid snapshot" of a knowledge base (i.e. a valid information term for its set of formulas) can be used to verify the set of constraints encoded by the KB and, in case of a violation, to constructively identify the source of inconsistencies, in order to amend the KB.

This ongoing work represents a first step towards the extension of this approach to more expressive description logics: for example, an interesting goal is to extend the discussed results to the full language of $\mathcal{ALC}$, exploiting the information terms semantics presented in [6]. An other issue to be investigated is the application of the mentioned formal results to representation and reasoning tasks, for example by extending the Datalog encoding and prototype discussed in [1] for ASP based generation of information terms. We also aim at developing procedures for the manipulation of information terms (see, e.g., [2]), in order to apply IT semantics in concrete problems.

An extended version of this paper is available at https://dkm.fbk.eu/technologies/constructive-description-logics.

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