A Labelling Semantics for Weighted Argumentation Frameworks*

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Abstract. Argumentation Theory provides tools for both modelling and reasoning with controversial information and is a methodology that is going to be proposed as a way to give explanations to results provided using machine learning techniques. In this context, labelling-based semantics for Abstract Argumentation Frameworks (AFs) allow for establishing the acceptability of sets of arguments, dividing them into three partitions: acceptable, rejected and undecidable (instead of classical Dung two sets IN and OUT partitions). This kind of semantics have been studied only for classical AFs, whilst the more powerful weighted and preference-based framework has been not studied yet. In this paper, we define a novel labelling semantics for Weighted Argumentation Frameworks, extending and generalising the crisp one.

Keywords: Argumentation theory · labelling-based semantics · weighted argumentation framework.

1 Introduction

Argumentation and its applications are receiving increasing interest in many fields of AI. For instance, argumentative processes are used in [21] to interpret online debates, while in [26] an argumentation system is devised to support expert opinion. Argumentation is also used to aid machine learning (see [16] for a survey) for both improving performances (e.g., classification accuracy) and providing explanations to the results. Argumentation problems are modelled through Abstract Argumentation Frameworks (AFs in short) [18], that consist of directed graphs in which the nodes are arguments that contain abstract information and the edges represent attack relations.

The acceptability of an argument of an AF can then be established following different criteria, formalised through the extension-based [18] and the labelling-based semantics [13]. Through the reasoning on the acceptability of the arguments according to a notion of defence, one can divide the set of arguments into two separated subsets, respectively containing acceptable and non-acceptable arguments. However, for certain applications of argumentation (especially those in which defeating an argument leads to the reinstatement of another one [13]), it is convenient to consider more degrees of acceptability in order for one to be able to further differentiate among arguments. The labelling defined in [13] refines the concept of acceptable argument and builds on the classical semantics for providing an additional acceptance status through the assignment of labels to the arguments.

In order to increase the expressiveness of AFs, attack relations between arguments can be endowed with a value (a weight) which indicates the strength of the attacks themselves. In this kind of frameworks, called weighted AFs, the acceptability criteria for the arguments also need to consider the weight of incoming and outgoing attacks. In two recent works [9, 10] the attacks from an argument to a set of arguments are grouped together as if they were a unique attack; in particular, the authors consider a weighted notion of defence that takes into account the weight associated to each attack, also generalising the approaches of [17] and [22]. In all these works, extension-based semantics have been used to identify sets of acceptable arguments. The correspondence between extension-based semantics and the labelled ones has been proved and showed important for the crisp framework [25]. The addition of such mapping for the weighted argumentation is an important result in the area that will be the initial brick for many additional result in the field. In this work, we extend the notion of labelling to Weighted Argumentation Frameworks and we provide a definition that generalises the original labelling [13]. For each weighted semantics, we give the conditions under which a labelling corresponds to a set of extensions. The rest of this paper is structured as follows: in Section 2 we summarise the main concepts of AFs, providing the definitions for extension-based semantics considering both weighted and non-weighted cases, and in Section 3 we present our definition of labelling for Weighted Argumentation Frameworks. Section 4 shows an implementation of the weighted labelling within a tool for argumentation problems. Finally, in Section 6 we conclude the paper discussing some of the possible future directions that we would like to investigate.

2 Preliminaries

In this section we recall the formal definition of AF and the related semantics introduced by Dung [18], together with the notion of labelling and labelling-based semantics [13, 2]. We also give the main definitions for Weighted AFs, relying on the definition of acceptability of arguments given in [9, 10].

2.1 Abstract Argumentation Frameworks

First of all, we recall the formal definition for an AF [18].

Definition 1 (Abstract Argumentation Framework). *An Abstract Argumentation Framework is a pair* $\langle \mathcal{A}, \mathcal{R} \rangle$ *where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{R}$ *is a binary relation on* $\mathcal{A}$*.*

Consider two arguments a, b belonging to an AF. We denote with $(a, b) \in \mathcal{R}$ (or simply $a \rightarrow b$) an attack from a to b; we can also say that b is defeated by a. We define the sets of arguments that attack (and that are attacked by) another argument as follows.

Definition 2 (Attacks). *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF, $a \in \mathcal{A}$ and $A \subseteq \mathcal{A}$. We define the sets $a^+ = \{b \in \mathcal{A} \mid a \rightarrow b\}$, $a^- = \{b \in \mathcal{A} \mid b \rightarrow a\}$, $A^+ = \cup\{a^+ \mid a \in A\}$ and $A^- = \cup\{a^- \mid a \in A\}$.*

In order for b to be acceptable, we require that every argument that defeats b is defeated in turn by some other argument of the AF. More formally, we have the following definition.

Definition 3 (Acceptable argument). *Given an AF $F = \langle \mathcal{A}, \mathcal{R} \rangle$, an argument $a \in \mathcal{A}$ is acceptable with respect to $D \subseteq \mathcal{A}$ if and only if $\forall b \in \mathcal{A}$ such that $b \in a^-$, $\exists c \in D$ such that $c \in b^-$, and we say that a is **defended** by D.*

Using the notion of defence as a criterion for distinguishing acceptable arguments in the framework, one can further refine the set of selected "good" arguments through semantics.

Definition 4 (Extension-based semantics). *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A set $E \subseteq \mathcal{A}$ is conflict-free in F if and only if there are no $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$. A conflict-free subset E is then*

- *admissible, if each $a \in E$ is defended by E;*
- *complete, if it is admissible and $\forall a \in \mathcal{A}$ defended by E, $a \in E$;*
- *stable, if $E \cup E^+ = \mathcal{A}$;*
- *preferred, if it is admissible and it is maximal (with respect to set inclusion);*
- *grounded, if it is complete and it is minimal (with respect to set inclusion).*

Strong admissibility is introduced in [3] as a refinement of the admissible semantics.

Definition 5. *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A set $E \subseteq \mathcal{A}$ is strongly admissible if and only if each $a \in E$ is defended by some $E' \subseteq E \setminus \{a\}$ which in its turn is again strongly admissible.*

The work in [13] describes how to assign labels to the arguments of an AF in such a way that the set of arguments is partitioned into three subsets, each representing a different degree of acceptance. Below, we report the labelling function and the characterisation for the various semantics.

Definition 6 (Labelling for AFs). *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A labelling L of F is a total function $L : \mathcal{A} \rightarrow \{\texttt{IN}, \texttt{OUT}, \texttt{UNDEC}\}$. For any $A \subseteq \mathcal{A}$, we denote $A|_{\texttt{IN}}$, $A|_{\texttt{OUT}}$ and $A|_{\texttt{UNDEC}}$ the set of all the arguments labelled `IN`, `OUT` and `UNDEC` by L, respectively.*

Given a labelling L, it is possible to identify a correspondence with the extension-based semantics [2]. The set of `IN` arguments coincides with an extension of acceptable arguments. We rephrase the semantics in [2] as follows.

Definition 7 (Labelling-based semantics). *Let L be a labelling of an AF $F = \langle \mathcal{A}, \mathcal{R} \rangle$ and $a \in \mathcal{A}$. Then*

- *L is a conflict-free labelling if and only if:*
  - $L(a) = \mathit{IN} \implies a^-|_{\mathit{IN}} = \emptyset$, *and*
  - $L(a) = \mathit{OUT} \implies a^-|_{\mathit{IN}} \neq \emptyset$
- *L is a admissible labelling if and only if:*
  - $L(a) = \mathit{IN} \implies a^- = a^-|_{\mathit{OUT}}$, *and*
  - $L(a) = \mathit{OUT} \implies a^-|_{\mathit{IN}} \neq \emptyset$
- *L is a complete labelling if and only if:*
  - $L(a) = \mathit{IN} \iff a^- = a^-|_{\mathit{OUT}}$, *and*
  - $L(a) = \mathit{OUT} \iff a^-|_{\mathit{IN}} \neq \emptyset$
- *L is a stable labelling if and only if:*
  - *L is a complete labelling, and*
  - $\mathcal{A}|_{\mathit{UNDEC}} = \emptyset$;
- *L is a preferred labelling if and only if:*
  - *L is an admissible labelling, and*
  - *$\mathcal{A}|_{\mathit{IN}}$ is maximal among all the admissible labellings*
- *L is a grounded labelling if and only if:*
  - *L is a complete labelling, and*
  - *$\mathcal{A}|_{\mathit{IN}}$ is minimal among all the complete labellings*

A labelling for the strongly admissible semantics is given in [14], where the author relies on a numbering on the arguments to assign the correct labels. In every labelling of the various semantics, arguments for which not every attacker is labelled OUT, and no attacker is labelled IN are labelled UNDEC. The admissible labelling that we consider in the definition above coincides with the interpretation given in [14], where IN arguments can attack both OUT and UNDEC arguments. Different definitions of labelling (as for instance the one given in [20] and surveyed in [15]) force arguments attacked by an IN to be OUT[1]. However, nothing changes in terms of extensions, since the set of IN arguments remains the same. Also, note that a complete labelling coincides with the reinstatement labelling given in [13].

2.2 Weighted Argumentation Frameworks

In order to compute the set of extensions of a particular AF, attack relations are used to determine the acceptability of the arguments. Since it is not possible to further diversify the relations among arguments, every attack in the AF has the same "strength", that is, the existence or not of an attack is the only thing that matters in determining the semantics. To overcome this limit, Dung's AFs have been extended to Weighted AFs (WAFs) by associating the attacks with a weight that represents the support of the relation [19].

[1] Hence $\iff$ is used instead of $\implies$ in the second condition for admissible labelling: $L(a) = \mathtt{OUT} \iff a^-|_{\mathtt{IN}} \neq \emptyset$.

In order to analyse a WAF in terms of sets of extensions, a definition of defence is required that encompasses the notion of weighted attack relations. In [10] the framework is equipped with a c-semiring [7, 8] that provides the operation for composing the weights in order to estimate the effectiveness of a defence. The acceptability of an argument is then determined by comparing the compositions of the attacks with the composition of the defences. C-semirings [7, 8] are absorptive, commutative semiring, that is commutative semirings with idempotent plus operator (also called tropical semirings) and top element. These structures allow expressing both the values of the weights and the aggregation operators and thus are parametric to the desired notion of defence.

Definition 8 (c-semirings). *A c-semiring is a tuple* $\mathbb{S} = \langle S, \oplus, \otimes, \bot, \top \rangle$ *such that* S *is a set,* $\top, \bot \in S$*, and* $\oplus, \otimes : S \times S \to S$ *are binary operators making the triples* $\langle S, \oplus, \bot \rangle$ *and* $\langle S, \otimes, \top \rangle$ *commutative monoids (semi-groups with identity), satisfying* i) $\forall s, t, u \in S.\ s \otimes (t \oplus u) = (s \otimes t) \oplus (s \otimes u)$ *(distributivity), and* ii) $\forall s \in S.\ s \otimes \bot = \bot$ *(annihilator). Moreover, we have that* $\forall s, t \in S.\ s \oplus (s \otimes t) = s$ *(absorptiveness). The operator* $\oplus$ *also defines a preference relation* $\leq_{\mathbb{S}}$ *over the set* S*, such that* $a \leq_{\mathbb{S}} b \iff a \oplus b = b$*, for* $a, b \in S$.

We list some of the most common instances of c-semirings.

- $\mathbb{S}_{boolean} = \langle \{false, true\}, \vee, \wedge, false, true \rangle$
- $\mathbb{S}_{fuzzy} = \langle [0, 1], \max, \min, 0, 1 \rangle$
- $\mathbb{S}_{probabilistic} = \langle [0, 1], \max, \times, 0, 1 \rangle$
- $\mathbb{S}_{weighted} = \langle \mathbb{R}^+ \cup \{+\infty\}, min, +, +\infty, 0 \rangle$

Different c-semirings can represent different notions of defence for WAF, by using the operators $\oplus$ and $\otimes$ for obtaining an ordering among the values in S. For simplicity, we refer to these values as weights. Note that the element $\top$ of the c-semiring (*e.g.*, 0 for the weighted and *true* for the boolean) coincides with having no relation between two arguments. We denote with $\text{WAF}_{\mathbb{S}}$ a WAF endowed with a c-semirings $\mathbb{S}$ and we call it a semiring-based WAF.

Definition 9 ($\text{WAF}_{\mathbb{S}}$). *A semiring-based WAF is a quadruple* $\langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$*, where* $\mathbb{S}$ *is a c-semiring* $\langle S, \oplus, \otimes, \bot, \top \rangle$*,* $\mathcal{A}$ *is a set of arguments,* R *the attack binary-relation on* $\mathcal{A}$*, and* $W : \mathcal{A} \times \mathcal{A} \longrightarrow S$ *is a binary function. Given* $a, b \in \mathcal{A}$ *and* $R(a, b)$*, then* $W(a, b) = s$ *means that* a *attacks* b *with a weight* $s \in S$*. Moreover, we require that* $R(a, b)$ *if and only if* $W(a, b) <_{\mathbb{S}} \top$.

Given a $\text{WAF}_{\mathbb{S}}$ we can evaluate the overall weight of all the attacks from a set of arguments towards another set through the **composition** operator $\otimes$ of the c-semiring $\mathbb{S}$ [9, 12]. In particular, we use $\bigotimes$ to indicate the $\otimes$ operator on a set of values (indeed $\otimes$ is a binary operator that composes two weights).

Definition 10 (Attacks). *Let* $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ *be a* $WAF_{\mathbb{S}}$*. A set of arguments* $\mathcal{B}$ *attacks a set of arguments* $\mathcal{D}$ *and the weight of such attack is* $k \in S$*, if*

$$W(\mathcal{B}, \mathcal{D}) = \bigotimes_{b \in \mathcal{B}, d \in \mathcal{D}} W(b, d) = k.$$

The previous definition also allows composing the attacks from a set of arguments to another single argument, and from a single argument towards a set of arguments. The notion of weighted defence (or w-defence) can then be expressed in the following terms.

Definition 11 (w-defence). *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a WAF$_\mathbb{S}$. Then $\mathcal{B} \subseteq \mathcal{A}$ w-defends $b \in \mathcal{A}$ if and only if $\forall a \in \mathcal{A}$ such that $R(a, b)$, we have that $W(a, \mathcal{B} \cup \{b\}) \geq_\mathbb{S} W(\mathcal{B}, a)$.*

According to [10], by using the notion of w-defence for checking the acceptability of the arguments in the weighted framework, it is possible to redefine all the extension-based semantics presented in Definition 4.

Definition 12 (Extension-based semantics for WAF$_\mathbb{S}$). *Given a WAF$_\mathbb{S}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$, a subset of arguments $\mathcal{B} \subseteq \mathcal{A}$ is w-conflict-free if $W(\mathcal{B}, \mathcal{B}) = \top$. A w-conflict-free subset $\mathcal{B}$ is then*

- *w-admissible, if $\forall a \in \mathcal{B}^-$. $W(a, \mathcal{B}) \geq_\mathbb{S} W(\mathcal{B}, a)$ (that is $\mathcal{B}$ w-defend itself from the arguments in $\mathcal{A} \setminus \mathcal{B}$);*
- *w-complete, if it is w-admissible and each argument $b \in \mathcal{A}$ such that $\mathcal{B} \cup \{b\}$ is w-admissible belongs to $\mathcal{B}$;*
- *w-stable, if it is w-admissible and $\forall a \notin \mathcal{B}$. $\exists b \in \mathcal{B}$ such that $W(b, a) <_\mathbb{S} \top$;*
- *w-preferred, if it is a maximal (with respect to set inclusion) w-admissible subset of $\mathcal{A}$;*
- *w-grounded, if it is the maximal (with respect to set inclusion) w-admissible extension included in the intersection of w-complete extensions;*
- *w-quasi-strongly admissible[2], if $\forall a \in \mathcal{B}^-$, $\forall b \in \mathcal{B}$. $\exists \mathcal{C} \subseteq \mathcal{B} \setminus \{b\}$ with $W(a, \mathcal{B}) \geq_S W(\mathcal{C}, a)$.*

The definition for w-quasi-strongly admissible extensions, first given in [12], states that a subset of arguments $\mathcal{B}$ is w-strongly admissible when for all $b \in \mathcal{B}$, $\mathcal{B}$ is defended by a subset of $\mathcal{B}$ that does not include b. In other words, each argument in $\mathcal{B}$ is defended by the rest of the arguments in $\mathcal{B}$.

Contrary from classical AFs, for which we can use the procedure in [13] for assigning labels to the arguments in such a way that there is a correspondence between the labelling and the set of extension, no work on this direction has been done for what concerns the weighted case.

3 Labelling for Weighted AFs

We extend the notion of labelling introduced in [13] to weighted AFs. In particular, we consider a WAF$_\mathbb{S}$ and we provide a definition for the labelling. Furthermore, we give the conditions for determining whether a labelling corresponds to

[2] The definition for the w-quasi-strongly admissible semantics is introduced in [12], where the authors refer to it by the term w-strongly admissible. However, differently from the classical case, the defending set $\mathcal{B}' \subseteq \mathcal{B} \setminus \{a\}$ is not required to recursively be w-strongly admissible, and thus we considered it more appropriate to use a different name.

a certain extension. In order to incorporate the notion of weighted defence in the labelling, we need to take into account the strength of the attack relations.

Definition 13 (Labelling for WAF$_{\mathbb{S}}$). *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a WAF$_{\mathbb{S}}$. A labelling L of F is a total function $L : \mathcal{A} \to \{$IN, OUT, UNDEC$\}$. For any $A \subseteq \mathcal{A}$, we denote $A|_{IN}$, $A|_{OUT}$ and $A|_{UNDEC}$ the set of all the arguments labelled IN, OUT and UNDEC by L, respectively. We also define, for each argument, the weight of attacks, incoming into and outgoing from an argument, as $w_{a^-|_{IN}} = W(a^-|_{IN}, a)$ and $w_{a^+|_{IN}} = W(a, a^+|_{IN})$.*

According to the definition of collective defence [9] we need to know the strength resulting from the composition of all the attacks towards an argument. In the weighted system, OUT arguments are associated with the $\bigotimes$ of the incoming attacks. An argument a with label OUT is attacked by the arguments in $a^-|_{\mathtt{IN}}$ with a total strength that is expressed by $w_{a^-|_{\mathtt{IN}}}$. With this information, one can easily compute the acceptability of defended arguments. The main issue one has to take into account when dealing with the study of semantics in WAF$_{\mathbb{S}}$ is the notion of weighted defences among the arguments. According to the classical notion of defence, an argument a is defended from the attack of another argument b if there exists a third argument c that attacks b in turn. On the other hand, when a weight is assigned to the attacks, the previous condition cannot ensure alone that the argument a will be defended by c: it can be the case that the attack $c \to b$ is not strong enough to defeat $b \to a$ and thus to justify a (see arguments a, c and d in Figure 1).

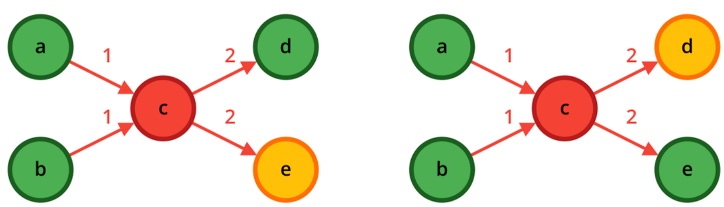

Fig. 1. Example of two labellings on a WAF$_{\mathbb{S}}$ with a weighted semiring where IN arguments are highlighted in green, UNDEC in yellow, and OUT in red. When d is IN, e is UNDEC (and vice versa) because arguments a and b can only defend one of the two.

According to the definition of collective weighted defence given in [9], a set of argument is defended from an attacker b only if the $\bigotimes$ of all the defending arguments is stronger than the $\bigotimes$ of the attacks coming from b. This means that the strength of the attacks of the defending arguments is distributed among the defended arguments, so it is not guaranteed for two arguments that are separately w-defended to sill be w-defended when considered together (this is what happens in the example in Figure 1 with arguments d and e).

In the following, we give a characterisation of the weighted semantics through the notion of labelling of WAF$_{\mathbb{S}}$. The intuition behind this representation is that when an argument a attacked by an OUT b cannot be labelled IN because of another IN argument that is "consuming" the attacks of the defending arguments towards b, then a is labelled UNDEC.

Fact 1 (w-conflict-free labelling) *The w-conflict-free labelling coincides with the conflict-free labelling.*

Indeed, since attacks are not allowed within a conflict-free set of arguments, one does not need to consider the weights. We now define the w-admissible labelling.

Definition 14 (w-admissible labelling). *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is a w-admissible labelling for F if and only if:*

- $L(a) = \mathit{IN} \implies a^- = a^-|_{\mathit{OUT}} \wedge \forall b \in a^-.\ w_{b^-|_{\mathit{IN}}} \leq_{\mathbb{S}} w_{b^+|_{\mathit{IN}}}$
- $L(a) = \mathit{OUT} \implies w_{a^-|_{\mathit{IN}}} <_{\mathbb{S}} \top$

The condition $w_{b^-|_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+|_{\mathtt{IN}}}$ makes sure that the composition of the attacks of the arguments defending a is stronger than the attack of b. For an argument to be OUT, we require $w_{a^-|_{\mathtt{IN}}} <_{\mathbb{S}} \top$, that is to say that that there must exist at least an attack coming from an IN argument (as for the classical admissible labelling). Indeed, $\top$ means that there is no attack between two arguments. The WAF$_{\mathbb{S}}$ used in Figure 1 admits six w-admissible labellings, corresponding to the sets of IN arguments $\{a\}$, $\{b\}$, $\{a, b\}$, $\{a, b, d\}$ and $\{a, b, e\}$ (depicted in Figure 1), and the empty set.

Definition 15 (w-complete labelling). *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is a w-complete labelling for F if and only if:*

- $L(a) = \mathit{IN} \iff a^- = a^-|_{\mathit{OUT}} \wedge \forall b \in a^-.\ w_{b^-|_{\mathit{IN}}} \leq_{\mathbb{S}} w_{b^+|_{\mathit{IN}}}$
- $L(a) = \mathit{OUT} \iff w_{a^-|_{\mathit{IN}}} <_{\mathbb{S}} \top$

The definition of the w-complete labelling is similar to the w-admissible one, with the exception that the conditions given for IN and OUT arguments are both necessary and sufficient. The two labellings in Figure 1 represent all and only w-complete labellings for the considered WAF$_{\mathbb{S}}$.

Definition 16 (w-stable labelling). *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$. L is a w-stable labelling for F if and only if*

- *L is a w-complete labelling and*
- $\mathcal{A}|_{\mathit{UNDEC}} = \emptyset$

According to the classical definition, a stable semantics partitions the arguments in two disjoint sets: one contains the arguments that are either not attacked or defended by other acceptable arguments, while the other contains the rest of the arguments (i.e., those that are attacked and not defended). In the weighted case, we obtain the same kind of partition through Definition 16. The examples in Figure 1 do not represent w-stable labellings since both of them have an UNDEC argument (respectively e and d). The labelling in Figure 2 is, instead, w-stable.

We next present the w-preferred labelling for WAF$_{\mathbb{S}}$.

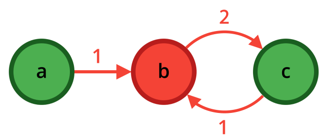

Fig. 2. Examples of a WAF$_{\mathbb{S}}$ with a w-stable labelling.

Definition 17 **(w-preferred labelling).** *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$. L is a w-preferred labelling for F if and only if*

– *L is a w-admissible labelling and*
– *$\mathcal{A}|_{IN}$ is maximal among all the w-admissible labellings*

As for the classical definition, also in the weighted case the w-preferred extensions is the largest admissible sets. The WAF$_{\mathbb{S}}$ in Figure 1 has only two w-preferred labellings, both represented in the picture.

Definition 18 **(w-grounded labelling).** *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is a w-grounded labelling for F if and only if:*

– $L(a) = \mathtt{IN} \iff$ *for all w-complete labellings L', $L'(a) = \mathtt{IN}$ and*
– $L(a) = \mathtt{OUT} \iff w_{a^-|_{\mathtt{IN}}} <_{\mathbb{S}} \top$

We know from [12] that the w-grounded extension always exists, is unique and corresponds to any maximal w-admissible extension included in the intersection of w-complete extensions. None of the labelling in Figure 1 is w-grounded. Indeed the intersection of IN arguments in the example WAF$_{\mathbb{S}}$ is $\{a, b\}$, that is neither d nor e should be IN. Figure 2, instead, shows an example of w-grounded labelling.

Definition 19 **(w-quasi-strongly admissible labelling).** *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is a w-quasi-strongly admissible labelling for F if and only if:*

– $L(a) = \mathtt{IN} \implies a^- = a^-|_{\mathtt{OUT}} \wedge w_{b^-|_{\mathtt{IN}} \setminus \{a\}} \leq_{\mathbb{S}} w_{b^+|_{\mathtt{IN}}}$
– $L(a) = \mathtt{OUT} \implies w_{a^-|_{\mathtt{IN}}} <_{\mathbb{S}} \top$

We obtain a w-quasi-strongly admissible labelling by imposing that every IN argument is always defended by other IN arguments. The labelling in Figure 2 is not a w-quasi-strongly admissible labelling: in fact, the attack of the IN argument a towards the OUT argument b is not sufficient alone to defend c. On the other hand, both the labellings in Figure 1 are w-quasi-strongly admissible.

The sets of arguments labelled IN by the above-defined labellings for WAF$_{\mathbb{S}}$ are equivalent to the extensions of the corresponding semantics.

Theorem 1. *A labelling L of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded, w-quasi-strongly admissible) labelling if and only if $\mathcal{A}|_{IN}$ is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded, w-quasi-strongly admissible) extension of F.*

Proof. We show for each semantics the correspondence between the IN arguments and the set of extensions. We refer to Definition 12 for the $\text{WAF}_{\mathbb{S}}$ semantics.

- (L is w-admissible $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-admissible.) The OUT arguments attacking $\mathcal{A}|_{\texttt{IN}}$ are defeated by $\mathcal{A}|_{\texttt{IN}}$. Thus, $\mathcal{A}|_{\texttt{IN}}$ is w-defend from the attacks coming from $\mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}}$ and so it is a w-admissible extension.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-admissible $\Rightarrow L$ is w-admissible.) $\mathcal{A}|_{\texttt{IN}}$ w-defends itself from the attacks of every $b \in \mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}}$, so $W(\mathcal{A}|_{\texttt{IN}}, b) \leq_{\mathbb{S}} W(b, \mathcal{A}|_{\texttt{IN}})$. Moreover, every $a \in \mathcal{A}|_{\texttt{IN}}$, is IN and thus L is a w-admissible labelling.
- (L is w-complete $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-complete.) When L is w-complete, then it is also w-admissible and it labels all the arguments w-defended by $\mathcal{A}|_{\texttt{IN}}$ as IN. Hence $\mathcal{A}|_{\texttt{IN}}$ is a w-complete extension.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-complete $\Rightarrow L$ is w-complete.) In this case $\mathcal{A}|_{\texttt{IN}}$ is a w-admissible extension where all the w-defended arguments belong to $\mathcal{A}|_{\texttt{IN}}$. Then L is w-complete labelling.
- (L is w-stable $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-stable.) L is a w-complete labelling in which no argument is labelled UNDEC. Thus, the set $\mathcal{A}|_{\texttt{IN}}$ attacks all the other arguments in $\mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}}$, and so $\mathcal{A}|_{\texttt{IN}}$ is a w-stable extension.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-stable $\Rightarrow L$ is w-stable.) We have that the set $\mathcal{A}|_{\texttt{IN}}$ is attacking all the arguments in $\mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}}$, so $\mathcal{A}|_{\texttt{UNDEC}} = \emptyset$. Then, since $\mathcal{A}|_{\texttt{IN}}$ is a w-admissible extension containing all the w-defended arguments, $\mathcal{A}|_{\texttt{IN}}$ is a w-complete extension and L a w-stable labelling.
- (L is w-preferred $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-preferred.) The set of arguments labelled IN by L coincides with a w-admissible extension which is maximal with respect to the set inclusion. Follows that $\mathcal{A}|_{\texttt{IN}}$ is a w-preferred extension.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-preferred $\Rightarrow L$ is w-preferred.) We have that $\mathcal{A}|_{\texttt{IN}}$ is a maximal w-admissible extension, so L is a w-preferred labelling.
- (L is w-grounded $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-grounded.) If L is w-grounded, all the arguments in $\mathcal{A}|_{\texttt{IN}}$ are also INin any w-complete labelling, thus $\mathcal{A}|_{\texttt{IN}}$ represents the maximal w-admissible extension included in the intersection of w-complete extensions.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-grounded $\Rightarrow L$ is w-grounded.) $\mathcal{A}|_{\texttt{IN}}$ contains all and only arguments that are included in the intersection of w-complete extensions, so L is a w-grounded labelling.
- (L is w-strongly admissible $\Rightarrow \mathcal{A}|_{\texttt{IN}}$ is w-strongly admissible.) The OUT arguments attacking any argument $a \in \mathcal{A}|_{\texttt{IN}}$ are defeated by $(\mathcal{A} \setminus \{a\})|_{\texttt{IN}}$. Thus, any argument in $\mathcal{A}|_{\texttt{IN}}$ is w-defend by the other arguments in $\mathcal{A}|_{\texttt{IN}}$ from the attacks coming from $\mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}}$ and so $\mathcal{A}|_{\texttt{IN}}$ is a w-strongly admissible extension.
- ($\mathcal{A}|_{\texttt{IN}}$ is w-strongly admissible $\Rightarrow L$ is w-strongly admissible.) Each argument $a \in \mathcal{A}|_{\texttt{IN}}$ is w-defends by $(\mathcal{A} \setminus \{a\})|_{\texttt{IN}}$ from the attacks of every $b \in a^- \cap (\mathcal{A} \setminus \mathcal{A}|_{\texttt{IN}})$, so $W((\mathcal{A} \setminus \{a\}), b) \leq_{\mathbb{S}} W(b, \mathcal{A}|_{\texttt{IN}})$. Hence L is a w-strongly admissible labelling.

□

We summarize in Table 3 the conditions specified in Definitions from 14 to 18 for obtaining weighted labellings corresponding to the Dung semantics.

Table 1. Summarisation of the introduced labellings for $WAF_{\mathbb{S}}$.

	conditions on $\mathtt{IN}$ arguments	conditions on $\mathtt{OUT}$ arguments	other conditions
w-cf	$L(a) = \mathtt{IN} \implies a^-\|_{\mathtt{IN}} = \emptyset$	$L(a) = \mathtt{OUT} \implies a^-\|_{\mathtt{IN}} \neq \emptyset$	
w-adm	$L(a) = \mathtt{IN} \implies a^- = a^-\|_{\mathtt{OUT}}$ $\wedge \forall b \in a^-.\ w_{b^-\|_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+\|_{\mathtt{IN}}}$	$L(a) = \mathtt{OUT} \implies w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	
w-com	$L(a) = \mathtt{IN} \iff a^- = a^-\|_{\mathtt{OUT}}$ $\wedge \forall b \in a^-.\ w_{b^-\|_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+\|_{\mathtt{IN}}}$	$L(a) = \mathtt{OUT} \iff w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	
w-stb	$L(a) = \mathtt{IN} \iff a^- = a^-\|_{\mathtt{OUT}}$ $\wedge \forall b \in a^-.\ w_{b^-\|_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+\|_{\mathtt{IN}}}$	$L(a) = \mathtt{OUT} \iff w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	$\mathcal{A}\|_{\mathtt{UNDEC}} = \emptyset$
w-pre	$L(a) = \mathtt{IN} \implies a^- = a^-\|_{\mathtt{OUT}}$ $\wedge \forall b \in a^-.\ w_{b^-\|_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+\|_{\mathtt{IN}}}$	$L(a) = \mathtt{OUT} \implies w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	$\mathcal{A}\|_{\mathtt{IN}}$ max w-adm
w-gde	$L(a) = \mathtt{IN} \iff \forall L'$ w-*com*, $L'(a) = \mathtt{IN}$	$L(a) = \mathtt{OUT} \iff w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	
w-qsa	$L(a) = \mathtt{IN} \implies a^- = a^-\|_{\mathtt{OUT}}$ $\wedge w_{b^-\|_{\mathtt{IN}} \setminus \{a\}} \leq_{\mathbb{S}} w_{b^+\|_{\mathtt{IN}}}$	$L(a) = \mathtt{OUT} \implies w_{a^-\|_{\mathtt{IN}}} <_{\mathbb{S}} \top$	

The conditions we give for the weighted semantics are a generalization of the classical case, and all the labellings for $WAF_{\mathbb{S}}$ corresponds to the respective classical semantics when the framework is instantiated with a boolean semiring. When the $WAF_{\mathbb{S}}$ is instantiated with a boolean semiring, all the attacks from an argument to another are associated with the value $false$ and also $w_{a^-|_{\mathtt{IN}}}$ always corresponds to $false$.

Theorem 2. *The labelling of a $WAF_{\mathbb{S}}$ instantiated with a boolean semiring corresponds to the classical labelling.*

Proof. By Definition 8, the weight of an attack between two arguments in a $WAF_{\mathbb{S}}$ F where $\mathbb{S}$ is boolean always correspond to the value $false$. Since the composition operator is $\wedge$, also the $\otimes$ of every pair of attacks in F is $false$, and thus assigning a labelling boils down to checking the existence of attacks between arguments, as for the crisp case. □

It follows that if L is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded) labelling of a $WAF_{\mathbb{S}}$ F, then L is an admissible (respectively complete, stable, preferred, grounded) labelling of F.

4 Implementation

To complete our study and facilitate the use of weighted labelling semantics for argumentation-based application, we provide a tool able to represent $WAF_{\mathbb{S}}$ and visualize the computed labellings for various semantics. For this purpose, we extend ConArg[3] [11], a suite of tools for argumentation, with a series of functionalities for handling weighted argumentation problems. The web interface, which

[3] ConArg website: http://dmi.unipg.it/conarg.

is shown in Figure 3, is implemented in JavaScript and relies on a server-side solver written in C. In the following, we describe an example of use of the tool for weighted argumentation.

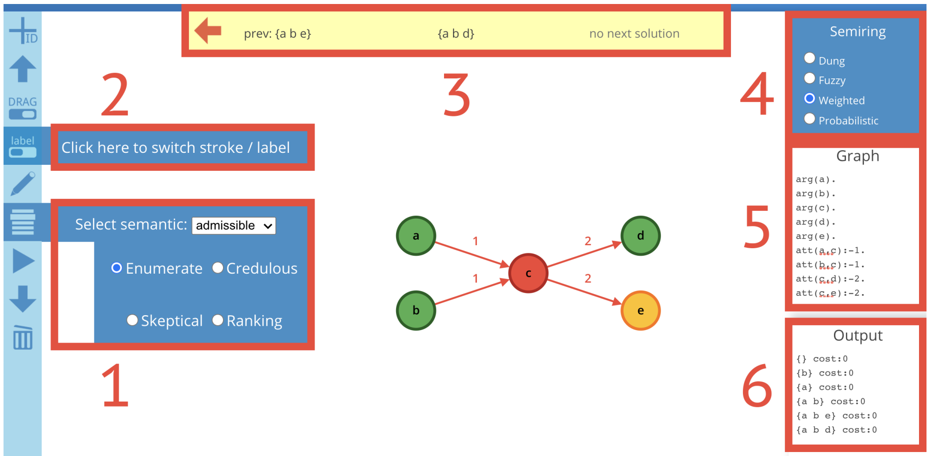

Fig. 3. ConArg web interface displaying a weighted labelling for a $\mathrm{WAF}_{\mathbb{S}}$. The highlighted areas corresponds to: 1) semantics selection, 2) representation of weights by stroke/label, 3) solution selection, 4) semiring selection, 5) input area, 6) output area.

First of all, we use panel 4 of Figure 3 to select a semiring: this determines both the representation of the AF (for instance classical, weighted, probabilistic) and the kind of solution provided by the solver. If weighted is chosen, it is possible to specify a $\mathrm{WAF}_{\mathbb{S}}$ by either using the input area (panel 5) or directly clicking on the canvas to draw arguments and attacks. The next step is to select the semantics (panel 1) for which we want obtain a labelling. Since we selected the weighted semiring, we will obtain a weighted labelling. The solver computes the sets of `IN` arguments, that are then displayed in panel 6. The labellings are directly visible on the $\mathrm{WAF}_{\mathbb{S}}$ through the usual colour scheme: `IN` arguments are green; any arguments attacked by an `IN` is red (that stands for `OUT`); all the remaining arguments (i.e., the `UNDEC` ones) are yellow. In case the solver returns more than one solution for the selected semantics (as happens in Figure 3), we can choose which labelling to visualise by using panel 3.

5 Related Work

The problem of extending classical AFs with values expressing the strength of arguments and attacks is widely studied, and many different approaches have been presented in the literature. In [1], the authors take into account preference orderings for comparing arguments, while in [6] the success of an attack conducted by an argument toward another one depends on an ordering among the "values" promoted by each argument. A study on bipolar WAFs is conducted in [24], where the authors present an extension for weighted frameworks that takes into account two different types of relations (one for attack and one for support). Another formalism based on a notion of strength is given in [5], were arguments in Quantitative Argumentation Debate Frameworks are evaluated through a score system. The main difference with our work lies in the fact that we take into account the basic definition of WAFs [19], without further refinements on the framework level. Moreover, our study is focused on the interpretation of the labelling in the weighted case.

For what concern the notion of weighted defence, many possible definitions can be considered: for instance, Martìnez, García and Simari [22] use the relative strength of the attacks in order to determine if some defence constraints are satisfied, while in [17] the authors aggregate the weights of the defence and check if this value is greater than the weight of the corresponding attack. We, on the other hand, use the notion introduced in [9], that also generalises the other two approaches mentioned above.

6 Conclusion and Future Work

With this work, we introduce a labelling for semiring-based WAFs (never done before), together with a set of labelling conditions corresponding to extensions for some semantics. We also show that our labelling function generalises the classical approach for the non-weighted case. We have also developed and made available online an implementation of the labelling for WAFs. We have considered the definition of collective defence provided in [9], for which an argument a of a $\text{WAF}_{\mathbb{S}}$ is defended by a set of arguments $a^-|_{\texttt{IN}}$ when $W(a^-|_{\texttt{IN}}, a) \geq_{\mathbb{S}} W(a, a^-|_{\texttt{IN}})$.

As future work, we plan to extend this work in different directions. For instance, since all the definitions we give for weighted semantics are parametric to a chosen notion of defence, it is possible to obtain labellings for semantics in which the weighted defence is differently declined. The definitions of the labelling-based semantics for WAFs, that we give in Section 3, do not include conditions for the `UNDEC` since they are obtained from `IN` and `OUT` arguments. In this sense, we would like to investigate the possible advantages of giving explicit conditions for labelling the `UNDEC` arguments, similarly to what is done in [23] for classical AFs. An interesting study could then be carried out on the *dont care* and *dont know* labels, that are used in [4] as further differentiation of `UNDEC` arguments. In our context, the difference between the two labels could be made more continuous by considering the weight on the attack relations. We also plan to give a definition of w-strongly admissible extension (generalising the one provided in [3] for the crisp case) and introduce the respective labelling.

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