Modeling the assessment of credit risk losses in banking

Katerina Larionova[0000-0003-4462-9858], Tetyana Donchenko[0000-0002-6929-4639], Andriy Oliinyk[0000-0003-4144-6052], Hennadii Kapinos[0000-0002-9995-1759], Oleg Savenko[0000-0002-4104-745X] and Olexander Barmak[0000-0003-0739-9678]

Khmelnytskyi National University, 11 Instytutska Str., Khmelnytskyi, 29016, Ukraine
k_larionova@i.ua, donchtv@gmail.com, oleynik_andrey@hotmail.com, p_kapinos@i.ua, savenko_oleg_st@ukr.net, Barmak@gmail.com

Abstract. The article develops a model of credit risk assessment within the scope of the variability concept that can be used for verification of new methods for borrowers' credit capacity estimation, the acceptable level of credit risk forecasting and its early prediction. It is aimed to be used during the automated banking systems development. The proposed model of credit risk assessment has been tested on the basis of the data from one of the Ukrainian banks. To determine the adequacy of this model has been proved by the comparison analysis of the proposed model with the results obtained by the National Bank of Ukraine methodology.

Keywords: credit risk, default risk, credit portfolio, risk management.

1 Introduction

The current state of the financial services market, which is unstable due to relevant external and internal factors influencing it, requires the solution of many problems, one of which is the assessment of risks, especially the risks of the most common lending banking operations. In addition, to maintain the stability of the banking system, which is an urgent task in today's conditions of national economy development, it is important to determine the main areas of research in this area and develop appropriate methodological and scientific support.

The questions of effective strategic and tactical bank management are directly associated with the credit risk, which should be acceptable. Taking into account the fact that it is not possible to avoid credit risk completely, the problem of its estimation is of great relevance, since its accuracy affects the minimization of this risk and the choice of the most effective strategy of its management. The appropriate credit risk estimation and the use of risk management methods can be a guarantee of bank financial firmness and stability.

The objective of the paper is to highlight the fundamental principles and develop a model for credit risk assessment using mathematical tools. To pursue this objective the next tasks should be solved: to distinguish the credit risk notion essence; to define the fundamental principles and to develop a model for credit risk assessment using mathematical models on the basis of the probability theory; to approbate the model of credit risk assessment through the Ukrainian bank data.

The concept of "assessment" is defined as the establishment of the level of the indicator. The combination of categories of "credit risk" and "assessment" is reflected in the definition of "credit risk assessment of financial institutions" as establishing the level of possibility of loss by the bank of partial or full loan to the borrower-financial institution due to bad faith, inability of the borrower own financial resources, low liquidity or insufficiency of collateral for the loan, non-performance of obligations by the guarantor or insurer under the loan insurance agreement. The proposed definition differs from the existing ones by taking into account the primary (financial resources of the financial institution) and secondary (collateral, guarantee and insurance collateral for the loan) sources of loan repayment, which helps to improve credit risk management in the bank. Thus, if there is a risk of improper fulfillment of obligations by the borrower to the bank at the expense of primary sources, the bank may take timely measures to minimize the identified threat by strengthening the collateral for the loan (collateral, surety, loan insurance agreement).

Taking into account the economic literature and practice, it can be concluded that the concept of risk is quite multifaceted and can have different meanings depending on the specific scenarios. However, risk is broadly defined as the uncertainty of a particular event occurrence in future. In the banking business the risk refers to financial losses that may take place during the certain transactions performance. In terms of mathematics the risk can be measured by the probability that an anticipated event will not happen (will happen) and it will lead to negative consequences or losses. It should be emphasized that banking business is rather specific, as it is characterized by high riskiness on the one hand, and assurance of reliability and trust, which are realized through providing certain guarantees to clients – on the other hand. The banking system of Ukraine as well as the world banking practice often faces the problem associated with estimation of credit risk, which can lead to financial losses in the result of default on borrower's commitments. According to the requirements of Basel Committee banks by their own should choose methodologies to estimate and model credit risk in their business [1]. Therefore, the problem of choosing modeling methods for credit risk assessment in banking business is of great relevance and controversy. It is obvious that banks have to create and implement their own risk management systems in order to achieve success in high-risk lending activities. It is these systems which help the management apparatus to identify, estimate and control corresponding risks. Nowadays the process of credit risk management is computer-aided, that means that it is based on the results of estimation, analysis and forecasting, obtained with the help of information systems. The main stages of the credit risk management process can be defined as follows: the identification of objective and subjective reasons for credit risk existence; estimation and forecasting of the risk magnitude; credit risks minimization or optimization; regular control of credit risks through credit operations monitoring.

The decision-making system should not only identify and assess the credit risk, but also determine its justifiability or tolerability in terms of the profitability of lending transaction. Acceptable risk is also possible, since it does not threaten the bank, and the eventual losses are less than expected profit and the amount of the compensation for possible losses reserve, which is specially created for the given transaction. Each banking institution defines the limits of acceptable risk by its own according to the chosen management strategy. However, the risk that exceeds the permissible limits is critical, that means that it is much higher than the amount of expected profits and the reserve created. The upper limit of the critical risk is determined by the bank's fund value, so in case of risk event the bank will not make a profit, as well as will lose its own fund. If the risk magnitude is even higher than the critical risk, the bank goes into bankruptcy which is succeeded by the liquidation procedure and the assets sale.

Credit risk is an important component of any bank portfolio and needs to be controlled and managed regularly. It is necessary to use the modern modeling methods and information technologies in order to reduce credit risk and determine its main indicators (probability of default, credit risk appetite, etc.).

The first who began to develop the credit risk assessment econometric models were Altman (Z-score model), Beaver and Tamar. Since then, the models have been improved by scientists and become more elaborate. Scholars [4; 5; 7; 9] in their papers on credit risk assessment develop and refine models, which help to determine according to the research purpose the probability of default, the level of loan portfolio losses, the variation amount of the loan portfolio value, the probability of non-redemption of nonperforming loans and others.

In recent years, large foreign financial institutions have developed a range of credit risk assessment models that vary in elaboration and application methodologies and are widely recognized throughout the world. *CreditRisk+, CreditMetrics, Moody's, KMV Portfolio Manager, Credit Monitor* and *CreditPortfolioView* (*McKinsey & Co., Inc.*) models are among them. In fact, these models are recommended standards to determine credit risks and the basis for the *VAR* approaches development. They allow to determine with different accuracy level the amount of losses associated with credit risk and to calculate the loan derivative value at risk (*value at risk,* thereinafter *VAR*). The models mentioned above have their own peculiarities, but their purpose is the same – to determine the losses apportionment in the credit risks portfolio and, on this basis, to calculate the expected loss in the portfolio at any confidence interval, the change in the extent of these losses and the amount of funds needed for the portfolio support.

The *VAR* determination models allow banks to estimate the difference between the funds needed for the portfolio support and the amount of capital required by the Basel Accords.

Moody's KMV Portfolio Manager model [13], in particular, provides the probability of default defining with the help of methods used in option pricing based on Black-Scholes and Merton models. The particular feature of *KMV Portfolio Manager* is the fact that it is based on the use of the empirical expected default frequencies (EDF), which is calculated with the help of *KMV Credit Monitor* software, developed by the same company.

The *CreditRisk+* model is aimed exclusively for the default risk assessment and does not estimate losses from other credit events. According to *CreditRisk+* model the default losses are estimated through the simple classification of assets by their size, however, the probability of default for each range falls under the gamma distribution, and then the values in ranges are aggregated into the losses distribution due to the default in all ranges.

The class of macroeconomic models includes the Wilson model [15; 16], which has provided the basis for the *CreditPortfolio View* software product, and is designated for credit risk assessment and has been developed by the McKinsey & Co. Consulting Group.

In recent years, SVM model developed by Bernhard E. Boser, Isabelle M. Guyon, and Vladimir N. Vapnik [3] has become more and more popular in the framework of the defaults forecasting as a nonlinear nonparametric algorithm for classification. Western commercial banks and rating agencies implement it in their business. However, despite its obvious advantages, SVM model has some limitations in loan scoring practice.

The review of existing techniques is extremely important for selection, implementation and adaption of the most appropriate model. At the same time, selecting the most appropriate approach, it is necessary to take into account the available mathematical tools, the nature and the quality of benchmark data, the planning horizon, the study objectives and the specifics of the bank portfolio. The personnel skill level, the extent of adoption and usage the latest IT systems and products in the bank are also of great importance. The usage of foreign models is rather complicated, as they have to be adjusted to the Ukrainian economy realities or the own models for credit risk assessment should be developed.

In Ukraine, the Resolution of the Board of the NBU No 351 [10] proposed a model of credit risk assessment for domestic banks, the application of which allows banks to provide a full and timely assessment of credit risk, which will facilitate the correct calculation of their capital and strengthen financial stability of the banking sector. Resolution of the NBU Board No 351 introduces improved approaches to assessing expected credit risk losses and is based on the Basel Principles of Banking Supervision [2], including the use of three components of credit risk (EAD – risk exposure, PD – probability of default of the debtor, LGD - loss in default). This Resolution and this credit risk assessment model itself have been developed for more than a year in cooperation with the banking community with the involvement of experts from the IMF, the World Bank, the international company Oliver Wyman, USAID.

This model is designed to address a number of significant gaps in the current requirements for credit risk assessment, which allowed banks to significantly underestimate the share of problem loans and the amount of credit risk on assets.

The model of credit risk assessment proposed by the Resolution of the NBU Board No 351 has a number of advantages, in particular:

- determining the amount of credit risk in monetary units, rather than interest, which allows you to estimate the actual amount of loss from lending to a particular borrower;
- taking into account when assessing the credit risk, the quality of credit collateral on the loan and the availability of guarantees, sureties or insurance protection under the loan agreement;

- ease of calculation (to determine the financial condition of borrowers – financial institutions that are large and medium-sized enterprises – only 6 indicators are used for financial institutions, small enterprises – 5 indicators, while in the methods of Fitch Ratings and Standard and Poor's such indicators are about 30);
- availability of criteria and scales for assigning the borrower's creditworthiness to the appropriate risk class;
- assessment of the level of credit risk is carried out on the basis of public information and financial statements of financial institutions.

However, some problems of adequate risk assessment of lending to financial institutions remain unresolved, in particular:

- subjectivity of determining the integral value of credit risk (the methodology does not substantiate exactly how qualitative criteria affect the level of credit risk);
- low formalization, taking into account qualitative criteria for credit risk assessment (there are no clear rules and scales in the methodology to determine which value of the qualitative criterion is low, medium or high);
- to assess the financial condition of borrowers of financial companies and credit unions, as well as enterprises of 15 other types of economic activity, the same logistics models are used, although the specifics of these companies differ significantly. Thus, there is an inconsistency of views on the appropriateness of using different approaches to assessing the risk of lending to financial institutions.

The proposed model allows to apply a complex credit risk assessment using the probability theory apparatus, integral calculations and differential equations, which enables to predict the credit risk level and make effective managerial decisions in risk management.

2 Method

A number of situations in which credit risk has been demonstrated have been investigated in the scope of the relevant study and mathematical models based on probability theory have been developed. In this case, the risk measurement procedure involves constructing of ρ function, which assigns a number to each random allocation:

$$\rho \colon Y \to R \tag{1}$$

where Y is the set of permissible in the particular problem probable distributions that represent risks; R is the set of real numbers.

The analysis and systematization of scientific sources and practical experience, at the present stage, distinguishes the next concepts in the structure of financial risks measurement procedures [8; 14]: losses in adversity; variability; financial risks measurement in the framework of within the expected utility theory; sensitivity.

The risks measurement according to the concept of losses in adversity is a relatively widespread phenomenon in world practice and the domestic banking system. Thus, it is proposed to estimate the expected losses assessed by the bank, depending on the probability of the borrower default, the amount of default losses and the loan extent at the time of default in order to assess credit risk under Basel II [1]:

$$EL = PD \times LGD \times EAD \tag{2}$$

where EL – expected losses; PD (probability of default) – the probability that the borrower will not be able to fulfill his obligations under the contract; LGD (loss given default) – the percentage of losses as relating to the credit amount that the bank will take at the time of the borrower default; EAD (exposure at default) – the loan amount granted at the moment of default.

Under this approach based on the Internal Rating (IRB) the bank will only assess the probability of default and will use data on losses in case of default and exposure to default provided by a credit or rating agency. The European Union has also adopted the methodology for the risk estimation according to the concept of losses in adversity by the Capital Adequacy Directive [6]. The National Bank of Ukraine (NBU) recommends this method to commercial banks for effective market risk management.

It is necessary in the framework of this paper to consider more precisely the concept of variability. It is based on the identification of so-called "location indicators", relating to which variability is calculated. As a rule, mathematical expectation, variance, median, mode, quantile can be used as "location indicators". In the scope of the variability concept loan risk has been constructed, as it is the most widespread and elaborate in the banking activity. Formalized approach to the credit financing has helped to come to conclusion that the lending process, especially at the stage of chargeback and repayment of deposit percentage, can be considered as a functional relationship to the time that is random variable. Therefore, it is reasonable to consider this process as a flow of borrower's payments at the certain point of time with a corresponding probability. Formalized approach to the credit operations also has allowed us to conclude that credit risk can be observed in three aspects. Firstly, the borrowers do not reimburse the bank loans entirely. Secondly, borrowers do not reimburse bank loans on schedule. Thirdly, borrowers reimburse bank loans entirely, but not on time and by partial amounts at one's own wish, breaking the loan amortization schedule both in time and amount payments. In practice, as a rule, all three aspects of credit risk can be observed simultaneously. Simultaneous action of all the aspects of credit risk will not be considered, as the situations when the loan is not reimbursed by the borrower and the payments are delayed in time are helpful in the analysis of the credit risk elementary components. It is supposed that the symbiosis of these components will help to simulate the third aspect of the credit risk. It can be argued that the process represents a functional relationship to time, as in case of incomplete credit repayment time latency will be observed. On the one hand, if the borrowers do not reimburse the entire amount of the credit, constructing a model it is necessary to take into account the reduction of the amounts which are returned according to maturity dates on schedule, and, on the other hand, in the case of the complete loan amortization, which is repaid with maturity dates break, it is necessary to consider the payments delay time when the repayment amounts are reimbursed completely.

It is obvious that, in its simplest form, the credit reimbursement process can be considered as a repayment of the credit sum S at the moment of time τ. Taking into account the fact that the credit reimbursement process is continuous in time, can change over time, and moreover, it can be of random character in terms of the sums of loan repayments, it is reasonable to introduce a function of time $\xi(t)$, which determines the loan repayment amount at time t. Thus, the appearance of a payment at time t, referring to the loan reimbursement, is a random variable and the value of the function $\xi(t)$ will be directly proportional to the loan repayment amount at time t. If the function $\xi(t)$ indicates by the flow of loan repayment amounts, each succeeding payment may lead to full or partial loan repayment. Therefore, it is reasonable to use the Boolean function $F(U)$ to define the flag indicator U according to which the loan is reimbursed completely. Correspondingly, if $F(U) = 1$, U possess the "truth" value, that means, that credit has been reimbursed completely, and if $F(U) = 0$, U possesses "false" value, that means, that the loan has not yet been repaid.

First of all, it is necessary to consider the situation of incomplete loan reimbursement. Since the process is of random nature, let's define the random variable μ, which denotes the proportion of the reimbursed loan and, correspondingly, takes the value on the interval [0,1]. Thus, the incomplete loan reimbursement process can be represented by the next formula:

$$\xi(t) = \mu \times S \times F\{t \geq \tau\} \tag{3}$$

where μ is a random variable; S is a loan repayment amount before reimbursement process; $F\{t \geq \tau\}$ is a Boolean function.

It is obvious that the random variable μ has a probabilistic nature and the parameters of its distribution form the variable which characterizes the risk of lending operations. Correspondingly, if the probability $P\{\mu = 1\} = 1$, there is no risk, and if $P\ \{\mu = 0\} = 0$, there is the risk event. The nature of this distribution is rather interesting, problematic and highly discussed among the scientists. In the scope of this paper the distribution of random variable will be considered to be normal. In general, the parameters of this distribution can be investigated:

– on the basis of the mathematical expectation (E) of a random variable:

$$E_{\xi(\tau)} = S \times E_{\mu} \tag{4}$$

where E is the mathematical expectation;

– on the basis of variance:

$$D_{\xi(\tau)} = S^2 \times D_{\mu} \tag{5}$$

where D is a variance.

In this case the payment at the time t is a random variable, correspondingly, $E_{\xi(\tau)}$ and $D_{\xi(\tau)}$ are the average and the variance of the payments flow, $S{\cdot}E_{\mu}$ is the loan repayment average amount.

Let's consider the next case, when loans are returned, but not on schedule, and there is the certain delay time relating to the loan amortization schedule, which can be defined by δ. Thus, the loan reimbursement process can be represented by the formula:

$$\xi(t) = S \times F\{t \geq \tau + \delta\} \tag{6}$$

It is obvious, that δ is random variable, which cannot be of negative value and the parameters of its distribution characterize the risk of credit operations. Thus, there is no risk in the case of $P\{\delta=0\}=1$, and the risk is very high in the case of $P\{\delta=\infty\}=0$, that means that the loan is not reimbursed. Simulating this process, it has been assumed that as the final result the loan would be reimbursed. The amount of payments at any moment of time $t \geq \tau$ can possess the values 0 or S. Regarding the payments flow, the loan repayment amount 0 or S can appear at different moment of times t with varying probability. Thus, the flow average value and variance will be correspondingly equal to:

$$E_{\xi(t)} = S \times P\{\delta \leq t + \tau\} \tag{7}$$

$$D_{\xi(\tau)} = S^2 \times P\{\delta \leq t - \tau\} \times (1 - P\{\delta \leq t - \tau\}) \tag{8}$$

In this model, the average delay time is δ, and the average loan repayment amount is S, that means, that the model presumes that sooner or later the loan will be reimbursed.

Further the model which would try to take into account all the aspects of credit risk, such as time delays and loan reimbursement payments has been built. The set of random variables possessing values on the interval [0, 1] has been defined as $\{\mu_t, t \geq 0\}$ and the following equation is true for it:

$$P\{\mu_0 > z\} > 0 \text{ to all } z<1 \tag{9}$$

Thus, it is possible to estimate the effect of all the credit risk aspects with the help of the next correspondence:

$$\xi(t) = \mu_{t-\tau} \times S \times F\{t \geq \tau\} \tag{10}$$

In this case, the possibility of loan reimbursement by repeated partial payments is taken into account. In addition, the equation (9) ensures a positive probability of complete loan reimbursement on schedule, since it is not rational to assume that the loan has been issued if there has been the confidence that the borrower would not able to return it. Thus, the next model is built:

$$E_{\xi(t)} = S \times E_{\mu_{t-\tau}} \tag{11}$$

$$D_{\xi(t)} = S^2 \times D_{\mu_{t-\tau}} \tag{12}$$

Correspondingly, the average return share is equal to:

$$\lim_{t\to\infty} E_{\mu_t} \tag{13}$$

Practical calculations for this model face the problem of determining the start values, for example, the probabilistic distribution of the loan repayment random variable μ. We believe that the normal (Gaussian) distribution of a random variable is appropriate for this, however μ should have a positive value and cannot greater than 1, as this will contradict the normal distribution condition. If, however, the given random variable is assumed to be Gaussian with some restrictions, and to characterize the amount of client's available assets with the help of which the loan can be reimbursed, its values which exceed mean, that a client has spare funds and the loan can be completely repaid, and if this random variable is of negative value, a client is insolvent or debtor.

Let's estimate the parameters of these models, since only if these parameters exist the model can be implemented in practice. For example, let's consider the first model and set the distribution of the partial loan repayment random variable μ. It is necessary to choose a set of distributions to which μ can be attributed for this purpose. The normal (Gaussian) distribution, which is defined by the mathematical expectation E and the variance D has been chosen. However, the usage of normal distribution is still controversial. Firstly, μ must be of positive value. Although it is not of great importance, since taking into account the typical values of μ makes the probability of its negative value so slight that it can be neglected. Secondly, the μ partial cannot be greater than 1, and such a probability for a normal distribution with typical values of E and D is very high. So, how this problem can be solved?

It can be assumed that there is a random Gaussian variable λ that characterizes the borrower's ability to repay the loan in order to solve this problem. Let's suppose, for example, that it is the indicator of borrower's available assets at the time of debt repayment. Thus, if λ is greater than 1, a borrower has even more funds than needed for complete loan reimbursement and a bank can receive its assets on schedule. If λ is less than 1, a borrower not only does not have enough money to repay the current loan, but probably has other debts. Thus, the next equity can be truth for any random variable λ, which characterizes the client's ability to reimburse the credit:

$$\mu = min(\, max(\lambda, 0), 1) \tag{14}$$

Further it is necessary to define the parameters E and D. It is sufficient to estimate the mathematical expectation of E_μ and the probability $P\{\mu = 0\}$ in order to do this. The usage of the bank balance data, that are turnover and balance data on the corresponding accounts, is the easiest way to do this. It is obvious that this estimation can be obtained by dividing the total amount of delayed loan payments by the total loan amount.

Let's denote the repayment amount of loan debt obligations, the obtaining of which is scheduled at the moment of time t by $P(t)$, and let's indicate the amount of delayed payments, which maturity date at the time t has already passed, by $B(t)$. Thus, the loan reimbursement process can be characterized by the functions $p(u)$,where $P(t)$ is the volume from the repayments amount received by the bank for the time increment u (from time t to time $t+u$), and $b(u)$ is the proportion from the repayments amount $P(t)$, which has not been paid at time t. It is obvious, that:

$$p(0) = b(0) = 0 \tag{15}$$

$$p(u) + b(u) = 1 \tag{16}$$

As it can be seen, these functions do not vary depending on time t and therefore, it is appropriate instead of $P(t)$ and $B(t)$ to take their mean values P and B for the time interval u. Thus, the E_μ can be estimated by the approximate formula:

$$E_\mu = \frac{B}{P} \tag{17}$$

where B is the mean value of the delayed payments amount; P is the mean value of the repayment amount of the debt obligations on the loan.

Formula (17) does not vary depending on the scale of the time measurement and, therefore, is correct, and the B and P variables are defined by the balance sheet. It is obvious, that this is not the only way to estimate E_μ, and, therefore, the question can be controversial. It should be noted that in this case the model parameters have been estimated in simplified way and therefore the initial results of the research can be extended in the direction of detalization, justification and elaboration. Let's try to define the result obtained above more exactly. Let's define the amount of bad debts that have been written off at the cost of the reserve at time t and are nonreturnable – by $Z(t)$ is. Thus, the loan reimbursement process can be characterized by the functions $p(u)$, where $P(t)$ is the volume from the repayments amount received by the bank for the time increment u (from time t to time $t+u$), $b(u)$ is the proportion from the repayments amount $P(t)$, which has not been paid at time t, and $z(u)$ is proportion of the payments amount $P(t)$ that has been written off. It is obvious, that:

$$p_t(0) = b_t(0) = z_t(0) \tag{18}$$

$$p_t(u) + b_t(u) + z_t(u) = 1 \tag{19}$$

If it is assumed that the nature of requirements for debt forgiveness does not depend on the time, they are scheduled for, that means, that the requirements quality are approximately the same, these functions do not vary depending on time t. Thus, the next estimation for the functions $B(t)$ and $Z(t)$ can be obtained:

$$B(t) = \int_0^\infty P(t-u) \cdot g^{'}(u)du \tag{20}$$

$$Z(t) = \int_0^\infty P(t-u) \times z^{'}(u)du \tag{21}$$

The process of loan payments obtaining and writing off in the general case can be represented by a system of differential equations:

$$p'(u) = \alpha \times b(u) \tag{22}$$

$$z'(u) = \beta \times b(u) \tag{23}$$

Thus, taking into account the formulas (9) and (10), the system of equations has been developed:

$$\begin{cases} p'(u) = \alpha \times b(u) \\ z'(u) = \beta \times b(u) \\ p_t(0) = b_t(0) = z_t(0) \\ p_t(u) + b_t(u) + z_t(u) = 1 \end{cases} \quad (24)$$

Let the average values of *P*(*t*), *B*(*t*), *Z*(*t*) to be correspondingly *P*, *B*, *Z* for a certain period of time. Thus, after substituting the mean values in formulas (20), (21) and finding the solution of the obtained system of differential equations, the following approximate formula for E_μ estimation can be developed:

$$E_\mu = \frac{1-\frac{Z}{P}}{1+\frac{B}{P}} \quad (25)$$

where *B* is the mean value of the delayed payments amount; *P* is the mean value of the repayments amount of the loan debt obligations; *Z* is the mean value of the bad debts amount that have been written off at the cost of reserves.

Moreover, investor's ration can be added to the model. Thus, overelaborating of the model gives the possibility to make more extended analysis of the information on bank credit risks elimination:

$$E_\mu = \frac{1-\frac{Z}{P}}{1+\frac{r\times B}{P}} = \frac{P-Z}{P+r\times B} \quad (26)$$

where *r* is the coefficient that can be defined as a market category, such as market risk.

At the last stage the level of credit risk is determined by E_μ variable interpretation with the help of linguistic characteristics of the bank credit risk levels scale (table 1). The proposed scale is based on the calculation of the integrated index of the financial status of a debtor – a corporate entity class, applying the correction factors specified in paragraph 22 Section II of the NBU Regulation №351. Loan payment by a debtor – a corporate entity in time is adopted as correction factors. However, the following demands should be hold to: if there is a delayed payment from 31 to 60 days – the bank gives a debtor the class that is not higher than 5; if this payment is delayed from 61 to 90 days, the class is not higher than 8; in the case of 91 and more days of delayed payment, a debtor gets the class not higher than 10 [16]. Taking into account the fact that the NBU methodology can define the credit risk losses volume, and the proposed model can determine the loan repayment level, it can be concluded, that estimation scales are of inverse character.

Thus, in our opinion, it is necessary to distinguish the following levels of credit risk: acceptable, moderate and critical.

The mathematical expectation of the credit risk level E_μ acquires a value in the range {0; 1}, and the closer it is to 1, the lower the bank credit risk level is. Thus, the formula for E_μ estimation has been developed. It does not depend on time and uses data that can be obtained from the bank's balance. It is obvious that the result obtained is not the only solution to this problem and can be clarified by different ways, which need to be discussed in further investigations.

Table 1. Characteristics of bank credit risk levels.

Credit risk level	Credit risk bounds	Characteristics
Acceptable	1,00– 0,90	Risk of low level, which can be temporarily ignored. Acceptable payment delay – from 31to 60 days.
Moderate	0,89 – 0,41	Moderate risk level, which should be carefully controlled by bank management. Acceptable payment delay – from 61 to 90 days.
Critical	0,40 – 0	Risk of high level, which can lead to bankruptcy. Acceptable payment delay – from 91 days.

It should be noted that the proposed model has certain advantages and limitations. The main advantages are: an integrated approach to the credit risk assessment modeling, which involves various loan repayment scenarios that are relevant from the aspect of early diagnosis of possible credit risk and the bank's decision of its minimization or taking. Taking into account the high share of problem loans in domestic banks, this advantage is particularly important; the possibility to use this model to predict the acceptable level of credit risk, on the basis of which the optimal credit portfolio can be formed; flexibility in application of this model, as it can be used both for the credit risk assessment of the bank credit portfolio in general, and for its components (for example, for a credit portfolio of corporate clients or individuals); ease of use and implementation of this model in bank information systems.

The main difficulties associated with the integrated credit risk assessment based on the proposed model are: insufficiency or total absence of historical data; the absence or inconsistency of statistical data due to the specifics of the banking business or the credit policy peculiarities. However, this should not prevent the banks from developing and applying the proposed model, the input data for which can be obtained from the open source information or based on the experts' reports. The development and implementation of the own methods and models for credit risk assessment, which will provide risk management with the input data for the formation of a bank credit strategy is an extremely important step for domestic banks. In general, it can be argued that there is a problem of effective modeling of credit risk according to the variability concept. This is primarily associated with the asymmetry of the lending activities process, which results into an asymmetric distribution of random variables and correspondingly increases the margin of error, as in the case of the Gaussian distribution. On the other hand, indicators in this model have a tendency to "dispersion" or "scattering" in different directions, that is not inherent in the risk assessment, since as a rule, deviation is considered under adversity. Despite the fact that there are problems referring model parameters assessment, they have been estimated with some restrictions and can be specified in other ways. As a result, the obtained formulas using the real balance data of the balance can be implemented in the banking systems development.

3 Results and discussion

Let's consider the proposed credit risk assessment model on the example of the Ukrainian joint-stock company commercial bank PrivatBank (JSC CB PrivatBank). The obtained results will be compared with the results obtained by the method proposed by NBU in order to determine the model adequacy.

Let's calculate and assess the integrated credit risk of the Ukrainian JSC CB PrivatBank according to the requirements and methods of the National Bank of Ukraine. The information on the credits' distribution by classes of corporate and private debtors, published by the National Bank of Ukraine on January 1, 2018, serves as an input indicator for the bank's credit risk calculation. According to the statistical reporting of corporate JSC CB PrivatBank we have made the table 2 on the credits distribution by classes of debtors [11; 12;].

Table 2. The credits distribution by classes of debtors in JSC CB PrivatBank.

Date	Class of a corporate debtor, billion UAH									
	1	2	3	4	5	6	7	8	9	10
01.02.18	1,6	0,32	0,2	0,78	0,19	0,1	0,44	0,01	0,003	209,8
01.03.18	1,6	0,38	0,24	0,1	0,22	0,12	0,45	0,02	0,003	175,2
01.04.18	2,0	0,37	0,42	0,22	0,07	0,02	0,45	0,01	0,14	202,9
01.05.18	2,2	0,46	0,45	0,28	0,1	0,04	0,42	0,01	0,19	202,9
01.06.18	2,5	0,47	0,41	0,31	0,15	0,21	0,43	0,005	0,041	202,8
01.07.18	2,7	0,61	0,4	0,5	0,13	0,23	0,42	0,003	0,58	203,2
01.08.18	2,6	0,56	0,49	0,44	0,17	0,25	0,44	0,007	0,044	204,5
01.09.18	2,65	0,58	0,57	0,57	0,26	0,24	0,44	0,008	0,041	206,9
01.10.18	2,7	0,67	0,77	0,73	0,25	0,3	0,43	0,006	0,028	215,2
01.11.18	2,7	0,73	0,85	0,94	0,28	0,27	0,01	0,47	0,04	215,5
01.02.19	3,31	0,74	2,49	0,96	0,57	0,04	0,09	0,03	0,25	216,6
01.03.19	3,27	1,03	2,4	0,97	0,55	0,05	0,09	0,03	0,13	215,9
01.04.19	3,54	0,9	2,4	1,23	0,55	0,1	0,16	0,01	0,21	216,8
01.05.19	3,76	0,96	2,28	0,98	1,22	0,46	0,03	0,01	0,19	216,1
01.06.19	3,94	0,82	2,34	1,05	1,35	0,15	0,09	0,01	0,22	216,4
01.07.19	4,22	0,86	2,58	0,97	1,37	0,05	0,12	0,01	0,25	215,5
01.08.19	3,93	0,63	2,52	0,86	1,63	0,09	0,09	0,01	0,29	214,01
01.09.19	4,01	1,53	2,05	0,46	1,77	0,09	0,02	0,05	0,22	214,47
01.10.19	4,01	1,37	2,33	0,86	1,22	0,13	0,01	0,01	0,25	212,80
01.11.19	3,93	1,61	2,29	0,99	1,15	0,13	0,02	0,01	0,27	214,59
01.12.19	4,31	1,59	2,06	1,19	0,58	0,73	0,02	0,01	0,24	213,26
01.01.20	4,35	1,61	2,22	1,18	0,55	0,03	0,03	0,01	0,22	212,85

The loan debtor's losses given default (LGD) granted to corporate entities of JSC CB PrivatBank according to the scenario of a compromise position of risky lending for 10 months of 2018 and 2019 is calculated in table 3.

Thus, the credit risk of JSC CB PrivatBank for loans granted to corporate entities in the scenario of a compromise position of risk lending for 10 months of 2018 is measured by the losses amount from UAH 175,456 billion at the date of March 1, 2018 to UAH 213,162 billion at the date of January 1, 2020.

Table 3. Credit risk for loans granted to corporate entities according to the scenario of a compromise position of JSC CB PrivatBank risky lending in 2018, 2019.

Date	Bank losses for loans, granted to corporate entities by a borrower's – a corporate entities classes, billion UAH										Total losses, billion UAH
	1 (0,5%)	2 (1,0%)	3 (2%)	4 (4%)	5 (7%)	6 (11%)	7 (18%)	8 (33%)	9 (60%)	10 (100%)	
01.02.18	0,008	0,003	0,004	0,031	0,001	0,011	0,079	0,003	0,002	209,8	209,942
01.03.18	0,008	0,004	0,005	0,004	0,015	0,13	0,081	0,007	0,002	175,2	175,456
01.04.18	0,01	0,004	0,008	0,009	0,005	0,02	0,081	0,003	0,084	202,9	203,124
01.05.18	0,011	0,005	0,009	0,011	0,007	0,004	0,077	0,003	0,114	202,9	203,141
01.06.18	0,012	0,005	0,008	0,012	0,011	0,023	0,077	0,002	0,025	202,8	202,975
01.07.18	0,014	0,006	0,008	0,02	0,009	0,025	0,077	0,001	0,35	203,2	203,836
01.08.18	0,013	0,006	0,01	0,018	0,012	0,028	0,079	0,002	0,026	204,5	204,694
01.09.18	0,014	0,006	0,011	0,023	0,018	0,026	0,079	0,003	0,025	206,9	207,105
01.10.18	0,014	0,007	0,015	0,029	0,018	0,033	0,077	0,002	0,017	215,2	215,43
01.11.18	0,014	0,007	0,017	0,038	0,02	0,03	0,002	0,155	0,024	215,5	215,807
01.02.19	0,017	0,007	0,050	0,038	0,040	0,004	0,016	0,010	0,15	216,6	216,933
01.03.19	0,016	0,010	0,048	0,039	0,039	0,006	0,016	0,010	0,078	215,9	216,162
01.04.19	0,018	0,009	0,048	0,049	0,039	0,011	0,029	0,003	0,126	216,8	217,132
01.05.19	0,019	0,010	0,046	0,039	0,085	0,051	0,005	0,003	0,114	216,1	216,472
01.06.19	0,020	0,008	0,047	0,042	0,095	0,017	0,016	0,003	0,132	216,4	216,779
01.07.19	0,021	0,009	0,052	0,039	0,096	0,006	0,022	0,003	0,15	215,5	215,896
01.08.19	0,020	0,006	0,050	0,034	0,114	0,010	0,016	0,003	0,174	214,01	214,438
01.09.19	0,020	0,015	0,041	0,018	0,124	0,010	0,004	0,017	0,132	214,47	214,851
01.10.19	0,020	0,014	0,047	0,034	0,085	0,014	0,002	0,003	0,15	212,8	213,170
01.11.19	0,020	0,016	0,046	0,040	0,081	0,014	0,004	0,003	0,162	214,59	214,975
01.12.19	0,022	0,016	0,041	0,048	0,041	0,080	0,004	0,003	0,144	213,26	213,658
01.01.20	0,022	0,016	0,044	0,047	0,039	0,003	0,005	0,003	0,132	212,85	213,162

Losses from credit risk on loans granted to corporate entities of JSC CB PrivatBank according to the scenario of the aggressive position of risky lending for the period of 10 months of 2018 and 2019 are represented in the table 4. Thus, the credit risk of JSC CB PrivatBank for loans granted to corporate entities according to the scenario of an aggressive position of risky lending for the research period is measured by the losses amount from UAH 175,565 billion at the moment of March 1, 2018 to UAH 213,351 billion at the date of January 1, 2020.

The calculation of the mathematical expectation of loan repayments according to the proposed model and the analysis of the results are represented in the table 5. Taking into account the results given in the table 5, it can be concluded that the E_μ value is the most closely approximate to zero, that gives evidence of the critical level of JSC CB PrivatBank credit risk in 2018-2019, which corresponds to the results of the NBU methodology. The results obtained with the help of the proposed model and the NBU methodology do not differ significantly by the majority of indicators. Thus, it can be concluded that the proposed model is an effective tool for credit risk assessment in banking business. This statement has been tested on actual data.

Table 4. Credit risk for loans granted to corporate entities according to the scenario of an aggressive position of JSC CB PrivatBank risky lending in 2018, 2019.

Date	Bank losses for loans, granted to corporate entities by a borrower's – a corporate entities classes, billion UAH										Total losses, billion UAH
	1 (0,9%)	2 (1,9%)	3 (3%)	4 (6%)	5 (10%)	6 (17%)	7 (32%)	8 (59%)	9 (99%)	10 (100%)	
01.02.18	0,144	0,006	0,006	0,047	0,019	0,017	0,141	0,006	0,003	209,8	210,189
01.03.18	0,144	0,007	0,007	0,006	0,022	0,02	0,144	0,012	0,003	175,2	175,565
01.04.18	0,18	0,007	0,013	0,013	0,007	0,003	0,144	0,006	0,139	202,9	203,412
01.05.18	0,198	0,009	0,014	0,017	0,01	0,007	0,134	0,006	0,188	202,9	203,479
01.06.18	0,225	0,009	0,012	0,019	0,015	0,036	0,138	0,003	0,041	202,8	203,334
01.07.18	0,243	0,012	0,012	0,03	0,013	0,039	0,134	0,002	0,574	203,2	204,247
01.08.18	0,234	0,011	0,015	0,026	0,017	0,043	0,141	0,004	0,436	204,5	205,427
01.09.18	0,239	0,011	0,017	0,034	0,026	0,041	0,141	0,005	0,041	206,9	207,455
01.10.18	0,243	0,013	0,023	0,044	0,025	0,051	0,138	0,004	0,028	215,2	215,769
01.11.18	0,243	0,014	0,026	0,056	0,028	0,046	0,003	0,28	0,039	215,5	216,235
01.02.19	0,030	0,014	0,075	0,058	0,057	0,007	0,029	0,018	0,248	216,6	217,134
01.03.19	0,029	0,020	0,072	0,058	0,055	0,009	0,029	0,018	0,129	215,9	216,318
01.04.19	0,032	0,017	0,072	0,074	0,055	0,017	0,051	0,006	0,208	216,8	217,332
01.05.19	0,034	0,018	0,068	0,059	0,122	0,078	0,010	0,006	0,188	216,1	216,683
01.06.19	0,035	0,016	0,070	0,063	0,135	0,026	0,029	0,006	0,218	216,4	216,997
01.07.19	0,038	0,016	0,077	0,058	0,137	0,009	0,038	0,006	0,248	215,5	216,127
01.08.19	0,035	0,012	0,076	0,052	0,163	0,015	0,029	0,006	0,287	214,01	214,685
01.09.19	0,036	0,029	0,062	0,028	0,177	0,015	0,006	0,030	0,218	214,47	215,070
01.10.19	0,036	0,026	0,070	0,052	0,122	0,022	0,003	0,006	0,248	212,8	213,384
01.11.19	0,035	0,031	0,069	0,059	0,115	0,022	0,006	0,006	0,267	214,59	215,201
01.12.19	0,039	0,030	0,062	0,071	0,058	0,124	0,006	0,006	0,238	213,26	213,894
01.01.20	0,039	0,031	0,067	0,071	0,055	0,005	0,010	0,006	0,218	212,85	213,351

Table 5. Comparative results of the JSC CB PrivatBank credit risk assessment according to the proposed model and the NBU methodology.

Date	According to the proposed model						According to the NBU methodology [12]	
	B, UAH billion	Z, UAH billion	P, UAH billion	E_μ	Repayments amount, UAH billion	Losses volume, UAH billion	Losses according to the compromise scenario, UAH billion	Losses according to the aggressive scenario, UAH billion
01.02.18	2,936	209,849	212,785	0,0136	2,894	209,891	209,942	210,189
01.03.18	3,216	208,903	212,119	0,0149	3,161	208,958	175,456	175,565
01.04.18	3,557	203,033	206,59	0,0169	3,491	203,099	203,124	203,412
01.05.18	3,991	203,064	207,055	0,0189	3,913	203,142	203,141	203,479
01.06.18	4,435	202,817	207,252	0,0209	4,332	202,920	202,975	203,334
01.07.18	4,936	203,253	208,189	0,0231	4,809	203,380	203,836	204,247
01.08.18	4,971	204,588	209,559	0,0232	4,862	204,697	204,694	205,427
01.09.18	5,314	206,986	212,3	0,0243	5,159	207,141	207,105	207,455
01.10.18	5,810	215,182	220,992	0,0256	5,657	215,335	215,43	215,769
01.11.18	6,194	215,488	221,682	0,0272	6,030	215,652	215,807	216,235

Date	According to the proposed model						According to the NBU methodology [12]	
	B, UAH billion	Z, UAH billion	P, UAH billion	E_{μ}	Repayments amount, UAH billion	Losses volume, UAH billion	Losses according to the compromise scenario, UAH billion	Losses according to the aggressive scenario, UAH billion
01.02.19	8,23	216,85	225,08	0,0353	7,945	217,135	216,933	217,134
01.03.19	8,39	216,03	224,42	0,0360	8,079	216,341	216,162	216,318
01.04.19	9,06	217,01	226,07	0,0385	8,704	217,366	217,132	217,332
01.05.19	9,70	216,29	225,99	0,0412	9,311	216,679	216,472	216,683
01.06.19	9,75	216,62	226,37	0,0413	9,349	217,01	216,779	216,997
01.07.19	10,18	215,75	225,93	0,0431	9,738	216,192	215,896	216,127
01.08.19	9,76	214,3	224,06	0,0417	9,343	215,327	214,438	214,685
01.09.19	9,98	214,69	224,67	0,0425	9,549	215,121	214,851	215,070
01.10.19	9,94	213,05	222,99	0,0427	9,522	213,468	213,170	213,384
01.11.19	10,13	214,86	224,99	0,0431	9,697	215,293	214,975	215,201
01.12.19	10,49	213,50	223,99	0,0447	10,012	213,978	213,658	213,894
01.01.20	9,98	213,07	223,05	0,0428	9,547	213,953	213,162	213,351

4 Conclusion

The proposed model of credit risk assessment according to the concept of variability is universal and allows us to predict the credit risk level and make effective management decisions due to the means of probability theory, integral calculations and differential equations. This model provides to determine the mathematical expectation E_{μ}, using data, which are not dependent on time and can be obtained from a banking institution's balance sheet. The model uses a comprehensive approach to credit risk assessment modeling, which involves different ways of loan repayment.

The main advantages of the proposed credit risk assessment model include: a complex approach to modeling credit risk assessment, which involves different kinds of credit repayment; the possibility to use this model for the acceptable credit risk level prediction, on the basis of which the optimal bank credit portfolio can be built; universality of this model (it can be used both for the assessment of bank porfolio credit risk taken as a whole, and for its components); ease of use.

The proposed model for credit risk assessment is one of a set of models, which is primarily can be used during development of automated banking systems, credit risk management systems and expert systems. This model can be also useful for verification of new methods for borrowers' credit capacity assessment and credit risk forecasting.

The model can be included to the so-called "block of models", which is a part of modern decision-making systems. Such systems can be used in banking business, that significantly increases the level of financial management in the area of credit risk management.

References

1. Basel Committee on Banking Supervision (BCBS): Basel II: International Convergence of Capital Measurement and Capital Standards: A revised framework, comprehensive version. http://www.bis.org/publ/bcbsc111.htm (2005). Accessed 27 Aug 2020
2. Basel Committee on Banking Supervision: Revisions to the Standardised Approach for credit risk, 2nd consultation. Bank for International Settlements. http://www.bis.org/bcbs/publ/d347.pdf (2015). Accessed 20 August 2020
3. Boser, B.E., Guyon, I.M., Vapnik, V.N.: A training algorithm for optimal margin classifiers. In: COLT '92: Proceedings of the fifth annual workshop on Computational learning theory, pp. 144–152 (1992). doi:10.1145/130385.130401
4. Cai, R., Zhang, M.: How does credit risk influence liquidity risk? Evidence from Ukrainian banks. Visnyk of the National Bank of Ukraine **241**, 21–33 (2017)
5. Chatterjee, S.: Modelling credit risk. Centre for Central Banking Studies, Bank of England, London. https://www.bankofengland.co.uk/-/media/boe/files/ccbs/resources/modelling-credit-risk (2015). Accessed 29 Jul 2019
6. Council Directive 93/6/EEC of 15 March 1993 on the capital adequacy of investments firms and credit institutions. Official Journal of the European Communities **36**(L 141), 0001–0026
7. Gostrik, O.A., Stepanenko, O.A.: Definition of permissible credit risk of commercial bank with using the method of system dynamics. Scientific Herald of Uzhgorod National University 6(3), 60–63 (2016)
8. Kaminsky, A.B.: Conceptual approaches to the measurement of financial risks. Finance of Ukraine 5, 78–85 (2006)
9. Sheu, H.-J., Liu, C.-L.: The Credit Risk Pricing with Particle Filter Approach. Proceedings of the 9th Joint Conference on Information Sciences, JCIS 2006, CIEF-89
10. The National Bank of Ukraine. Regulation No 351 on approval of the Regulation on the Ukrainian banks determination of the credit risk magnitude for active banking transactions. https://zakon2.rada.gov.ua/laws/show/v0351500-16 (2016). Accessed 17 Aug 2020
11. The National Bank of Ukraine: Statistics. financial statements of Ukrainian banks. https://bank.gov.ua/files/stat/Inc_Exp_Banks_2020-09-01.xlsx (2020). Accessed 22 Aug 2020
12. The National Bank of Ukraine: Statistics. The main indicators of the banks of Ukraine. https://bank.gov.ua/files/stat/Indicators_Banks_2020-09-01.xlsx (2020). Accessed 22 Aug 2020
13. Vasicek, O.A.: Probability of Loss on Loan Portfolio. KMV Corporation, San Francisco. http://www.moodyskmv.com/research/files/wp/Probability_of_Loss_on_Loan_Portfolio.pdf (1987). Accessed 17 Aug 2020
14. Vitlinskyi, V.V., Velykoivanenko, H.I.: Financial risk and methods of its measurement. Finance of Ukraine 12, 13–23 (2000)
15. Wilson, T.: Portfolio Credit Risk (I). Risk **10**(9), 56–61 (1997)
16. Wilson, T.: Portfolio Credit Risk (II). Risk **10**(10), 111–117 (1997)