The cryptocurrencies risk measure based on the Laplace distribution

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Abstract. Current research has led to a rejection of the hypothesis of a normal distribution of financial assets returns. Under these conditions, portfolio variance cannot serve as a good risk measure. In this paper analyzed the daily returns of the most common cryptocurrencies: Bitcoin, Ethereum, XRP, USDT, Bitcoin Cash, Litecoin. It is shown that the asset returns are not normally distributed, but with good precision follow the Cauchy distribution and Laplace distribution. The analytical expressions for risk measure were obtained using the distribution function and the VaR technique. However, the risk assessment of the return obtained on the basis of the Cauchy distribution is twice as high as the risk assessment obtained on the basis of the Laplace distribution. Therefore, the question arises: what distribution law to use to measurement the cryptocurrency risk? The paper shows that the Laplace distribution is the most adequate basis for measuring of cryptocurrencies risk.

Keywords: cryptocurrency, expected return, return distribution, risk measure, portfolio of assets.

1 Introduction

The first complete cryptographic currency appeared in 2008 thanks to the efforts of Satoshi Nakamoto. It was named Bitcoin. New varieties of digital currency appear each year due to the information technology active development and the globalization processes spread. The main advantages of cryptography are that the user controls them without any regulatory rules in the transaction. Third party costs on a transaction can be greatly reduced. This has been the main reason for the rapid development of the market for virtual currencies (crypto-currency) over the past 10 years. More than 2000 varieties of digital money have appeared on the market since the birth of Bitcoin for 5 years. Bitcoin (BTC) remains the most widespread cryptocurrency: there is the largest market capitalization among other digital currencies (about $220 billion) [23]. The first positions of the market capitalization rating as of July 2020 are the following cryptocurrencies: ETH (Ethereum) – about $45 billion, XRP (Ripple) – about $12 billion, USDT (Tether) – about $10 billion, LTC (Litecoin) and BCH (Bitcoin Cash) – $4-5 billion each.

But investments in cryptocurrency can be quite risky as their price is very volatile [5; 12; 18; 19; 20]. Thus, during the period from July 2018 to July 2019 there were significant changes in the exchange rate. Initially, the cost of one Bitcoin was $6,600 (July 2018). There was a significant dropping in mid-December 2018 in the price – to $3,200. Then there was a sharp increasing at the end of June 2019 – to $13,000. The price of Bitcoin Cash fluctuated from $869 per unit (July 2018) to $77 per unit (mid-December 2018) to $400 per unit in June 2019. The price of the unit XRP demonstrated a sharp jump from $0.26 to $0.58 during three weeks in September 2018. Then it began to fall with slight fluctuations. The course of the ordinary currency (dollars, euros, etc.) strongly depends on inflation, politic factors and other economic conditions. Thus, its calculations can be performed fairly accurately, taking into account the influence factors changing. Instead, fluctuations in the price of cryptocurrency are very difficult to forecast. Therefore, making the correct decisions in investing and trading cryptocurrency in order to get the most return is a rather difficult task. The interaction between supply and demand, the attractiveness for investors, macroeconomic conditions and financial events are important factors in the formation of the cryptocurrency price [10]. In addition, investors rely vastly on speculation and rumors that also affect the cryptocurrency price change.

2 Literature review

Diversification is an important risk reduction tool. Creating a portfolio of financial assets is one of its instruments. In this paper, the formation of cryptocurrency investment portfolio based on the Markowitz model is investigated [17]. By changing the proportion of certain assets in a portfolio, it can be managed to maximize return or to minimize risk. The Markowitz model relies on the hypothesis of a normal distribution of returns. This hypothesis significantly simplifies the problem of choosing a portfolio for investing, since it allows you to compare alternative portfolios by just two criteria: standard deviation and mathematical expectation. However, numerous theoretical researches in the field of finance [2; 13; 15; 16; 21; 24] and the events in the financial market at the end of 2008 – early 2009 are doubted the hypothesis of a normal distribution of return.

It has been shown that the distribution of financial assets contains so-called “heavy tails”. It indicates a high likelihood of realization of very large and very small return values. The task of this work is investigating the distribution of the return of virtual currencies and using it to minimize the risk of working with portfolios of cryptocurrencies. The results of the study [11] are shown that the inclusion in the investment portfolio of several cryptocurrencies brings to investors the advantages of diversification for short term investments.

Building a portfolio solely on the basis of cryptocurrencies [8] shows that a cryptocurrencies set increases investment opportunities with a low level risk. In contrast to our research, this work does not take into account the possible deviation of the distribution of the cryptocurrency return from the normal one. In the work [1] researchers apply a portfolio diversification strategy that is based on several models of

portfolio formation. So, on the basis of the modern portfolio theory, an optimal risk portfolio has been established and the effect of cryptocurrency on the usual investment portfolio of assets has been investigated. The results, obtained in [6], show that the expected return on the cryptocurrency portfolio is greater than the return of separate cryptocurrency. The risk assessment was carried out according to the quantile method, but unlike our research, the distribution of assets return does not determine.

The authors of [4] emphasize the importance of modeling nonlinearity and taking into account the behavior of tail distribution in analyzing the causal relationships between Bitcoin revenues and trading volume. For analysis the Bitcoin behavior in the study [7] taking into account heavy tails of return distribution, quantile regression is used. This made it possible to determine that Bitcoin does act as a hedge against market uncertainty. Yet, the quantile method is applied only to Bitcoin analysis without specifying the asset return distribution [4; 7]. The authors of the article [9] analyzed some statistical properties of the largest cryptocurrencies, in particular their distribution law. In the study accentuated that the return is clearly non-normal. Several types of distribution have been identified, which are subject to certain cryptocurrencies. These are the generalized hyperbolic distribution (Bitcoin and Litecoin), and the normal inverse Gaussian distribution, the generalized *t* distribution, and the Laplace distribution for smaller cryptocurrencies. The article [22] showed that the profitability of Bitcoin after risk adjustment, depending on the specific measure of risk, can be compared with the profitability of shares based on Sharpe and Sortino ratios using. In the paper [3] another approach is offered. It considers the decision-making process related to technological innovation is considered in the conditions of uncertainty and risk arising from incomplete information about the explored system. The proposed model allows describing the dynamics of multi-stage control of the technological innovation process, depending on investment resources receipt.

3 Methods

Thus, as shown by the analysis of literary sources, in present-day conditions, not only currencies and valuable metals are used for investment, but also cryptocurrency assets are added to the portfolio. Our analysis was done on the basis of historical data on prices of 6 cryptocurrency (Bitcoin, Bitcoin Cash, Litecoin, XRP, Ethereum, Tether) for the period from January 1, 2018 to June 30, 2020. This data are freely available from the www.coinmarketcap.com site – CoinMarketCap Analytical Services contains historical and actual data about cryptocurrency. The data set is divided into 6 parts, each of which refers to a specific quarter of the study period. The volume of quarterly data is 90–92 records, the total amount of data – 912 records. For comparison, we included in our analysis a study of the stock prices of such leading companies as Amazon and Google. In this case quarterly data volume is 61–64 records, the total amount of data – 628 records.

For further processing, the calculation of the corresponding normalized cryptocurrency return is performed according to following equation

$$x_{ni} = (C_{ni+1}/C_{ni} - 1) \cdot 100\%, \quad (1)$$

where x_n is the daily return of the n-th asset, C_n is the daily closing price of the n-th asset, i is the observation number.

The dynamics of cryptocurrency Bitcoin return is presented in fig. 1. The main characteristics of the investigated cryptocurrency return for the observed period are given in table 1. As well, for comparison, in table 1, we introduced the statistical characteristics of the two successful companies' stocks. The analysis of statistical characteristics, given in table 1, showed that the daily stock return of the represented companies is higher than the similar investigated cryptocurrencies return. At the same time, their risk (if we consider the risk as a standard deviation) is much lower (except for the cryptocurrency USDT). From the correlation matrix (table 2) it can be seen that the return of the cryptocurrency is sufficiently correlated with each other (except USDT).

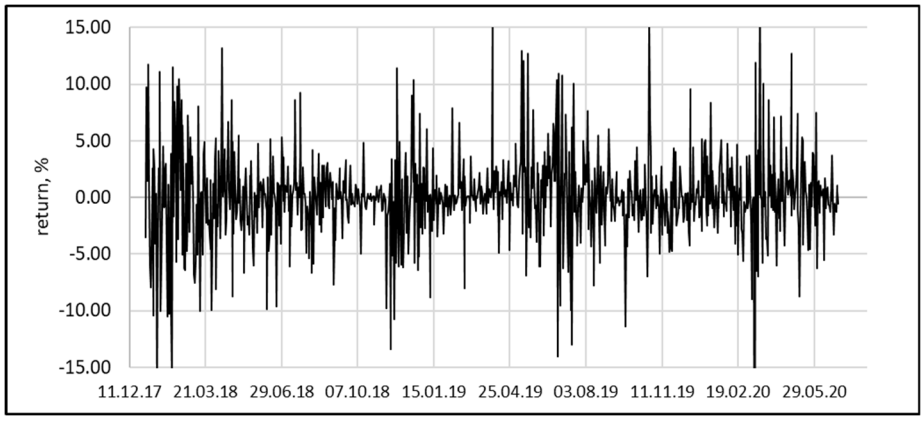

Fig. 1. Dynamics of day return cryptocurrency Bitcoin (01.01.2018 – 30.06.2020).

Table 1. Statistical characteristics of cryptocurrency return (%) for the period 01/01/2018 to 06/30/2020.

	Minimum	**Maximum**	**Average**	**Median**	**Standard deviation**	**Skewness**
Google	-11.101	10.449	0.067	0.130	1.970	-0.109
Amazon	-7.922	9.445	0.158	0.208	2.043	-0.098
BTC	-37.170	18.188	0.037	0.100	4.054	-0.662
ETH	-42.347	18.940	0.001	-0.048	5.083	-0.566
XRP	-32.899	37.989	-0.141	-0.286	5.301	0.465
USDT	-5.121	5.484	0.000	-0.009	0.540	0.511
BCH	-42.956	51.214	-0.055	-0.300	6.532	0.776
LTC	-36.177	33.722	-0.050	-0.225	5.268	0.455

Let's introduce the concept of the risk zone frontier [14]. In this capacity we will use the 5% quantile of return. To determine the risk zone frontier, it is necessary to identify the distribution of returns. Under the investor risk we understand the difference between

the most expected value of cryptocurrency return and 5% quantile of return (risk zone frontier L), which is determined using the corresponding return distribution. If the distribution is normal, the most expected return value is the average value of sample $\overline{x}$. If the distribution is different from the normal one and is asymmetric, we will use the median return μ as an expected return. A significant asymmetry in the return distribution (last row of table 1) prompts as the most expected return value to choose the median sample, rather than the average value of sample.

Table 2. Correlation matrix of cryptocurrency return (%) for the period 01/01/2018 to 06/30/2020.

	BTC	**ETH**	**XRP**	**USDT**	**BCH**	**LTC**
BTC	1	0.84	0.69	-0.01	0.78	0.81
ETH	0.84	1	0.77	-0.07	0.79	0.84
XRP	0.69	0.77	1	-0.07	0.67	0.72
USDT	-0.01	-0.07	-0.07	1	-0.03	-0.04
BCH	0.78	0.79	0.67	-0.03	1	0.79
LTC	0.81	0.84	0.72	-0.04	0.79	1

Consequently, the value of the asset risk, in accordance with the above definition, can be estimated by the ratio

$$V_j = \mu_j - L_j. \tag{2}$$

For statistical research, we divided the data set into 10 time intervals, each of which corresponds to one quarter. As a result of research of the cryptocurrencies Bitcoin, Bitcoin Cash, Litecoin, XRP, Ethereum, Tether using the Pearson, Kolmogorov-Smirnov, and Shapiro-Wilk tests, in most cases the hypothesis of return normal distribution was rejected (fig. 2). Computer experiments showed that the return of the investigated cryptocurrency with good accuracy is described by both Cauchy distribution and Laplace distribution (fig. 3).

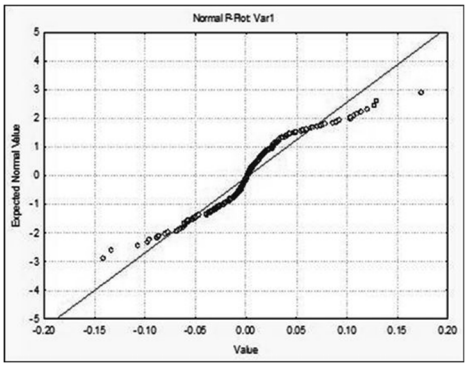

Fig. 2. Hypothesis testing on normal distribution of the Bitcoin return.

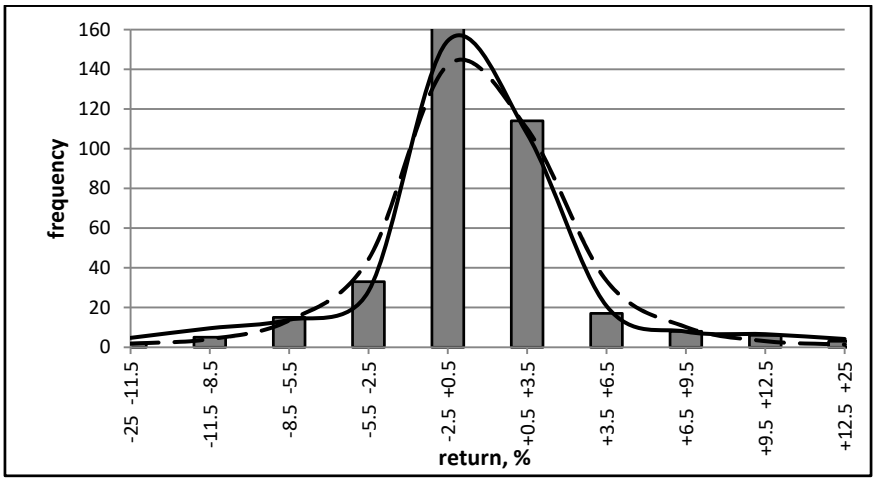

Fig. 3. Actual form of Bitcoin return distribution (gray columns), Cauchy distribution (solid line), Laplace distribution (dashed line).

To test the hypothesis of the Cauchy (Laplace) distribution of cryptocurrency returns, we used Pearson's chi-squared test (χ^2). To apply this criterion, it is necessary to calculate Pearson statistics using the formula

$$Q^2 = \sum_{i=1}^{k} \frac{(n_i - m_i)^2}{m_i}, \tag{3}$$

and compare it with tabular values $\chi^2_{crit}(\alpha, k-3)$. Here k is the number of intervals, m_i – the theoretical number of the random variable values in the i-th interval, n_i – the actual number of the random variable values in the i-th interval, $\alpha = 0.05$ – the level of significance of the test. In our case $\chi^2_{crit}(0.05, 10-3) = 14.07$. If $Q^2 \leq \chi^2_{crit}$ the hypothesis of Cauchy (Laplace) distribution is accepted, otherwise it is rejected. The results of test of hypothesis for the cryptocurrency return distribution are shown in the tables 3, 4. It is seen that for most cases the hypothesis of the corresponding distribution is accepted at the level $\alpha = 0.05$. The tables 3, 4 also show the results of testing the hypothesis of the return distribution for Google stocks and Amazon stocks. Comparing table 3 and table 4, we can conclude that the Laplace distribution more accurately describes the distribution of cryptocurrency return compared to the Cauchy distribution.

The Cauchy distribution function has the form

$$F(x) = \frac{1}{\pi} arctg\left(\frac{x-\mu}{\gamma}\right) + \frac{1}{2}. \tag{4}$$

Here μ is the mathematical expectation (median) of return, γ is the coefficient of distribution function chosen by us for each case in accordance with the least squares method.

The Laplace distribution function $F(x)$ has the form

$$F(x) = \begin{cases} \frac{1}{2}e^{\gamma(x-\mu)}, & x \le \mu \\ 1 - \frac{1}{2}e^{-\gamma(x-\mu)}, & x > \mu. \end{cases} \quad (5)$$

Here x is the return on financial assets, μ is the mathematical expectation (median) of return, γ is the coefficient of distribution function chosen by us for each case in accordance with the least squares method.

Table 3. Pearson's chi-squared test for Cauchy distribution.

	Google	**Amazon**	**BTC**	**ETH**	**XRP**	**USDT**	**BCH**	**LTC**
18_Q1	9.15	11.68	12.09	4.86	8.14	5.35	10.7	11.22
18_Q2	12.11	7.54	14.52	12.1	10.33	13.89	12.12	14.51
18_Q3	15.07	12.63	8.93	10.6	5.28	13.14	9.24	14.09
18_Q4	2.92	5.7	10.85	4.84	12.34	3.59	7.44	5.03
19_Q1	7.92	5.18	14.07	5.41	3.22	10.19	11.1	8.84
19_Q2	7.03	6.94	14.07	8.95	4.32	13.32	5.41	10.02
19_Q3	10.19	15.19	8.25	13.79	8.64	14.99	10.69	4.87
19_Q4	13.23	8.34	8.21	4.76	14.37	14.11	13.82	14.29
20_Q1	4.99	2.15	8.82	5.17	7.02	12.75	5.14	7.72
20_Q2	3.11	5.78	6.21	6.24	5.84	26.67	8.23	9.1
Average	*8.57*	*8.11*	*10.6*	*7.67*	*7.95*	*12.8*	*9.39*	*9.97*

Table 4. Pearson's chi-squared test for Laplace distribution.

	Google	**Amazon**	**BTC**	**ETH**	**XRP**	**USDT**	**BCH**	**LTC**
18_Q1	5.43	3.64	6.04	2.15	7.2	3.58	7.74	6.95
18_Q2	5.79	4.66	11.31	8.08	3.8	5.55	5.43	8.47
18_Q3	6.79	4.6	5.23	2.77	3.2	4.15	8.53	6.21
18_Q4	0.76	4.7	14.25	7.73	9.55	1.35	9.99	2.69
19_Q1	3.06	1.89	10.41	9.81	5.11	4.35	8.46	4.53
19_Q2	3.73	5.96	6.14	6.12	2.74	6.31	4.09	7.1
19_Q3	5.15	8.08	3.43	6.29	4.78	6.69	6.74	5.77
19_Q4	5.78	4.87	5.49	3.84	10.81	5.8	4.99	10.24
20_Q1	4.58	0.53	6.67	2.96	6.07	9.73	4.14	3.67
20_Q2	5.43	3.64	6.01	3.07	3.07	12.4	2.06	2.98
Average	*4.65*	*4.26*	*7.5*	*5.28*	*5.63*	*5.99*	*6.22*	*5.86*

To determine the coefficient γ an interval distribution table was constructed. The role of the minimized value was the sum of the squares of the differences between the theoretical and actual values of the frequency at different intervals (equation 3). The parameter μ (median) for the various cryptocurrencies and periods are shown in table 1.

Using the form of the Cauchy distribution function (4), we can find an analytic expression for the frontier of risk zone L_α at a given confidence level α [14]:

$$L_\alpha = \mu + \gamma \cdot tg\left(\pi\left(\alpha - \frac{1}{2}\right)\right). \quad (6)$$

Similarly, for the Laplace distribution, from relation (4) we determine an analytic expression for the frontier of risk zone

$$L_\alpha = \mu + \frac{ln(2\alpha)}{\gamma}. \quad (7)$$

Using (2), (6), (7) we calculated the risk value V at the level of 5% for each cryptocurrency at the appropriate period of time (quarter). However, the risk value calculated on the basis of the Cauchy distribution (riskC) is twice the value of the risk calculated on the basis of the Laplace distribution (riskL). For example, for Bitcoin in the 1st quarter of 2018 the value risk Cauchy $V_C = 25.89\%$, the value risk Laplace $V_L = 12.60\%$. A similar situation is observed for other cryptocurrencies and periods (table 5). For comparison, the Table 5 also shows statistics for Google stocks of and Amazon stocks. The standard deviation, which is a measure of risk in the normal distribution, almost halves the risk compared to the estimate obtained from the Laplace distribution (table 5). In this regard, the question arises: which of the two distributions described above most adequately describes the risks of cryptocurrencies: the Cauchy distribution or the Laplace distribution?

Table 5. Statistical characteristics of cryptocurrency risks, %.

Year	**2018**				**2019**				**2020**		**Average**
Quarter	**Q1**	**Q2**	**Q3**	**Q4**	**Q1**	**Q2**	**Q3**	**Q4**	**Q1**	**Q2**	
Google											
Mediane	0.38	0.05	0.09	-0.18	0	0.16	0.19	0.05	0.15	0.5	*0.14*
StDev	1.99	1.42	1.19	2.3	1.5	1.69	1.82	0.94	3.42	2.36	*1.86*
RiskC	6.87	5.84	5.3	8.49	5.47	4.54	5.09	3.71	10.55	5.69	*6.15*
RiskL	3.97	2.64	2.34	4.17	2.65	2.39	2.47	1.76	5.55	3.52	*3.15*
StDev/RiskL	0.5	0.54	0.51	0.55	0.56	0.71	0.74	0.53	0.62	0.67	*0.59*
RiskC/RiskL	1.73	2.21	2.26	2.03	2.06	1.9	2.06	2.11	1.9	1.62	*1.99*
Amazon											
Mediane	0.48	0.3	0.34	-0.38	0.29	0.1	0.01	0.02	0.09	0.74	*0.2*
StDev	1.94	1.65	1.31	3.52	1.92	1.43	1.3	1	2.88	2.14	*1.91*
RiskC	7.71	5.39	5.48	13.32	6.59	4.03	6.02	2.92	10.71	6.5	*6.87*
RiskL	4.27	2.9	2.45	6.56	3.4	2.47	2.67	1.8	5.21	3.46	*3.52*
StDev/RiskL	0.45	0.57	0.53	0.54	0.56	0.58	0.49	0.56	0.55	0.62	*0.55*
RiskC/RiskL	1.81	1.86	2.23	2.03	1.94	1.63	2.26	1.63	2.06	1.88	*1.93*
BTC											
Mediane	-0.11	0.17	0.36	-0.12	0.12	1.32	-0.18	-0.2	-0.18	0.17	*0.14*
StDev	5.97	3.7	2.81	3.87	2.21	4.59	3.9	2.98	5.61	3.16	*3.88*
RiskC	25.89	12.37	9.33	10.08	4.49	13.22	14.02	8.71	13.87	9.27	*12.13*
RiskL	12.6	6.42	4.84	6.15	3.4	6.94	6.68	4.79	7.04	5.34	*6.42*
StDev/RiskL	0.47	0.58	0.58	0.63	0.65	0.66	0.58	0.62	0.8	0.59	*0.62*
RiskC/RiskL	2.05	1.93	1.93	1.64	1.32	1.9	2.1	1.82	1.97	1.74	*1.84*
ETH											
Mediane	-0.41	0.31	-0.6	-0.18	-0.15	0.68	-0.1	-0.4	0.19	0.33	*-0.03*
StDev	6.66	5.23	4.83	5.59	4.14	4.69	4.31	3.08	6.98	4.06	*4.96*

Year	2018				2019				2020		Average
Quarter	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	
RiskC	28.71	21.57	16.52	16.15	10.91	15.63	15.15	9.34	17.98	12.88	*16.48*
RiskL	13.17	9.56	7.94	9.27	6.83	8.16	7.08	5.27	8.77	6.52	*8.26*
StDev/RiskL	0.51	0.55	0.61	0.6	0.61	0.58	0.61	0.58	0.8	0.62	*0.61*
RiskC/RiskL	2.18	2.26	2.08	1.74	1.6	1.92	2.14	1.77	2.05	1.98	*1.97*
XRP											
Mediane	-0.41	0.31	-0.6	-0.18	-0.15	0.68	-0.1	-0.4	0.19	0.33	*-0.03*
StDev	6.66	5.23	4.83	5.59	4.14	4.69	4.31	3.08	6.98	4.06	*4.96*
RiskC	28.71	21.57	16.52	16.15	10.91	15.63	15.15	9.34	17.98	12.88	*16.48*
RiskL	13.17	9.56	7.94	9.27	6.83	8.16	7.08	5.27	8.77	6.52	*8.26*
StDev/RiskL	0.51	0.55	0.61	0.6	0.61	0.58	0.61	0.58	0.8	0.62	*0.61*
RiskC/RiskL	2.18	2.26	2.08	1.74	1.6	1.92	2.14	1.77	2.05	1.98	*1.97*
USDT											
Mediane	-0.41	0.31	-0.6	-0.18	-0.15	0.68	-0.1	-0.4	0.19	0.33	*-0.03*
StDev	6.66	5.23	4.83	5.59	4.14	4.69	4.31	3.08	6.98	4.06	*4.96*
RiskC	28.71	21.57	16.52	16.15	10.91	15.63	15.15	9.34	17.98	12.88	*16.48*
RiskL	13.17	9.56	7.94	9.27	6.83	8.16	7.08	5.27	8.77	6.52	*8.26*
StDev/RiskL	0.51	0.55	0.61	0.6	0.61	0.58	0.61	0.58	0.8	0.62	*0.61*
RiskC/RiskL	2.18	2.26	2.08	1.74	1.6	1.92	2.14	1.77	2.05	1.98	*1.97*
BCH											
Mediane	-0.41	0.31	-0.6	-0.18	-0.15	0.68	-0.1	-0.4	0.19	0.33	*-0.03*
StDev	6.66	5.23	4.83	5.59	4.14	4.69	4.31	3.08	6.98	4.06	*4.96*
RiskC	28.71	21.57	16.52	16.15	10.91	15.63	15.15	9.34	17.98	12.88	*16.48*
RiskL	13.17	9.56	7.94	9.27	6.83	8.16	7.08	5.27	8.77	6.52	*8.26*
StDev/RiskL	0.51	0.55	0.61	0.6	0.61	0.58	0.61	0.58	0.8	0.62	*0.61*
RiskC/RiskL	2.18	2.26	2.08	1.74	1.6	1.92	2.14	1.77	2.05	1.98	*1.97*
LTC											
Mediane	-0.41	0.31	-0.6	-0.18	-0.15	0.68	-0.1	-0.4	0.19	0.33	*-0.03*
StDev	6.66	5.23	4.83	5.59	4.14	4.69	4.31	3.08	6.98	4.06	*4.96*
RiskC	28.71	21.57	16.52	16.15	10.91	15.63	15.15	9.34	17.98	12.88	*16.48*
RiskL	13.17	9.56	7.94	9.27	6.83	8.16	7.08	5.27	8.77	6.52	*8.26*
StDev/RiskL	0.51	0.55	0.61	0.6	0.61	0.58	0.61	0.58	0.8	0.62	*0.61*
RiskC/RiskL	2.18	2.26	2.08	1.74	1.6	1.92	2.14	1.77	2.05	1.98	*1.97*

4 Risk zone testing

Analysis of relations (4) and (5) showed that the Cauchy distribution has very long and heavy tails (fig. 4). In this regard, the risk zone frontier determined on the basis of the Cauchy distribution will be significantly smaller than the risk zone frontier determined on the basis of the Laplace distribution. In this case, the number of return cases that fall into the risk zone determined on the Cauchy distribution basis will be significantly less than the number of cases that fall into the risk zone determined on the Laplace distribution basis. After counting the number of cases that fall into the risk zone, we

can conclude which of the two distribution laws more adequately describes the distribution of cryptocurrency returns in the negative return zone. The results of counting the number of critical cases (the case where the return falls into the risk zone) at the confidence level 5% are shown in table 6 and table 7.

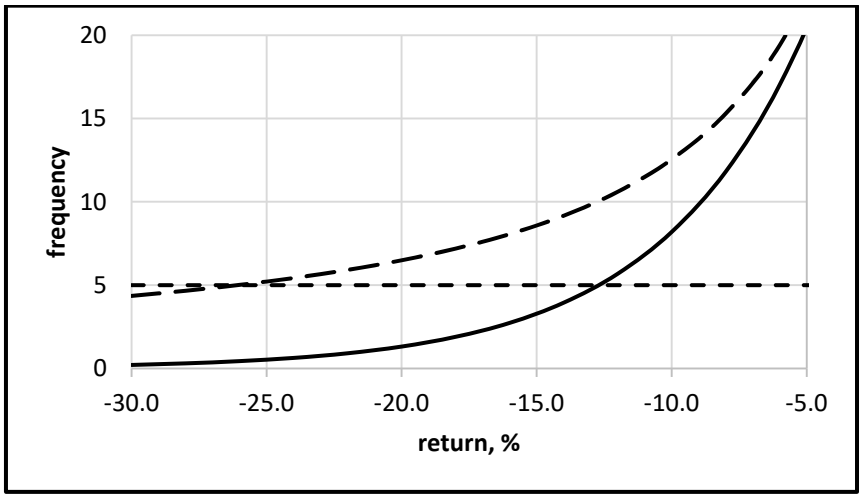

Fig. 4. The Laplace distribution (solid line), the Cauchy distribution (dashed line).

Table 6. Number of critical cases for Cauchy distribution. The frontier of the risk zone was determined at the confidence level 5%.

Year	**2018**				**2019**				**2020**		**All cases**	**All days**	**Frequency, %**
Quarter	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**			
Google	0	0	0	0	0	2	0	0	1	1	4	628	0.64
Amazon	0	1	0	0	0	1	0	0	0	1	3	628	0.48
BTC	0	0	0	2	2	1	0	0	1	0	6	912	0.66
ETH	0	0	1	0	2	0	2	0	1	0	6	912	0.66
XRP	0	0	0	1	2	0	0	1	1	1	6	912	0.66
USDT	0	1	0	0	1	0	0	0	3	0	5	912	0.55
BCH	0	0	0	1	2	0	2	1	1	0	7	912	0.77
LTC	0	0	0	0	0	0	1	0	1	0	2	912	0.22
Stocks	0	1	0	0	0	3	0	0	1	2	7	1256	0.56
Crypto-currencies	0	1	1	4	9	1	5	2	8	1	32	5472	0.58

As can be seen from Table 6, in the case where the risk area is determined based on the Cauchy distribution at the confidence level 5%, the average probability for falling of cryptocurrency return into the risk area is 0.6% – 0.7% (except Litecoin). For the stocks return the average probability for falling into the risk zone is 0.5% – 0.6%. That is, the actual frequency of critical cases is 10 times less than theoretically predicted. Hence the conclusion about the inadequate description of the distribution of cryptocurrency (stock) returns in the negative return zone using the Cauchy distribution.

Table 7. Number of critical cases for Laplace distribution. The frontier of the risk zone was determined at the confidence level 5%.

Year	2018				2019				2020		All cases	All days	Frequency, %
Quarter	1	2	3	4	1	2	3	4	1	2			
Google	5	2	1	4	2	4	3	2	3	5	31	628	4.94
Amazon	4	4	3	1	2	3	3	0	3	3	26	628	4.14
BTC	2	5	6	5	3	5	4	2	2	4	38	912	4.17
ETH	2	5	7	6	3	3	6	6	4	3	45	912	4.93
XRP	3	4	4	4	3	1	5	4	5	4	37	912	4.06
USDT	5	4	6	5	2	1	3	2	4	3	35	912	3.84
BCH	3	2	2	5	3	4	5	6	4	3	37	912	4.06
LTC	2	3	4	4	3	1	5	6	4	3	35	912	3.84
Stocks	9	6	4	5	4	7	6	2	6	8	57	1256	4.54
Crypto-currencies	17	23	29	29	17	15	28	26	23	20	227	5472	4.16

In the case where the risk area is determined based on the Laplace distribution at the confidence level 5% (Table 7), the average probability for falling of cryptocurrency return (and stock return) into the risk area is 4% – 5%. Thus, the actual frequency of critical cases is close to the theoretically predicted. Thus, the Laplace distribution is an adequate basis for the risk measure of negative returns of cryptocurrency's and stock returns.

5 Formation of a cryptocurrency portfolio

When forming a cryptocurrencies portfolio, first of all it is necessary to take into account their return. Figure 5 shows the average return of cryptocurrencies for the two quarters of 2020. The best cryptocurrencies in terms of profitability are BTC, ETH, XRP and BCH. As can be seen from fig. 5, the average quarterly return on stocks significantly exceeds the average quarterly cryptocurrency return. This means that stocks are more attractive for long-term investments. Cryptocurrencies are a tool for speculative transactions.

Another important aspect of portfolio formation is taking into account the risks of cryptocurrencies and taking into account the correlations of their profitability. The minimum risk is typical for cryptocurrency USDT.

From the correlation matrix (table 8) it can be seen that the return of the cryptocurrency is sufficiently correlated with each other (except USDT). It is clear that the cryptocurrency USDT is the most important component of the portfolio, which will reduce its risk (fig. 6).

For the building the cryptocurrencies portfolio, let's used the technique, described in previous research [14]. Assuming that cryptocurrency returns $r_i(t)$ are poorly stationary random processes, each of which is characterized by mathematical expectations μ_i and a degree of risk V_i, then for portfolio optimization, a modified Markowitz model can be used. In this case, the mathematical description of the problem at the maximum portfolio return will have the form:

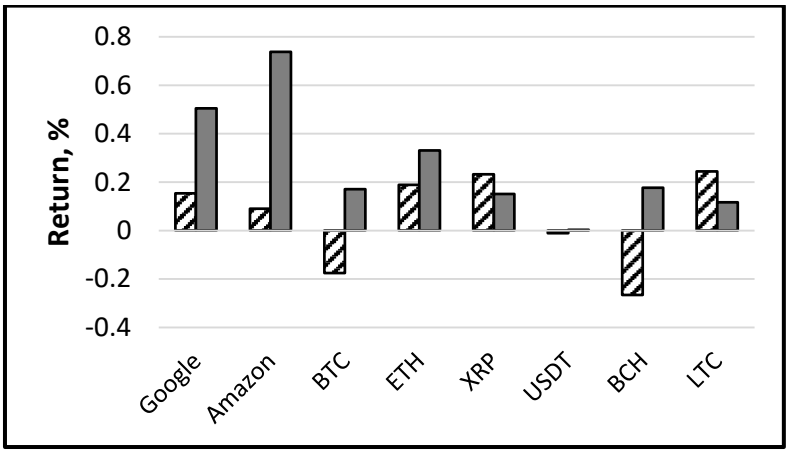

Fig. 5. Cryptocurrencies return for the I Quarter 2020 (dashed) and II Quarter 2020 (solid).

Table 8. Correlation matrix of cryptocurrency return (%) for the period 01/01/2020 to 06/30/2020.

	BTC	ETH	XRP	USDT	BCH	LTC
BTC	1	0.85	0.79	-0.01	0.82	0.84
ETH	0.85	1	0.86	-0.11	0.91	0.91
XRP	0.79	0.86	1	0.01	0.85	0.91
USDT	-0.01	-0.11	0.01	1	-0.02	0.00
BCH	0.82	0.91	0.85	-0.02	1	0.91
LTC	0.84	0.91	0.91	0.00	0.91	1

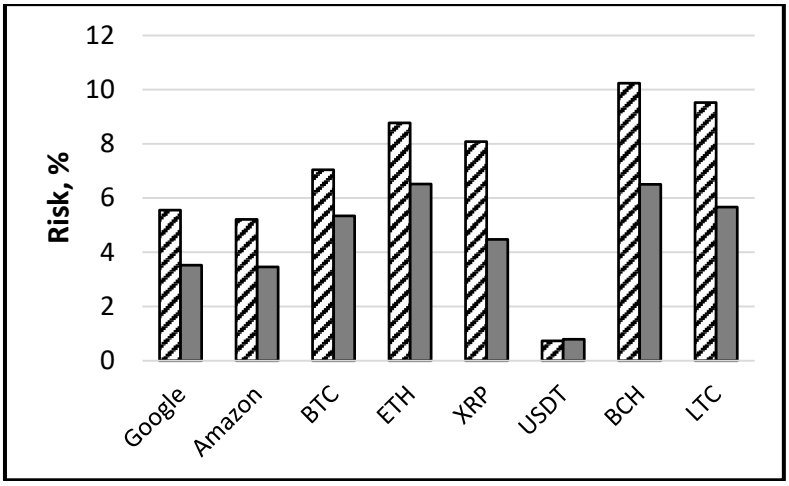

Fig. 6. Average cryptocurrencies risk for the I Quarter 2020 (dashed) and II Quarter 2020 (solid).

$$\begin{cases} R_p = w_i \times \mu_i \to max; \\ V_p = \sqrt{\sum_{i=1}^{6}\sum_{j=1}^{6}\left(w_i \times V_i \times w_j \times V_j \times \rho_{ij}\right)} \le V_{req}; \\ w_i \ge 0;\ \sum w_i = 1. \end{cases} \quad (8)$$

To assess of portfolio risk V_p, we used an approach similar to the Markowitz approach, but for the risk measure we used definition (2), rather than the standard deviation of the cryptocurrency return.

So, using the obtained above cryptocurrency risk estimates RiskL (table 5, column 20_Q2), we constructed the set of optimal portfolios (the efficient frontier). Each such portfolio gives maximum return at the established risk level. The table 9 presents the portfolio structure for each, obtained by us, optimal solution. The analysis of the table confirms the well-known statement that a higher return level always requires a higher risk degree. As you can see, the main role in the formation of the portfolio is played by cryptocurrencies ETH and USDT. The first provides high profitability, the second guarantees low risk. Other cryptocurrencies play the role of extras and do not participate in the formation of the portfolio.

Table 9. Set of optimal portfolios (the risk measure, based on the Laplace distribution).

BTC	ETH	XRP	USDT	BCH	LTC	Risk, %	Return, %
0.000	0.027	0.000	0.973	0.000	0.000	0.768	0.012
0.000	0.123	0.000	0.877	0.000	0.000	1.000	0.043
0.000	0.221	0.000	0.779	0.000	0.000	1.500	0.075
0.000	0.305	0.000	0.695	0.000	0.000	2.000	0.103
0.000	0.385	0.000	0.615	0.000	0.000	2.500	0.129
0.000	0.463	0.000	0.537	0.000	0.000	3.000	0.155
0.000	0.540	0.000	0.460	0.000	0.000	3.500	0.180
0.000	0.617	0.000	0.383	0.000	0.000	4.000	0.205
0.000	0.694	0.000	0.306	0.000	0.000	4.500	0.230
0.000	0.770	0.000	0.230	0.000	0.000	5.000	0.255
0.000	0.846	0.000	0.154	0.000	0.000	5.500	0.280
0.000	0.922	0.000	0.078	0.000	0.000	6.000	0.305
0.000	1.000	0.000	0.000	0.000	0.000	6.518	0.331

To increase profitability, the portfolio can include shares of well-known companies. We will introduce Amazon stocks into the previous portfolio instead of the low-yield cryptocurrency LTC. Similar to the above, we obtained the set of optimal portfolios presented in table 10. The main role in the formation of the portfolio is played by stocks Amazon and cryptocurrency USDT. The stocks provide high profitability, the cryptocurrency guarantees low risk. The introduction of Amazon's stock to the portfolio halved the portfolio's risk and doubled its profitability. Thus, we conclude that optimal portfolios should be built by combining cryptocurrencies and stocks of

highly profitable stable companies. The sets of optimal portfolios presented in tables 9 and 10 are illustrated in fig. 7 and fig. 8.

Table 10. Set of optimal portfolios (the risk measure, based on the Laplace distribution).

BTC	ETH	XRP	USDT	BCH	LTC	Risk, %	Return, %
0.000	0.024	0.000	0.920	0.000	0.056	0.741	0.052
0.000	0.038	0.000	0.723	0.000	0.239	1.000	0.191
0.000	0.045	0.000	0.624	0.000	0.330	1.250	0.260
0.000	0.052	0.000	0.537	0.000	0.411	1.500	0.322
0.000	0.057	0.000	0.455	0.000	0.487	1.750	0.380
0.000	0.063	0.000	0.375	0.000	0.561	2.000	0.436
0.000	0.069	0.000	0.297	0.000	0.634	2.250	0.491
0.000	0.075	0.000	0.220	0.000	0.705	2.500	0.546
0.000	0.080	0.000	0.144	0.000	0.776	2.750	0.600
0.000	0.086	0.000	0.068	0.000	0.846	3.000	0.653
0.000	0.078	0.000	0.000	0.000	0.922	3.250	0.706
0.000	0.000	0.000	0.000	0.000	1.000	3.458	0.738

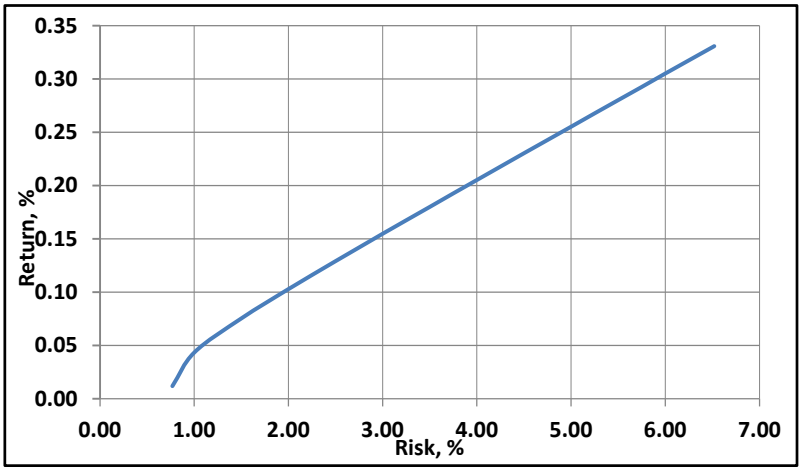

Fig. 7. Set of optimal cryptocurrency portfolios (table 9).

6 Conclusion

Due to its volatility, cryptocurrencies are an attractive tool for short-term investments. However, high volatility is a source of great risk. For assessing of cryptocurrencies risk, it is necessary to identify the return distribution. Numerous studies show that cryptocurrencies return and stocks return are not subject to normal distribution. The aim of our research is to compare the application of the Cauchy distribution and the Laplace distribution to the description of the actual distribution of cryptocurrency

yields. A comparison of the actual return frequency in the critically low zone with its theoretical value was used as an evaluation criterion. Calculations performed for six cryptocurrencies over a 30-month period showed that the Cauchy distribution describes well the return distribution in the central part, but greatly overestimates the probability of marginal values of return. In our opinion, using the Laplace distribution is the most adequate approach to measuring the risk of cryptocurrencies (stocks).

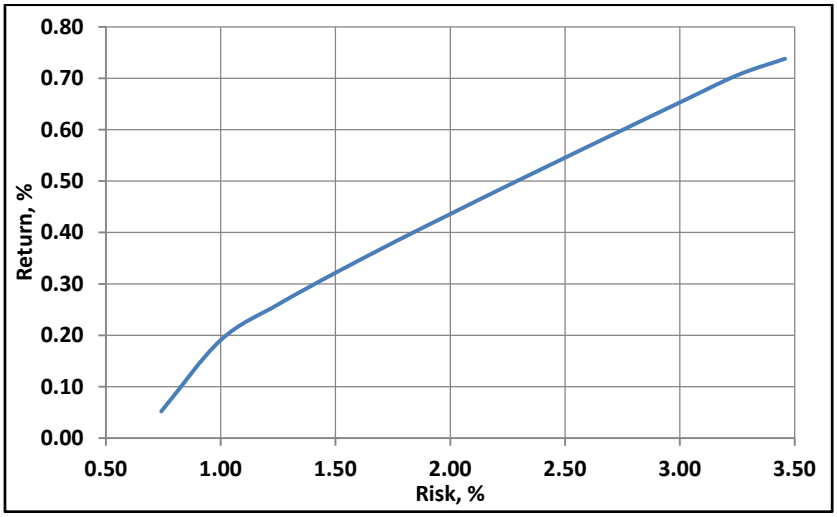

Fig. 8. Set of optimal combined portfolios (table 10).

A comparison of cryptocurrencies returns with the stocks return of leading companies showed that the average quarterly return of cryptocurrencies is low. Thus, it can be concluded that stocks are more attractive for long-term investments. Cryptocurrencies are a tool for speculative trans-actions. Inclusion of stocks of high-yield companies in the cryptocurrency's portfolio allows in-creasing portfolio profitability and reducing portfolio risk. We have shown that inclusion AMZN stocks into the cryptocurrency portfolio can double the portfolio's yield and halve its risk.

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