Econophysics of sustainability indices

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Abstract. In this paper, the possibility of using some econophysical methods for quantitative assessment of complexity measures: entropy (Shannon, Approximate and Permutation entropies), fractal (Multifractal detrended fluctuation analysis – MF-DFA), and quantum (Heisenberg uncertainty principle) is investigated. Comparing the capability of both entropies, it is obtained that both measures are presented to be computationally efficient, robust, and useful. Each of them detects patterns that are general for crisis states. The similar results are for other measures. MF-DFA approach gives evidence that Dow Jones Sustainability Index is multifractal, and the degree of it changes significantly at different periods. Moreover, we demonstrate that the quantum apparatus of econophysics has reliable models for the identification of instability periods. We conclude that these measures make it possible to establish that the socially responsive exhibits characteristic patterns of complexity, and the proposed measures of complexity allow us to build indicators-precursors of critical and crisis phenomena.

Keywords: Dow Jones Sustainability Index, measures of complexity, precursors of stock market crashes.

1 Introduction

Current economic trends have convincingly demonstrated that green development is a necessary condition for sustainable development, which is essential for a better life in the future [40]. Economists have described climate change as a global market failure estimating that without action, the rising overall costs of climate could result in losing at least 5% of global GDP each year. A growing number of financial institutions are joining in a constructive dialogue on the relationship between economic development, environmental protection, and sustainable development. Financial institutions, including banks, insurers, and investors, work with the United Nations Environment Programme – Finance Initiative to better understand environmental, social, and governance challenges, why they matter to finance, and how to take steps to address them [30].

The availability of stock indexes based on sustainability screening makes increasingly viable for institutional investors the transition to a portfolio based on a Socially Responsible Investment (SRI) benchmark at a relatively low cost.

The 2008 subprime crisis and increased social awareness have led to a growing interest in topics related to Socially Responsible Investment. SRI is a long-term investment that integrates environmental, social, and corporate governance criteria (ESG). According to the Global Sustainable Investment Alliance (GSIA), SRI reached 24 trillion euro's in 2016, registering a growth of 25.2% between 2014 and 2016. So, green and sustainable finance is more important nowadays than ever before [14].

This increased social interest coincides with international initiatives aimed at developing environmental and social policies on sustainable finance issues, such as the Action Plan on sustainable finance adopted by the European Commission in March 2018. This plan has three main objectives:

(i) to redirect capital flows towards sustainable investment to achieve sustainable and inclusive growth,

(ii) to manage financial risks stemming from climate change, environmental degradation, and social issues, and

(iii) to foster transparency and long-termism in financial and economic activity. Therefore, the main purpose is to enhance the role of finance and to build an economy that enables the goals of the Paris Agreement (2015) and the EU for sustainable development to be reached [15].

The Dow Jones Sustainability Index (DJSI) comprises global sustainability leaders as identified by SAM. It represents the top 10% of the largest 2,500 companies in the S&P Global BMI based on long-term economic, environmental, and social criteria [59]. Founded in 1995, RobecoSAM is an investment specialist focused exclusively on Sustainability Investing [38].

The S&P Global Broad Market Index (BMI) is the only global index suite with a transparent, modular structure that has been fully float-adjusted since 1989. This comprehensive, rules-based index series employs a transparent and consistent methodology across all countries and includes more than 11,000 stocks from 25 developed and 25 emerging markets [39]. The SAM Corporate Sustainability Assessment (CSA), established by RobecoSAM, is now issued by S&P Global. RobecoSAM, an asset manager focused entirely on sustainable investing, established the CSA in 1999. The CSA has become the basis for numerous S&P ESG Indices over the last two decades attracting billions of USD in assets. Besides, S&P Global acquired RobecoSAM's ESG Ratings and Benchmarking businesses which operate out of S&P Global Switzerland. SAM is a registered trademark of S&P Global. ESG is a generic term used in capital markets and used by investors to evaluate corporate behavior and to determine the future financial performance of companies. In the conditions of a wide variety of sustainable development indices, investors need to have a comparative characteristic of traditional indices with sustainable development indices obtained by *quantitative methods*. At the same time, the set of tools of modern financial analysis took shape in a separate rapidly growing applied science – *fintech*. Financial technology ('fintech') is emerging as a core disruptor of every aspect of today's financial system. Fintech covers everything from mobile payment platforms to high-frequency trading, and from crowdfunding and virtual currencies to blockchain. In combination, such forceful innovations will threaten the viability of today's financial sector business models, and indeed the effectiveness of current policies, regulations, and norms that have shaped modern finance.

The use of financial technology innovations is of course not new – but a step change is now expected with the novel application of several technologies in combination, notably involving blockchain, the 'Internet of things', and artificial intelligence [6]. The widespread introduction of fintech makes it possible to talk about *green finance* as a strategy for the financial sector and broader sustainable development that is relevant around the world [1; 7; 24; 33]. Green economy, green finance, and green development are the peculiar coordinates of the phase space in which today it is generally accepted to evaluate the sustainable development of world civilization.

Financial systems are complex systems and consist of a plurality of interacting agents possessing the ability to generate new qualities at the level of macroscopic collective behavior, the manifestation of which is the spontaneous formation of noticeable temporal, spatial, or functional structures [54]. For many years financial markets have been attracting the attention of many scientists like engineers, mathematicians, physicists, and others for the last two decades. Such vast interest transformed into a branch of statistical mechanics – *econophysics* [25]. Physics, economics, finance, sociology, mathematics, engineering, and computer science are fields of science which, as a result of cross-fertilization, have created the multi-, cross-, and interdisciplinary areas of science and research such as econophysics and sociophysics, thriving in the last two and a half decades. These mixed research fields use knowledge, methodologies, methods, and tools of physics for modeling, explaining, and forecasting economic and social phenomena and processes. Accordingly, econophysics is an interdisciplinary research field, applying theories and methods originally developed by physicists to solve problems in economics, usually those including uncertainty or stochastic processes, nonlinear dynamics, and evolutionary games. Obviously, quantitative econophysical methods for studying financial markets are an interesting and promising area of fintech.

Our research structured as follows. Section 2 contains a brief description of socially responsive indexes and an analysis of previous work on a comparative quantitative analysis of this variety of indices. Section 3 describes algorithms for constructing econophysical measures of complexity based on the informational, (multi-)fractal and quantum physical properties of a time series. These measures are calculated based on the DJSI index. Section 4 summarizes the results obtained and indicates the direction of subsequent studies.

2 Review of previous research

In the last 20-25 years, a huge number of social responsibility or sustainability indices have been created and their number continues to grow [13; 53]. Briefly consider the most commonly used.

The Dow Jones Sustainability Indices are a family of best-in-class benchmarks for investors who have recognized that sustainable business practices are critical to generating long-term shareholder value and who wish to reflect their sustainability convictions in their investment portfolios (http://www.sustainability-indices.com/). The family was launched in 1999 as the first global sustainability benchmark and tracks the stock performance of the world's leading companies in terms of economic, environmental, and social criteria. Dow Jones Sustainability World Index, the most important global stock market valuation index of corporate social responsibility.

FTSE4Good was created by the FTSE Group to facilitate investments in companies that meet globally recognized corporate responsibility standards and constitutes an important reference point for the establishment of benchmarks and ethical portfolios. Companies in the FTSE4Good Index have met stringent environmental, social, and governance criteria, and are therefore potentially better positioned to capitalize on the benefits of responsible business practice (http://www.ftse.com/).

MSCI is a leading provider of investment decision support tools to investors globally, including asset managers, banks, hedge funds, and pension funds. MSCI Global Sustainability Indexes include companies with high ESG ratings relative to their sector peers (http://www.msci.com/).

CDP (formerly the "Carbon Disclosure Project") is one of the world's leading not-for-profit climate change organizations, assessing transparency in the disclosure of information on climate change and greenhouse gas emissions, as well as in the management of water resources (http://www.cdp.net/).

United Nations Global Compact 100 ("GC 100"), a global stock index developed and released by the UN Global Compact in partnership with the research firm Sustainalytics (https://www.unglobalcompact.org/). The index lists the 100 companies which globally outstand for executive leadership commitment and consistent baseline profitability, as well as their adherence to the Global Compact's ten principles, on human rights, labor, environment, and anti-corruption issues.

STOXX Global ESG Leaders Indices, a group of indices based on a fully transparent selection process of the performance, in terms of sustainability, of 1,800 companies worldwide (http://www.stoxx.com/). The ratings are calculated for three sub-areas – environmental, social, and governance – and are then combined to form the overall index. The indices are managed by STOXX, the owner of some of the most important international stock indices, such as the STOXX50.

In our previous work [11], we performed a comparative analysis of the index DJSI [62] with its classic and traditional counterpart – the index Dow Jones Industrial Average (DJIA) [61].

In a comparative analysis of structural and dynamic properties of traditional stock market indices and social responsibility indices, descriptive statistics methods are used in most works [2; 31; 34; 43].

Descriptive statistics (mean, maximum, minimum, and standard deviation) of the financial information required to apply the Ohlson [34] valuation model reviewed in [31]. They were examining whether sustainability leadership – proxied by the membership of the Dow Jones Sustainability Index Europe – is value relevant for investors on the 10 major European stock markets over the 2001–2013 period. These results reveal that there exist significant differences across markets.

The article [43] analyzes rate-of-return and risk related to investments in socially responsible and conventional country indices. The socially responsible indices are the DJSI Korea, DJSI US, and Respect Index, and the corresponding conventional country indices are the Korea Stock Exchange Composite KOSPI, DJIA, and WIG20. Shown, that conclude that investing in the analyzed SRI indices do not yield systematically better results than investing in the respective conventional indices, both in terms of neoclassical risk and return rate.

The authors [2] examined sustainable investment returns predictability based on the US DJSI and a wide set of uncertainty and financial distress indicators for the period January 2002 to December 2014. They employ a novel nonparametric causality-in-quantile approach that captures nonlinearity in returns distribution. The authors conclude that the aggregate Economic Policy Uncertainty indicator and some components have predictive ability for real returns of the US sustainable investments index. Paper [55] explores the relationship between sustainability performance and financial performance by looking at the impact of sustainability index changes on the market value of a company. The author has studied the price effects of changes in the DJSI and FTSE4Good Index. He failed to observe statistically significant positive abnormal returns for companies being added to a sustainability index. On the opposite, he finds negative abnormal returns for companies being deleted from the FTSE, however not in the case of the DJSI. This can be explained by studying the volume effects and the behavior of investment managers.

However, the first works appeared using more modern methods of analysis, using the achievements of nonlinear dynamical systems and complexity theory [16; 26; 29; 32; 58]. The authors [58] constructed a sustainable regional green economy development index system from five aspects - economic, social, technological, resources, and environmental - using DPSIR (drivers, pressures, state, impact, response model) and entropy-TOPSIS (a technique for order preference by similarity to an ideal solution) coupling coordination to horizontally and vertically quantitatively analyze the sustainable green economy development. The model was verified by the actual situation of green economy development in Shandong Province from 2010 to 2016, which confirmed the feasibility of the method.

A sustainable development capacity measure model for Sichuan Province was established by applying the information entropy calculation principle and the Brusselator principle [26]. Each subsystem and entropy change in a calendar year in Sichuan Province were analyzed to evaluate Sichuan Province's sustainable development capacity. It was found that the established model could effectively show actual changes in sustainable development levels through the entropy change reaction system, at the same time this model could clearly demonstrate how those forty-six indicators from the three subsystems impact on the regional sustainable development, which could make up for the lack of sustainable development research.

A similar approach is implemented to measure the tourist attractiveness of the region [16]. And in work [29] information and entropy theory used for the sustainability of coupled human and natural systems.

Authors of [32] used R/S analysis to calculate the Hurst exponent as a measure of persistence (efficiency of traditional stock market indices and social responsibility stock market indices). The presence of persistence was evidence in favor of less efficiency. According to empirical results, SRI has lower efficiency, in particular the Dow Jones Sustainability Index. Lower efficiency was also observed in the emerging markets with a responsible investment segment, compared to the traditional stock market indices.

In paper [27] authors suggest three new indicators based on an engineering approach of irreversibility. They allow evaluating both the technological level and the environmental impact of the production processes and the socio-economic conditions of the countries. Indeed, they are based on the energy analysis and on the irreversible thermodynamic approach, in order to evaluate the inefficiency both of the process and of the production systems, and the related consequences. Three applications are summarized in order to highlight the possible interest from different scientists and researchers in engineering, economy, etc. [19; 23], in order to develop sustainable approaches and policies for decision-makers.

All mentioned measures can capture nonlinearity and complexity that peculiar even for sustainability indices. Analysis of previous papers [47; 48; 49; 51; 52] shows that indicators have theoretical perspectives and, in accordance with other studies, such approaches are presented to be robust and computationally efficient. In some aspects, the results of the multifractal analysis are presented to be better, but the computational costs leave a lot to be desired. Therefore, due to computationally efficiency and ability to monitor, and prevent crisis events in advance, the entropy measures present to be the most attractive. However, empirical results of quantum and multifractal measures present to be optimal that motivate further research work.

3 Econophysical measures of DJSI complexity and precursors of crisis states

In a series of recent works [4; 44; 45; 50], we have demonstrated the possibility of using the theory of complex systems and a set of developed analysis tools to calculate the corresponding measures of system complexity. These complexity measures make it possible to differentiate systems according to the degree of their functionality, to identify and prevent critical and crisis phenomena.

Since the DJSI index is used as a calculation base, we will provide more detailed information for it. DJSI measures the performance of companies selected for economic, environmental, and social criteria that weighted by market capitalization using a best-in-class approach. In assessing sustainability, the key factor in selecting components for any DJSI index is the overall company sustainability rating (TSS). The first CSA was undertaken in 1999 with the launch of the original DJSI family of indexes. The annual CSA process begins in March of each year and is published with new estimates in September. The index is calculated using the divisor methodology that is used for all Dow Jones Index stock indices. Indices are calculated daily throughout the calendar year. The exception is those days when all exchanges on which the index constituents are quoted are officially closed or if the WM Reuters exchange rate services are not published.

The table 1 and figure 1 provide information on the key companies in the index basket and the weight of the respective economic sectors to which they belong.

Table 1. Top 10 components by index weight.

No	Constituent	Symbol	Sector
1	Microsoft Corp	MSFT	Information
2	Technology Alphabet Inc C	GOOG	Communication Services
3	Nestle SA Reg	NESN	Consumer
4	Staples United health Group Inc	UNH	Health Care
5	Taiwan Semiconductor Manufacturing Co Ltd	2330	Information
6	Technology Roche Hldgs AG Ptg Genus	ROG	Health Care
7	Adobe Inc.	ADBE	Information
8	Technology Novartis AG Reg	NOVN	Health Care
9	Cisco Systems Inc	CSCO	Information
10	Technology Bank of America Corp	BAC	Financials

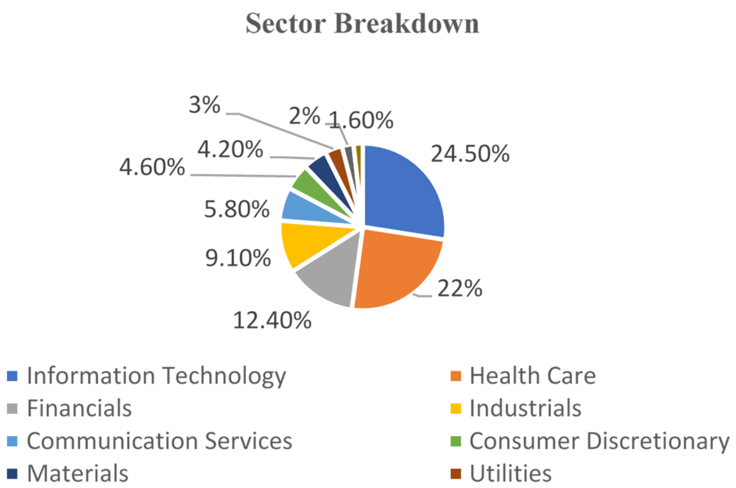

Fig. 1. Weights for each sector of the index, %.

For the daily DJSI time series $\{x(t)|t = 1, \dots, N\}$ we will carry out calculations of the corresponding measures of complexity within the framework of the moving window algorithm. For this purpose, the part of the time series (window), for which there were calculated measures of complexity, was selected, then the window was displaced along with the time series in a predefined value, and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the cryptocurrency. If this or that measure of complexity behaves in a definite way for all periods of crashes, for example, decreases or increases during the pre-crashes or pre-critical period, then it can serve as their indicator or precursor.

The returns over some time scale Δt is defined as the forward changes in the logarithm of the corresponding time series: $G(t) \equiv ln\, x\,(t + \Delta t)/\, ln\, x\,(t)$. We will determine standardized returns $g(t) \equiv [G(t) - \langle G \rangle]/\sigma$, where $\sigma \equiv \sqrt{\langle G^2 \rangle - \langle G \rangle^2}$ is the standard deviation of G, and $\langle \dots \rangle$ denotes the average over the time period under study.

In our previous paper [11] we devoted to a comparative analysis complexity of traditional stock market indices and social responsible indices in the example Dow Jones Sustainability Indices and Dow Jones Industrial Average. As measures of complexity, the entropies of various recurrence indicators are chosen – the entropy of the diagonal lines of the recurrence diagram, recurrence probability density entropy and recurrence entropy. It is shown that these measures make it possible to establish that the socially responsive Dow Jones index is more complex. In this paper, we will continue to use econophysical measures of complexity, considering other than recurrent entropy measures, as well as fractal and quantum measures of complexity in relation to the index DJSI.

4 Entropy complexity measures for an index DJSI

The most important quantity that allows us to parameterize complexity in deterministic or random processes is entropy. Originally, it was introduced by Clausius [8], in the context of classical thermodynamics, where according to his definition, entropy tends to increase within an isolated system, forming the generalized second law of thermodynamics. Then, the definition of entropy was extended by Boltzmann and Gibbs [5; 18], linking it to molecular disorder and chaos to make it suitable for statistical mechanics, where they combined the notion of entropy and probability.

After the fundamental paper of Shannon [42] in the context of information theory, where entropy denoted the average amount of information contained in the message, its notion was significantly redefined. After this, it has been evolved along with different ways and successful enough used for the research of economic systems [57].

A huge amount of different methods, as an example, from the theory of complexity, the purpose of which is to quantify the degree of complexity of systems obtained from various sources of nature, can be applied in our study. Such applications have been studied intensively for an economic behavior system.

The existence of patterns within the series is the core in the definition of randomness, so it is appropriate to establish such methods that will be based on the different patterns and their repetition [9]. In this regard, Pincus described the methodology *Approximate* *entropy* (ApEn) [37] to gain more detail analysis of relatively short and noisy time series, particularly, of clinical and psychological. Pincus and Kalman [36], considering both empirical data and models, including composite indices, individual stock prices, the random-walk hypothesis, Black-Sholes, and fractional Brownian motion models to demonstrate the benefits of ApEn applied to the classical econometric modeling apparatus. This research the usefulness of ApEn on the example of three major events of the stock market crash in the US, Japan, and India. During the major crashes, there is significant evidence of a decline of ApEn during and pre-crash periods. Based on the presented results, their research concludes that ApEn can serve as a base for a good trading system. Duan and Stanley [12] showed that it is possible to effectively distinguish the real-world financial time series from random-walk processes by examining changing patterns of volatility, ApEn, and the Hurst exponent. The empirical results prove that financial time series are predictable to some extent and ApEn is a good indicator to characterize the predictable degree of financial time series. Alfonso Delgado-Bonal [10] gives evidence of the usefulness of ApEn. The researcher quantifies the existence of patterns in evolving data series. In general, his results present that degree of predictability increases in times of crisis.

Permutation entropy (PEn), according to the previous approach, is a complexity measure that is related to the original *Shannon entropy* (ShEn) that applied to the distribution of ordinal patterns in time series. Such a tool was proposed by Bandt and Pompe [3], which is characterized by its simplicity, computational speed that does not require some prior knowledge about the system, strongly describes nonlinear chaotic regimes. As an example, Henry and Judge [20] applied PEn to the Dow Jones Industrial Average to extract information from this complex economic system. The result demonstrates the ability of the PEn method to detect the degree of disorder and uncertainty for the specific time that is explored.

4.1 Approximate entropy

When ApEn is calculated, for a given time series $\{x(i)|i = 1, \dots, N\}$, non-negative embedding parameter d_E, with $d_E \leq N$, and a filter r we construct subsequences $\vec{X}(i)$=$[x(i), x(i+1), \dots, x(i+d_E-1)]$ and $\vec{X}(j)$=$[x(j), x(j+1), \dots, x(j+d_E-1)]$. The relative neighborhoods in phase space are measures by L_∞ norm between all pairs of $\vec{X}(i)$ and $\vec{X}(j)$. Then, for each $i = 1, \dots, N-d_E+1$ we count the number of $j = 1, \dots, N-d_E+1$ that lie within a suitable distance r and define it as ${N_i}^{d_E}(r)$. For further estimations, we need to define the probability of finding such patters of the length d_E that will be similar to the given pattern:

$$ {C_i}^{d_E}(r) = \frac{{N_i}^{d_E}(r)}{(N-d_E+1)}, $$

or it can be presented in an equivalent form

$$ {C_i}^{d_E}(r) = \frac{1}{N-d_E+1} \sum_{j=1}^{N-d_E+1} \Theta(r - d[\vec{X}(i),\ \vec{X}(j)]), $$

where $\Theta(\cdot)$ is the Heaviside function which counts the instances where the distance d

is below the threshold r.

Next, we define the logarithmic average over all the vectors of the ${C_i}^{d_E}(r)$ probability as

$$F^{d_E}(r) = \frac{1}{(N-d_E+1)} \sum_{i=1}^{N-d_E+1} log(\,{C_i}^{d_E}(r))$$

and ApEn of a corresponding time series is defined as an increment of the absolute entropy $F^{d_E}(r)$ during the transition from a sequence of patterns of length d_E to a sequence of length $d_E + 1$ according to the following formula:

$$ApEn(d_E, r) = F^{d_E}(r) - F^{d_E+1}(r), \quad (1)$$

i.e., equation (1) measures the logarithmic likelihood that sequences of patterns that are close for d_E observations will remain close after further comparisons. Therefore, if the patterns in the sequence remain close to each other (high regularity), the ApEn becomes small, and hence, the time series data has a lower degree of randomness. High values of ApEn indicate randomness and unpredictability. But it should be considered that ApEn results are not always consistent, thus it depends on the value of r and the length of the data series. However, it remains insensitive to noise of magnitude if the values of r and d_E are sufficiently good, and it is robust to artefacts and outliers. Although ApEn remains usable without any models, it also fits naturally into a classical probability and statistics frameworks, and, generally, despite its shortcomings, it is still the applicable indicator of system stability, which significantly increased values may prognosticate the upcoming changes in the dynamics of the data.

The empirical results for the corresponding measure of entropy of DJSI are presented in figure 2:

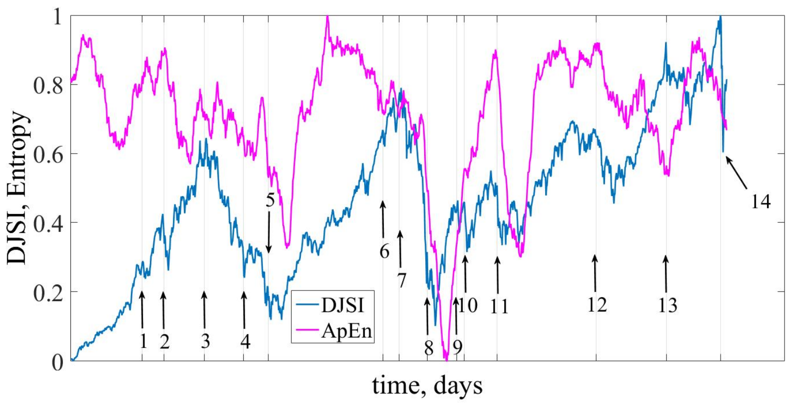

Fig. 2. ApEn dynamics of the entire time series of DJSI.

Long before the crisis, the value of this type of entropy begins to decrease, the complexity of the system decreases. This measure, in our opinion, is one of the earliest precursors of the crisis.

4.2 Permutation entropy

According to this method, we need to consider “ordinal patterns” that consider the order among time series and relative amplitude of values instead of individual values. For evaluating PEn, at first, we need to consider a time series $\{x(i)|i = 1, \dots, n\}$ which relevant details can be “revealed” in d_E-dimensional vector:

$$\vec{X}(i) = [x(i),\ x(i+\tau),\ .\ .\ .\ ,\ x(i+(d_E-1)\tau)],$$

where $i = 1, 2, \dots, N-(d_E-1)\tau$, and τ is an embedding delay of our time delayed vector. By the ordinal pattern, related to the time i, we consider the permutation $\pi_l(i) = (k_0, k_1, \dots, k_{d_E-1})$ of $[0, 1, \dots, d_E-1]$ where $1 \le l \le d_E!$. Then each of the subvectors is arranged in ascending order:

$$x(i+k_0\tau) \le x(i+k_1\tau) \le \cdots \le x(i+k_{d_E-1}\tau).$$

We will use ordinal pattern probability distribution as a basis for entropy estimation. Further, the relative frequencies of permutations in the time series are defined as

$$p(\pi_l) = \frac{the\ number\ of\ patterns\ thathas\ type\ \pi_l}{N-(d_E-1)\tau},$$

where the ordinal pattern probability distribution is given by $P = \{p_l(\pi_l)|l = 1, \dots, d_E!\}$. The Permutation entropy (denoted by $S[P]$) of the corresponding time series presented in the following form:

$$S[P] = -\sum_{l=1}^{d_E!} p_l\, log\, p_l\,.$$

Then, to take more convenient values, we calculate *Normalized permutation entropy* as

$$E_s[P] = \frac{S[P]}{S_{max}}$$

whose $S\ log\ d_{Emax}$ represents the maximum value of $E_s[P]$ (a normalization constant), and normalized entropy restricted between 0 and 1. Here, the maximal entropy value is realized when all $d_E!$ possible permutations are uniformly distributed (more noise and random data). With the much lower entropy value, we get a more predictable and regular sequence of the data. Therefore, the PEn gives a measure of the departure of the time series from a complete noise and stochastic time series.

In figure 3 we can observe the empirical results for permutation entropy, where it serves as indicator-precursor of the possible crashes and critical events.

Information measures of complexity due to their initial validity and transparency, ease of implementation and interpretation of the results occupy a prominent place among the tools for the quantitative analysis of complex systems.

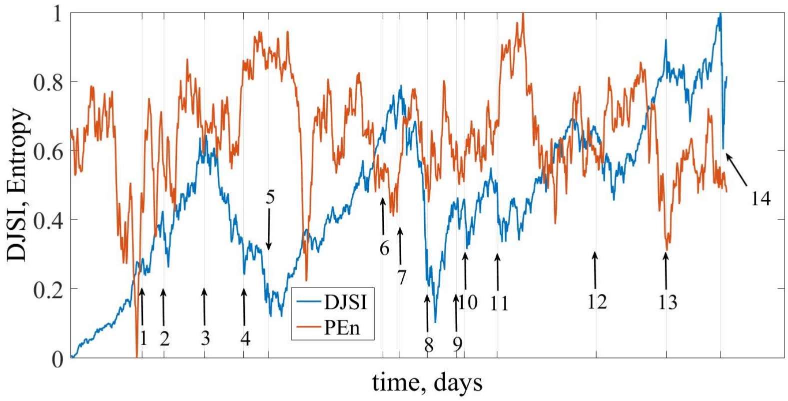

Fig. 3. PEn dynamics of the entire time series of DJSI.

5 Fractal and multifractral measures of complexity

The economic phenomena that cannot be explained by the traditional efficient market hypothesis can be explained by the fractal theory proposed by Mandelbrot [28]. Before, fractal studies focus on the Rescaled Range (R/S) analysis were proposed by Hurst [21] in the field of hydrology. Peng et al. [35] proposed a widely used Detrended Fluctuation Analysis (DFA) that uses a long-range power-law to avoid significant long-range autocorrelation false detection. As a multifractal extension (MF) of the DFA approach, Kantelhardtet et al. [22] introduced the MF-DFA method that for a long time has been successfully applied for a variety of financial markets. An especially interesting application of multifractal analysis is measuring the degree of multifractality of time series, which can be related to the degree of efficiency of financial markets [56].

Similarly to our article [17] where we applied the MF-DFA method to Ukrainian and Russian stock markets, we use it here to explore the multifractal property of DJSI and construct a reliable indicator for it.

As an extension to the original DFA, the multifractal approach estimates the Hurst exponent of a time series at different scales. Based on a given time series $\{x(i)|i = 1, \dots, N\}$, the MF-DFA is described as follows:

1. Construct the profile $Y(i)$ (accumulation) according to the equation below

$$Y(i) = \sum_{j=1}^{i}(g(j) - \langle g \rangle),$$

where $\langle g \rangle$ denotes the average of returns.

2. Then we need to divide the profile $\{Y(i)\}$ into $N_s \equiv int(N/s)$ non-overlapping segments of equal length s, and the local trend Y_v^{fit} for each segment is calculated by the least-square fit. Since time scale s is not always a multiple of the length of the time series, a short period at the end of the profile, which is less than the window size, may be removed. For taking into account the rejected part and, therefore, to use

all the elements of the sequence, the above procedure is repeated starting from the end of the opposite side. Therefore, the total $2N_s$ segments are obtained together, and the variance is computed as

$$F^2(v,\ s) = \frac{1}{s}\sum_{i=1}^{s}[Y((v-1)s+i) - Y_v^{fit}(i)]^2, \quad \text{for } v = 1,\ \dots\ ,\ N_s$$

and

$$F^2(v,\ s) = \frac{1}{s}\sum_{i=1}^{s}[Y(N-(v-N_s)s+i] - Y_v^{fit}(i)]^2, \quad \text{for } v = N_s+1, \dots, 2N_s.$$

Various types of MF-DFA such as linear, quadratic, or higher order polynomials can be used for eliminating local trend in segment v.

3. Considering the variability of time series and the possible multiple scaling properties, we obtain the q-th order fluctuation function by averaging over all segments:

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,\ s)]^{\frac{q}{2}}\right]^{\frac{1}{q}}.$$

The index q can take any non-zero value. For $q = 0$, $F_q(s)$ is divergent and can be replaced by an exponential of a logarithmic sum

$$F_0(s) = exp\left[\frac{1}{4N_s}\sum_{s=1}^{2N_s} ln(\,F^2(v,\ s))\right].$$

4. At least, we determine the scaling behavior of the fluctuation function by analyzing $log\,F_q\,(s)$ vs $log\,s$ graphs for each value of q. Here, $F_q(s)$ is expected to reveal power-law scaling

$$F_q(s) \sim s^{h(q)}.$$

The scaling exponent $h(q)$ can be considered as generalized Hurst exponent. With $q = 2$ MF-DFA transforms into standard DFA, and $h(2)$ is the well-known Hurst exponent.

5. Another way of characterizing multifractality of a time series is in terms of the multifractal scaling exponent $\tau(q)$ which is related to the generalized Hurst exponent $h(q)$ from the standard multifractal formalism and given by

$$\tau(q) = qh(q) - 1. \tag{2}$$

Equation (2) reflects temporal structure of the time series as a function of moments q i.e., it represents the scaling dependence of small fluctuations for negative values of q and large fluctuations for positives values. If (2) represents linear dependence of q, the time series is said to be monofractal. Otherwise, if it has a non-linear dependence on q, then the series is multifractal.

6. The different scalings are better described by the singularity spectrum $f(\alpha)$ which can be defined as:

$$\alpha = \frac{d\tau(q)}{dq} = h(q) + q\frac{dh(q)}{dq},$$
$$f(\alpha) = q[\alpha - h(q)] + 1,$$

with α is the Hölder exponent or singularity strength. Following the methods described above, we present results that reflect multifractal behavior of the DJSI time series.

Fig. 4(a) presents $F_q(s)$ in the log-log plot. The slope changes dependently on q, which indicates the multifractal property of a time series. As it was pointed out, multifractality emerges not only because of temporal correlation, but also because DJSI returns distribution turns out to be broad (fat-tailed), and this distribution could contribute to the multifractality of the time series. The same dependence can be observed in the remaining plots. The scaling exponent $\tau(q)$ remains nonlinear, as well as generalized Hurst exponents that can serve as evidence that Bitcoin exhibit multifractal property.

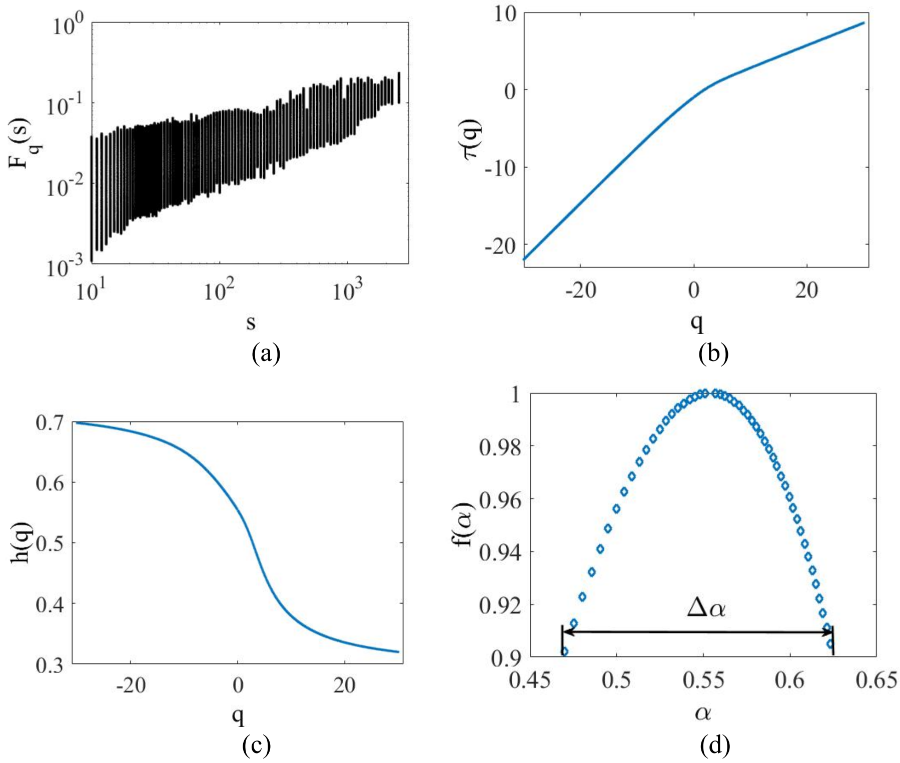

Fig. 4. The fluctuation function $F_q(s)$ (a), multifractal scaling exponent $\tau(q)$ (b), $h(q)$ versus q (c), and singularity spectrum $f(\alpha)$ (d) of the DJSI return time series obtained from MF-DFA.

In the case of multifractals, the shape of the singularity spectrum typically resembles an inverted parabola (see Fig. 4(d)); furthermore, the degree of complexity is straightforwardly quantified by the width of $f(\alpha)$, simply defined as

$\Delta\alpha = \alpha_{max} - \alpha_{min}$, where α_{min} and α_{max} correspond to the opposite ends of the α α values as projected out by different q-moments.

In the figure below we present the width of the spectrum of multifractality that changes over time accordingly to the sliding window approach. The whole figure consists of both a three-dimensional plot (singularity spectrum) and two-dimensional representation of its surface (fig. 5).

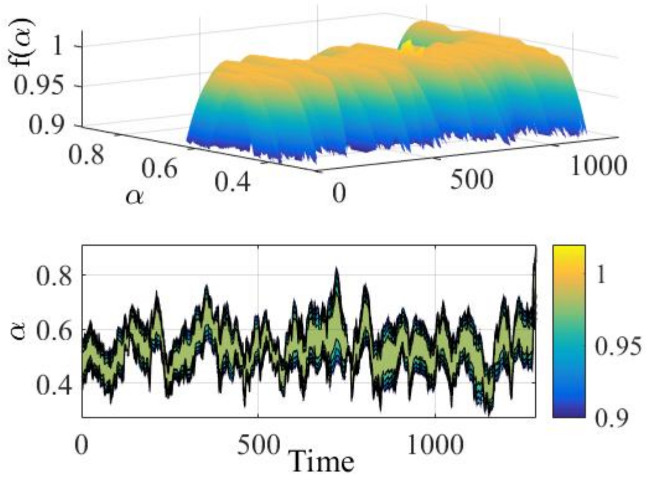

Fig. 5. Changes in the spectrum of multifractality in time.

If the series exhibited a simple monofractal scaling behavior, the value of singularity spectrum $f(\alpha)$ would be a constant. As can be observed, here our series exhibits a simple multifractal scaling behavior, as the value of singularity spectrum $f(\alpha)$ changes dependently on α, i.e., it exhibits different scalings at different scales. Moreover, with the sliding window of the corresponding length, we understand that at different time periods DJSI becomes more or less complex. The value of $\Delta\alpha$ gives a shred of additional evidence on it (fig. 6).

As we can see from the presented results, the width of the singularity spectrum after the crisis starts to increase, which tells us that more violent price fluctuations are usually expected. With the decreasing width of the singularity spectrum, the series is expected to hold the trend. As the rule, it reaches its minimum before the crash of DJSI value.

6 Heisenberg uncertainty principle and economic "mass" as a quantum measure of complexity

In this section, we will demonstrate the possibilities of quantum econophysics on the example of the application of the Heisenberg uncertainty principle [46]. In our paper [41], we have suggested a new paradigm of complex systems modeling based on the ideas of quantum as well as relativistic mechanics. It has been revealed that the use of quantum-mechanical analogies (such as the uncertainty principle, the notion of the operator, and quantum measurement interpretation) can be applied to describing socio-economic processes. Methodological and philosophical analysis of fundamental physical notions and constants, such as time, space, and spatial coordinates, mass, Planck's constant, light velocity from modern theoretical physics provides an opportunity to search for adequate and useful analogs in socio-economic phenomena and processes.

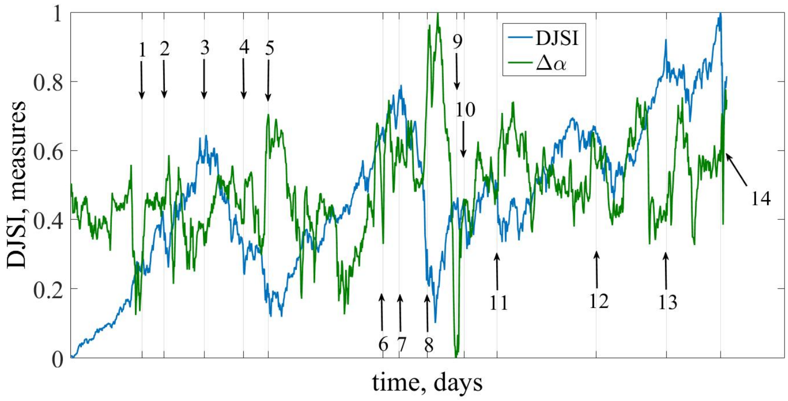

Fig. 6. The comparison of the DJSI time series with the width of the multifractality spectrum measure.

To demonstrate it, let us use the known Heisenberg's uncertainty ratio which is the fundamental consequence of non-relativistic quantum mechanics axioms and appears to be

$$\Delta x \cdot \Delta v \geq \frac{\hbar}{2m_0}, \tag{3}$$

where Δx and Δv are mean square deviations of x coordinate and velocity v corresponding to the particle with (rest) mass m_0, $\hbar$ – Planck's constant. Considering values Δx and Δv to be measurable when their product reaches their minimum, according to equation (3) we derive:

$$m_0 = \frac{\hbar}{2 \cdot \Delta x \cdot \Delta v},$$

i.e., the mass of the particle is conveyed via uncertainties of its coordinate and velocity – time derivative of the same coordinate.

Economic measurements are fundamentally relative, local in time, space and other socio-economic coordinates, and can be carried out via consequent and/or parallel comparisons "here and now," "here and there," "yesterday and today," "a year ago and now," etc.

Due to these reasons constant monitoring, analysis, and time series prediction (time series imply data derived from the dynamics of stock indices, exchange rates, cryptocurrencies prices, spot prices, and other socio-economic indicators) become relevant for the evaluation of the state, tendencies, and perspectives of global, regional, and national economies.

Suppose there is a set of K time series, each of N samples, that correspond to the single distance T, with an equally minimal time step Δt_{min}:

$$X_i(t_n), t_n = \Delta t_{min} n, \text{ for } n = 0, 1, 2, \ldots, N-1, \text{ for } i = 1, 2, \ldots, K.$$

To bring all series to the unified and non-dimensional representation, accurate to the additive constant, we normalize them, have taken a natural logarithm of each term of the series. Then, consider that every new series $X_i(t_n)$ is a one-dimensional trajectory of a certain fictitious or abstract particle numbered i, while its coordinate is registered after every time span Δt_{min}, and evaluate mean square deviations of its coordinate and speed in some time window $\Delta T = \Delta N \cdot \Delta t_{min} = \Delta N$, $1 \ll \Delta N \ll N$. The "immediate" speed of i particle at the moment t_n is defined by the ratio:

$$v_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{\Delta t_{min}} = \frac{1}{\Delta t_{min}} ln \frac{X_i(t_{n+1})}{X_i(t_n)},$$

with variance D_{v_i} and mean square deviation Δv_i.

After some transformations, we can write an uncertainty ratio for this trajectory:

$$\frac{1}{\Delta t_{min}} \left(< ln^2 \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} - \left(< ln \frac{X_i(t_{n+1})}{X_i(t_n)} >_{n,\Delta N} \right)^2 \right) \sim \frac{h}{m_i},$$

where m_i – economic "mass" of an X_i series, h – value which comes as an economic Planck's constant.

Since the analogy with physical particle trajectory is merely formal, h value, unlike the physical Planck's constant $\hbar$, can, generally speaking, depend on the historical period, for which the series are taken, and the length of the averaging interval (e.g., economical processes are different in the time of crisis and recession), on the series number i etc. Whether this analogy is correct or not depends on the particular series' properties.

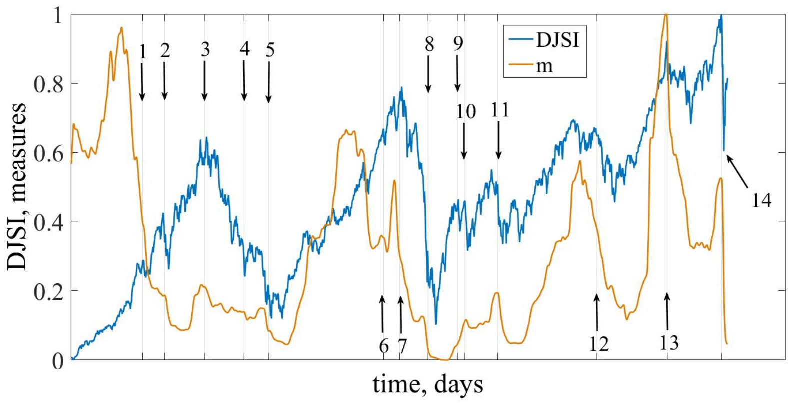

Fig. 7. Dynamics of measure m, and its dynamics with the window size of 250 days and step of 5 days.

Obviously, there is a dynamic characteristic values m depending on the internal dynamics of the market. In times of crashes and critical events marked by arrows, mass m is significantly reduced in the pre-crash and pre-critical periods (fig. 7). Obviously, m remains a good indicator-precursor even in this case. Value m is considerably reduced before a special market condition. The market becomes more volatile and prone to changes.

7 Conclusions

In this paper, for the first time, econophysical measures of complexity based on the analysis of entropy, multifractal, and quantum properties of time series are used for the analysis of sustainable development indices. Using the DJSI index as an example, it is shown that, firstly, all econophysical measures are complex measures and, secondly, they respond to critical and crisis conditions of the stock market.

In the future, a similar study for a set of other indices would be of interest, as well as a comparison with the results of using other quantitative measures of complexity.

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