The Criterion of Optimality in the Convex Vector Problem of Optimization

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Abstract
The article deals with the solution of the problem in vector optimization. It is shown there is a solution which is optimal according to Lagrange in the problem with convex particular functions of target that is characteristic to organizational and economic systems.

Keywords[1]
Vector objective/target function, Lagrange method, optimality, priorities of Pareto, vector optimization

1. Introduction

In the history of the development and use of information technologies, three stages of the use of models and methods of decision-making can be distinguished.

At the first stage, attempts were made to solve mainly optimization problems in a continual set of alternatives. This area has been developed in connection with the need to obtain an economic effect in industrial and transport technologies, both civil and military [1]. The massive nature of certain areas of use and development stimulated the interest of industrialists and the military in solving problems, and scientists in finding methods for solving optimization problems. At this stage, an important role was played by such scientists as L.I. Kantorovich, A.N. Kolmogorov, E.S. Wentzel, Dangitz, Ford, Bellman, etc [2-5].

The second stage of development is associated with the emergence and widespread penetration of computer technology into almost all spheres of human activity.

In the second half of the twentieth century, automated control systems for various purposes were developed [6]. When solving the problem of human-machine interaction, much attention was paid to the role of the human factor, which stimulated the emergence of the development of expert methods and the theory of decision-making, appropriate technical means and technologies based on these methods.

The third stage is associated with the rapid growth of databases, knowledge bases in general and special-purpose information systems, which stimulated research to identify regularity that could be used for decision-making. Within the framework of this new paradigm of data mining (knowledge discovery), systems began to be developed for analyzing large amounts of data. Among other regularities, a significant proportion is the identification of preferences on a set of objects, which is directly related to decision making.

The increased role of the human factor in creating knowledge bases and solving the problem of formalizing the conclusion of conclusions was the basis for including the theory of decision-making in the new scientific direction "Artificial Intelligence", which unites research in the field of brain-like structures. Decision support systems that have combined optimization and expert methods, databases

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and knowledge have become a practical tool for decision-makers.

Among the works that influenced the formation of a new scientific direction, one can single out the works of N. Nilsson on the creation of a universal problem solver, G. Simon, who studied the problem of uncertainty in the science of artificial intelligence proposed by him, L. Zade, who proposed the formalization of subjective uncertainty, D.A. Pospelov, who studied the problem of choice within the framework of situational management. Selection of the best objects based on their attributive representation. Zagoruiko in the framework of pattern recognition [7-14].

Thus, at present, it becomes possible to create brain-like computing structures for ultra-high performance computers [15].

One of the most important tasks, especially in demand at the second and third stages of the development of decision-making theory, is the study of methods of multicriteria choice on a finite set of alternatives (objects). These include vector and scalar optimization.

Vector optimization methods are based on establishing preferences on a set of vector object estimates. In the early twentieth century, Pareto and Edgeworth proposed the dominance relationship, later called Pareto-dominance. Research in this direction was continued by B. Rua, V.V. Podinsky, V.D. Nogin [16-19]. The ordering of objects based on the qualitative values of features was formed in the direction of the verbal analysis of solutions, developed in the works of O.I. Laricheva, A.B. Petrovsky and others [1, 20, 21].

Methods for scalarization of vector estimates are based on transforming a multi-criteria optimization problem into a single-criteria optimization problem using a multi-criteria utility function. The issues studied in the framework of this direction were formalized into a multicriteria utility theory. In the works of J. von Neumann, O. Morgenstern, an axiomatic approach was developed and the main directions of the multicriteria utility theory were formulated, which were then developed in the works of H. Rife, R. Keene, P. Fishburne, W. Armstrong, S.V. Emelyanova, N.V. Khovanova and others [22-28].

Of the Ukrainian scientists who are fruitfully working in solving this scientific direction, we consider it useful to single out the scientific school of E.G. Petrov [29].

Modern information technologies are largely based on the use of optimization procedures, which ensures the efficiency of their implementation [30].

At the same time, the complexity and coherence of the problems being solved causes the appearance of significant time consumption for optimization, which manifests itself as the effect of the dynamics of optimization procedures [31].

It should be considered, that with the development and improvement of information technology, the problem of finding the best solutions will take an increasing share among all processes.

The desire to increase the productivity and accuracy of information processing processes led to a revision of the approach to determining the amount of information and the transition to the analysis and synthesis of information systems in the information space [32].

The development of the information theory was historically associated with the works of R. Hartley, A. N. Kolmogorov, K. Shannon, A. Ya. Khinchin, V.A. Kotelnikov, V.D. Goppa, A.M. Iahlom [33-42]. Currently, there is a lack of elaboration of information processes and processes of forming models of systems and processes.

Human activity is associated with different types and methods of decision-making management. These include, for example, the problem of finding optimal options for managing organizational systems, the problem of designing systems that provide a control law for a given object or a certain control sequence of actions that provide either the maximum or minimum of a given set of necessary system quality criteria.

The assumed principles in the plotting and analysis of informational systems are saving principle, optimality principle, and the principle of unidirectional flow of time. In fact, saving principle and unidirectional flow of time processes are feasibility criteria that allow avoiding mistakes in the analysis and synthesis; and optimality principle is a tool providing protection from mistakes of "simple" solutions.

That`s why it is logical to thoroughly analyze initial criteria of optimality before studying intellectual systems where you deal with quite combined systems [43].

Organizational control systems have an important place among the control systems in total. Here it refers to the organizational control which differs from control of technological processes most notably in its object which is not machinery (equipment) but people, groups of people. It stands to reason the border defining the difference is rather conditional. Owing to it the control is implemented by people and it should be considered as organizational control [44].

We are interested first and foremost in optimal control of organizational systems. The optimal control of organizational systems is a problem of designing a system that provides for the predefined object of control or a process the law of control or controlling consequence of influences providing maximum and minimum of the predefined criteria manifold for the system quality [45].

The problems in optimal control of organizational systems require solving problems of multicriteria or vector optimization. It seems reasonable to use an approach where convexity of particular target functions is viewed as definite peculiarity of the control system for solving the tasks of vector optimization.

2. The modern state of the vector optimization problem

To solve the problems of choice, you need to know:

- selectable alternatives;
- a set of functions that characterize the properties of an object;
- a set of goals targeted to the evaluated features;
- a group of people taking part in decision-making;
- preference ration;
- the degree of conformity of the alternatives to the target state [44].

The optimality principle, in its implementation, is associated with the concept of the goal function. For the problem of choosing a variant or value of an impact, the goal function is a mapping over a set of numbers - a function. In this case, for the goal function *f(x)*, the value of the vector x is sought, which determines the solution to the optimization problem.

The problem of mutual optimization for several particular functions of objective is referred to as multiparameter optimization [46]:

$$x^* \to \min f_i(x), \quad i = \overline{1,n}. \tag{1}$$

The most accurate name of the problem (1) - vector optimization - emphasizes that it is simultaneous optimization of all components of objective function vector that the issue is about:

$$x^* \to \min f(x) \tag{2}$$

Indeed, since particular objective functions (1) can form the vector where components are particular objective functions (2) the problem can be called multicriteria because the objective function can be called a criterion though the notion “criterion” has a reasonably definite meaning in modeling, the theory of automatic control and the decision theory. In the endeavor to decrease the complexity of the problem (2), we define it as the problem with many functions of objective or the problem of vector optimization.

It is an outwardly simple problem – you have to find vector x* brining the minimum immediately to all components of the target vector. Since *dimx=n*, and *dimf=m*, stationary condition in the form of matrix equation [47,48]:

$$\frac{\partial f}{\partial x} = 0, \tag{3}$$

induce the gradient matrix:

$$\frac{\partial f}{\partial x} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \vdots & \frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_m}{\partial x_1} & \vdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix} \tag{4}$$

with dimension n to m. Solving the set of equations (3) lets us find new particular optima but it doesn’t make it possible to find the optimum for the vector function of objective since m of links among the components of objective function vector is absent. Consequently, to determine the optimum point it is essential to have m of links.

Methods for solving problems of the vector optimization differ in the way of additional organization [48].

The global criterion method implies ranging of criteria and objectives significance. When solving problems by the method of the main criterion, there is no listing of alternatives, one objective function and a set of restrictions are used, that is, one criterion is formed as the main target, and the rest as restrictive. The objective function plays the role of the main criterion and it is believed that the problem of multi-criteria optimization is solved by the main criterion method. A one-to-one correspondence between alternatives and outcomes is characteristic, the considered tasks are solved in conditions of certainty[44,49].

In such a case it is solved simply, but you have to decide by yourself what a global criterion is.

$$x^* \to \min f_k(x) = \alpha_k \tag{5}$$

Method of forming global objective function [50] uses polynomial models of the kind:

$$\begin{gathered} f(x) = \sum_{i=1}^{k} a_i f(x); \\ \left. \begin{aligned} a_i > 0,\ \ i = \overline{1,k} \\ \sum_{i=1}^{k} a_i = 1 \end{aligned} \right\} \end{gathered} \tag{6}$$

Weight coefficients α_i determine sensitivity of global objective function f to the change of particular objective functions f_i:

$$a_i = \frac{\partial f}{\partial f_i},\ a_i > 0,\ i = \overline{1,k} \tag{7}$$

Obviously coefficients α_i are normalization multipliers of Lagrange in the problem:

$$\begin{gathered} x^* \to \min f(x), \\ f_i(x) - c_i = 0, \quad i = \overline{1,k}. \end{gathered} \tag{8}$$

With Lagrange function:

$$L(x,\lambda) = \lambda_0 f(x) - \sum_{i=1}^{k} \lambda_i (f_i(x) - c_i) \tag{9}$$

which leads to Lagrange problem with minimization of constraints influence [48]. Obviously the appointment of weights is possible and “based on the problem requirements”.

It is also possible to use posynomial models of the form:

$$\begin{gathered} f(x) = \prod_{i=1}^{k} f_i^{a_i}(x); \\ f_i > 0, \quad i = \overline{1,k} \end{gathered} \tag{10}$$

Or with structural links and feedbacks present it is reasonable to use a posynomial model:

$$\begin{gathered} f(x) = \sum_{j=1}^{p} C_j \prod_{i=1}^{k} f_i^{a_{ij}}(x); \\ f_i > 0,\ \ i = \overline{1,k}; \\ C_j \geq 0,\ \ j = \overline{1,p} \end{gathered} \tag{11}$$

The given approach is called “sygnomial” optimization and based on the method of geometrical programming [51].

The Min-max methods is based on a game-theoretic formulation, in which the researcher and the external environment are considered as a pair of players with mutually contradictory interests. The goal of the researcher, as before, is to minimize the criterion by choosing an estimation operator from a certain class, but with the Min-max methods, the estimate is sought based on the worst state of the system under study. Thus, the estimation problem is reduced to minimizing the exact upper bound of the criterion, calculated from a given set of uncertainty. Therefore, unlike asymptotic estimation methods, Min-max methods are designed to provide the best quality of recovery of unknown parameters and processes from a fixed volume of observations [52].

Min-max methods are based on forming "benchmarks" [53, 54]:

$$\left.\begin{array}{c} f_i(x) \le t_i \quad i = \overline{1,k} \\ g_i(x) = 0, \;\; i = \overline{1,m} \\ g_i(x) \le 0, \;\; i = \overline{m+1,l} \end{array}\right\} \tag{12}$$

which provides no negativeness of the difference $t_i - f_i(\boldsymbol{x})$ for all valid values $\boldsymbol{x}$.

In such a case one can pass on to the minimum of the maximal deviation from metrics C_0 in order to find a strong optimum:

$$x^* \to \min\max\big(t_i - f(x)\big), \quad i = \overline{1,k}. \tag{13}$$

Priorities of Pareto are the method based on determining of an extremum or having necessary condition.

When optimizing by the Pareto method, an agreement is used that preference is given to one object over another only if the first object is not worse than the second in all respects and at least one of them is better. When this condition is met, the first object is considered dominant, the second dominated. Finding the Pareto optimum is assumed to be the equilibrium of all criteria, therefore, it can be considered the preferred optimization criterion, since the "rights" of any of them cannot be infringed in favor of other criteria [44, 55].

In the problem:

$$\begin{array}{c} x^* \to \min f_i(x), \quad i = \overline{1,k}; \\ \left.\begin{array}{c} g_i(x) = 0, \;\; i = \overline{1,m} \\ g_i(x) \le 0, \;\; i = \overline{m+1,l} \end{array}\right\} \end{array} \tag{14}$$

Let`s introduce optional constraints:

$$\begin{array}{c} f_i(x) \le f(x_d), \quad i = \overline{1,s}; \\ f_i(x) < f(x_d), \quad i = \overline{s+1,k} \end{array} \tag{15}$$

Thus, we come to the problem:

$$\begin{array}{c} x^* \to \min f_i(x), \qquad i = \overline{1,k}; \\ g_i \le 0, \;\; \overline{1,l}; \\ f_i(x) - f(x_d) \le 0, \qquad i = \overline{1,k} \end{array} \tag{16}$$

which leads to Lagrange problem with Lagrange function [48]:

$$L(x,\lambda) = \lambda_0 f(x) - \lambda \mathrm{g} - \lambda(f(x) - f(x_d)) \tag{17}$$

where $\boldsymbol{\lambda}$ is a matrix of Lagrange multipliers – coefficients of objective function sensitivity to the constraints:

$$\lambda = \begin{pmatrix} \frac{\partial f_1}{\partial f_1} & \vdots & \frac{\partial f_1}{\partial f_k} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_k}{\partial f_1} & \vdots & \frac{\partial f_k}{\partial f_k} \end{pmatrix} = \begin{pmatrix} 1 & \lambda_{12} & \cdots \lambda_{1k} \\ \cdots & 1 & \cdots \\ \lambda_{k1} & \lambda_{k2} & \cdots 1 \end{pmatrix} \tag{18}$$

There with the condition $\lambda_{ij}=0$ determines the location of an optimum within the area. In the given case the admissible points are considered to be the points to which the conditions of absolute inequality are applied. The realization of the inequality can be described as the realization of preference relation. The excess in admissible points leads to the local optimum which is called an effective solution or Pareto – an optimal solution. If one or several constraints can be hardly transformed into the strict ones, the decision is called weakly efficient. As a matter of fact, the method of Pareto comes to the procedure of solution choice with minimal mutual influence of particular targets or to the analysis of matrix $\boldsymbol{\lambda}$ in the problem (16).

Thus, we can draw a conclusion that modern methods in solving problems of vector optimization can be considered as variants of solution for Lagrange problem which allows us to set up the problem of strict argumentation for the optimization criterion in the problem of vector optimization.

3. The statement of the investigation problems

The purpose of the investigation is the analysis and refinement of optimality criterion offered in the given article for the convex vector problem [47].

4. The subject matter of the investigation problems

In accordance with the formulated investigation, the following was proposed: the problems of optimal management of organizational systems are the problem of designing a system that provides for a control object or a control process a control law or a control sequence of actions that provide a maximum or minimum of a given set of system quality criteria.

We consider the simple problem of vector optimization [46]:

$$x^* \to \min f(x) \tag{19}$$

where *dim**x***=*n*, *dim**f***=m.

To be more specific we will talk about the minimum of the function as it is generally accepted. In the case the stationary condition as a necessary condition has the form of the matrix equation:

$$\begin{gathered}\nabla f(x) = \begin{bmatrix}\nabla f_1(x)\\ \vdots \\ \nabla f_m(x)\end{bmatrix} = \begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \vdots & \frac{\partial f_1}{\partial x_n}\\ \cdots & \cdots & \cdots \\ \frac{\partial f_m}{\partial x_1} & \vdots & \frac{\partial f_m}{\partial x_n}\end{pmatrix} = \\ = \begin{pmatrix}0 & \vdots & 0\\ \cdots & \cdots & \cdots\\ 0 & \vdots & 0\end{pmatrix} \Leftrightarrow \frac{\partial f_{i,j}}{\partial x_j} = 0, (i,j = \overline{1,n,m})\end{gathered} \tag{20}$$

As it is shown above to find the minimum of the vector objective function it is required to introduce additionally *m* of relations.

We apply the standard method of constraints variation except that the objective vector itself is used as a constraints vector:

$$f(x) - \alpha = 0, \tag{21}$$

where vector **α** is determined by the value of objective vector in the point of minimum:

$$f(x^*) = \alpha, \tag{22}$$

In such a case we have the vector problem with the constraint of equality type by analogy with (17):

$$\left.\begin{aligned}x^* \to \min f(x)\\ f(x) - \alpha = 0\end{aligned}\right\}. \tag{23}$$

Lagrange function for the given problem has the following form:

$$L(x,\lambda) = \lambda_0 f(x) - \lambda(f(x) - \alpha). \tag{24}$$

In this case the necessary conditions of the minimum take form [56]:

$$\left.\begin{aligned}\nabla L(x^*,\lambda^*) = 0\\ f(x^*) - \alpha^* = 0\end{aligned}\right\} \tag{25}$$

The obtained set of equations doesn't have a solution since the quantity of variables doesn't correspond to the quantity of links.

As an additional link we will introduce the assumption about convexity of all particular target functions [57]. In this case due to dual features we obtain:

$$\left.\begin{aligned}x^* \xrightarrow[\lambda=\lambda^*]{} \min L(x,\lambda)\\ \lambda^* \xrightarrow[x=x^*]{} \max L(x,\lambda)\end{aligned}\right\} \tag{26}$$

Due to stationarity of the problem and use of the first condition from (26) we'll have:

$$x^* \xrightarrow[\lambda=\lambda^*]{} \min \{\lambda_0 f(x) - \lambda(f(x) - \alpha)\} = const \tag{27}$$

Since the target vector ***f*** is equal to the vector of optimal values in the point of minimum the expression can be written in the form:

$$\lambda_0 f(x^*) - \lambda^*(f(x^*) - \alpha^*) = \lambda_0 \alpha^* \tag{28}$$

Thus, constraints take on the form of conditions of additional slackness [58, 59]:

$$(\lambda_0 - \lambda)(f(x) - \alpha) = 0 \tag{29}$$

Considering minimality of the mutual influence in target functions the condition of additional slackness appears quite naturally in Lagrange function.

Actually the same requirements occur under Pareto optimality.

Hence, you can write down Lagrange function with constraints in the form of slackness conditions in the form:

$$x^* \xrightarrow[\lambda=\lambda^*]{} \min\{\lambda_0 f(x) - (\lambda - \lambda_0)(f(x) - \alpha)\} = 0 \tag{30}$$

Consequently, according to Lagrange in the stationary convex problem of vector optimization the necessary condition of optimality has the form:

$$\left.\begin{array}{c} \nabla L(x^*, \lambda^*) = 0) \\ f(x^*) - \alpha^* = 0 \\ \lambda_0 f(x^*) - (\lambda^* - \lambda_0)(f(x^*) - \alpha^*) = 0 \end{array}\right\} \tag{31}$$

Introducing the matrix of adduced vectors for Lagrange multipliers λ^{**}:

$$\lambda^{**} = \begin{pmatrix} 1 & \lambda_{12} & \lambda_{13} \cdots & \lambda_{1m} \\ \lambda_{21} & 1 & \lambda_{23} \cdots & \lambda_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \lambda_{m1} & \lambda_{m2} & \lambda_{m3} \cdots & 1 \end{pmatrix} \tag{32}$$

we can write down the optimality condition in the form:

$$\left.\begin{array}{c} \nabla L(x^*, \lambda^*) = 0) \\ f(x^*) - \alpha^* = 0 \\ \lambda^{**}(f(x^*) - \alpha^*) = 0 \end{array}\right\} \tag{33}$$

Actually the additional condition is the condition of additional slackness.

5. Practical part

Let us consider the well-known example [60].

The target function vector:

$$f(x) = \begin{bmatrix} x^2 \\ (x-1)^2 \end{bmatrix} \tag{34}$$

Using the condition (20) we obtain:

$$\nabla f(x) = \begin{bmatrix} \dfrac{d(x^2)}{dx} \\ \dfrac{d((x-1)^2)}{dx} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Which provides two points of particular minima $x_1^*=0$ and $x_2^*=1$, fig. 1. To solve the problem with vector function of target we will introduce the constraints:

$$\begin{array}{c} f_1(x) - \alpha_1 = 0 \\ f_2(x) - \alpha_2 = 0 \end{array} \tag{35}$$

In the problem Lagrange functions have the following form:

$$\begin{array}{c} L_1(x, \lambda_{12}) = x^2 - \lambda_{12}((x-1)^2 - \alpha_2); \\ L_2(x, \lambda_{21}) = (x-1)^2 - \lambda_{21}(x^2 - \alpha_1) \end{array} \tag{36}$$

necessary conditions of the optimum:

$$\left.\begin{array}{c} \dfrac{\partial L_1}{\partial x} = 2x - \lambda_{12} 2(x-1) = 0 \\ \dfrac{\partial L_2}{\partial x} = 2(x-1) - \lambda_{21} 2x = 0 \\ x^2 - \alpha_1 = 0 \\ (x-1)^2 - \alpha_2 = 0 \\ x^2 - \lambda_{12}((x-1)^2 - \alpha_2) = 0 \\ (x-1)^2 - \lambda_{21}(x^2 - \alpha_1) = 0 \end{array}\right\} \tag{37}$$

from the link (35):

$$\begin{array}{c} 2x - \lambda_{12} 2(x-1) = 0; \\ 2(x-1) - \lambda_{21} 2x = 0. \end{array} \tag{38}$$

We obtain $\lambda_{12}=1/\lambda_{21}$. Since the optimum point is unique from constraints we have α_1-α_2=2x-1.

Consequently, we can write it down:

$$x^2 - \frac{x}{x-1}\left((x-1)^2 - \alpha\right) = 0;$$
$$(x-1)^2 - \frac{x-1}{x}\left(x^2 - 2x + 1 - \alpha\right) = 0. \tag{39}$$

from which we obtain the connection:

$$(x-1)^2 = x^2 \tag{40}$$

Consequently, optimum is reached in x*=1/2 point and target functions have the value which is equal to α*=1/4 in the optimum, Fig. 1.

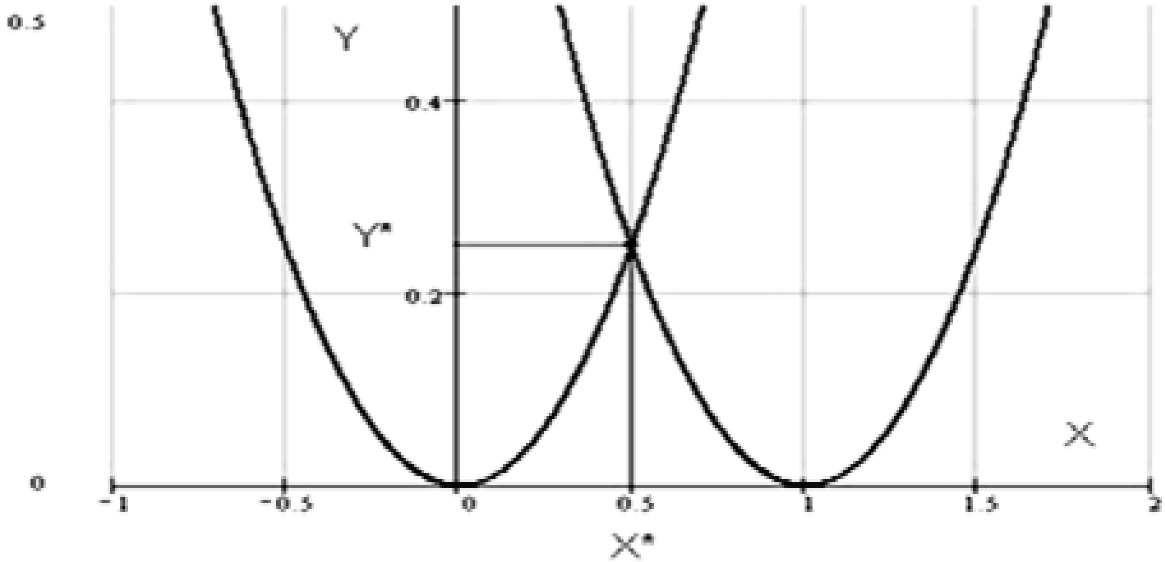

Figure 1: Graphic Solution of the Vector Optimization Problem

Hence, the assumption about convexity of particular target functions lets us obtain the optimal solution of the problem without invoking compromises and expert estimation.

6. Conclusions

The analysis and research carried out made it possible to make an assumption about the convexity of the partial functions. This assumption, in turn, made it possible to obtain optimal solutions to the problem without involving compromises and expert assessments.

All of the above made it possible to obtain the following important conclusions that can be used in the search for optimal solutions in building management systems for organizational systems.

It is shown that methods used for solving the problem of vector optimization can be kept to the necessary conditions in Lagrange problem.

It is shown that the problem of vector optimization has the solution based on duality or the optimal one according to Lagrange in the case of convexity of particular target functions.

The condition of optimality contains the condition of additional slackness.

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