Method of Diagnostic of Non-Positional Code Structures in the System of Residue Classes Basing on the Usage of an Alternative Number Set Informativeness

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Abstract

A method of diagnosis of data represented in the system of residue classes (SRC) is suggested in the article. It is shown, that the main disadvantage of existing methods of diagnosis data in SRC is a significant time of data diagnosis while the necessity of entering heavy informational redundancy to non-positional code structure (NCS) in SRC. The considered in the article method of diagnosis data in SRC allows increasing operability of a diagnosis procedure while entering minimal informational redundancy. The time of data diagnostic, compared to known methods, is decreasing firstly due to excluding the procedure of transforming numbers in SRC to positional notation as in known methods, i. e. eliminating a positional operation of numbers comparing. Secondly, the time of data diagnostic is decreased by reducing the quantity of SRC bases, which are giving the possibility of mistakes. Thirdly, the time of data diagnostic is decreased due to the usage of tabular sample value of an alternative set (AS) of numbers in SRC in one beat. The quantity of additionally entered informational redundancy is decrease by effective usage of inner informational redundancy existing in NCS. A specific example of the usage of the suggested method of diagnosis data in SRC is given. Therefore, the suggested method allows reducing the time of diagnosis of data errors in NCS, represented in SRC, which is increasing the diagnostic operability while entering minimal informational redundancy.

Keywords

Non-positional code structures, system of residue classes, alternative number set informativeness.

1. Introduction

A foundation of some modern specialized informational and telecommunication systems is based on computer systems (CS) of handling integer data, represented in non-positional notation in residue classes (SRC) [1–4]. In this case, one of the main ways of achieving high effectiveness of functioning of telecommunication systems while handling integer data in real-time is an improvement, firstly, such features of CS in SRC as reliability and performance of data handling [5–8].

It is known, that usage of such features of SRC as independence, rights equality, and low-discharge of residues $\{a_i\}$ defining non-positional code structure (NCS) of data

 $A_{SRC} = (a_1 \parallel a_2 \parallel ... \parallel a_{i-1} \parallel a_i \parallel a_{i+1} \parallel ... \parallel a_n \parallel ... \parallel a_{n+k})$

provides high user performance of implementing in CS calculation algorithms, which consist of a set of integer arithmetical operations [9–12]. The largest effectiveness of SRC usage can be achieved in case if implemented algorithms consist of a set of such arithmetical operations as addition, multiplication, and subtraction [8, 13, 14].

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On the other hand, a necessity of providing fault-tolerant functioning of CS in SRC requires the development and deployment of methods of quick control, diagnostic, and data error correction, which are different from methods, used in regular binary positional notations (PN) [15–17].

Thus, researches, devoted to the development and improvement of quick (operative) methods of diagnostic of errors of data in CS, functioning in SRC, are important and relevant.

The aim of the article is the development of the method of quick diagnostic of data in SRC while entering minimal informational redundancy.

In the general case, the diagnosis of data in SRC is being understood as a process of defining distorted residues in NCS as

$$A_{SRC} = (a_1 \| a_2 \| \dots \| a_{i-1} \| a_i \| a_{i+1} \| \dots \| a_n \| \dots \| a_{n+k}),$$

where n and k are quantity of informational and control bases m_i (i = $\overline{1, n+k}$) in ordered ($m_i < m_{i+1}$) SRC, correspondingly.

The diagnostic of NCS is being performed after the data control for further probable error correction. In the article, the method of data diagnostic in the case of entering minimal (k = 1) informational redundancy is considered. The minimal code distance equals two. The method is based on the concept of an alternative number set and the usage of features of NCS in SRC [10,14]. Due to those the procedure of increasing informativeness of AS in SRC is developed.

2. The Method of Diagnostic of Non-Positional Code Structures in the System of Residue Classes

Consider the method of NCS diagnostic, based on obtaining additional information about probably distorted residues of incorrect number \tilde{A} . This information is contained in all possible AS of number \tilde{A} .

Let SRC is specified by ordered $(m_i < m_{i+1})$ bases m_1, \dots, m_{n+1} . And let an incorrect number \tilde{A} is defined in the process of calculations.

For increasing informativeness about placement and error measures, it is suggested to additionally define AS of number as

 $W_{k_{\rho_i}}(\tilde{A}) = \{m_{k_1}, m_{k_2}, \dots, m_{k_{\rho_i}}\},\$

Tentatively calculate the value of the interval (j+1) of the number \tilde{A} occurrence in order to define the set of values (1)

$$j_k = \overline{m}_k \cdot \gamma_k \left(\mod m_k \right), \tag{2}$$

for $k = \overline{1, n+1}$.

Also, due to value k = n + 1 the $W_{n+1_{on+1}}(\tilde{A})$ equals

$$W_{n+1_{\rho_{n+1}}}\left(\tilde{A}\right) = W\left(\tilde{A}\right).$$

According to (2) the formation of k tables is performed, where values γ_k are matched against Δa_i . After defining AS $W_{k_{\rho_i}}(\tilde{A})$, which called primary ASs, the secondary ASs are defined as vectors, components of which are possible values of errors Δa_i as:

$$W_{1}^{(1)}\left(\tilde{A}\right) = \left\{\Delta a_{1}^{(1)}, \Delta a_{2}^{(1)}, ..., \Delta a_{n+1}^{(1)}\right\},\$$

$$W_{1}^{(\psi_{1})}\left(\tilde{A}\right) = \left\{ \Delta a_{1}^{(\psi_{1})}, \Delta a_{2}^{(\psi_{1})}, \dots, \Delta a_{n+1}^{(\psi_{1})} \right\};$$

$$W_{2}^{(2)}\left(\tilde{A}\right) = \left\{ \Delta a_{1}^{(2)}, \Delta a_{2}^{(2)}, \dots, \Delta a_{n+1}^{(2)} \right\},$$

$$\dots$$

$$W_{2}^{(\psi_{2})}\left(\tilde{A}\right) = \left\{ \Delta a_{1}^{(\psi_{2})}, \Delta a_{2}^{(\psi_{2})}, \dots, \Delta a_{n+1}^{(\psi_{2})} \right\};$$

and so on to the value of vectors in the form of:

$$W_{n}^{(\psi_{n})}\left(\tilde{A}\right) = \left\{\Delta a_{1}^{(\psi_{n})}, \Delta a_{2}^{(\psi_{n})}, ..., \Delta a_{n+1}^{(\psi_{n})}\right\},$$

and completely to value of vector as:

$$W_{n+1}(\tilde{A}) = \{\Delta a_1, \Delta a_2, \dots, \Delta a_{n+1}\}.$$

Components of the vector $W_{n+1}(\tilde{A})$ are compared to according components of all vectors $W_i^{(\psi_i)}(\tilde{A})$ for $i = \overline{1, n}$. The matching the measure components of vectors are chosen and the bases of SRC are defined, and their set defines resulted AS in the form of

$$W'(\tilde{A}) = \left\{ m_{z_1}, m_{z_2}, \dots, m_{z_{\rho}} \right\}.$$

Indeed, among AS $W_{k_{\rho}}(\tilde{A})$ there is always a basis m_i , which gives an error Δa_i , and that basis can be only among bases, which are common for the set (1)

$$W\left(\tilde{A}\right) \ge W'\left(\tilde{A}\right). \tag{3}$$

When an Δa_i has such value, that number A starts to belong to the interval, then an equation is fulfilled

$$W\left(\tilde{A}\right) = W'\left(\tilde{A}\right) \tag{4}$$

where

 $M=\prod_{i=1}^n m_i$

and

$$M_1 = M \cdot m_{n+1}$$

Thus, the idea of the suggested method lays in the following: all possible ASs are defined on each of the intervals of number A occurrence. After this, the common for these intervals bases

$$m_{z_1}, \ldots, m_{z_{\rho}},$$

which possibly give errors, are defined.

That set of bases define sought AS. The reduction of the number of bases in AS increases the informativeness of AS $W(\tilde{A})$ about place and measure of error. It decreases the time of reducing AS to an incorrect basis (the amount of steps of tentatively AS defining is decreasing) and increases operability of diagnostic of data in SRC.

The structure scheme of the process of AS reduction is presented in Fig. 1.

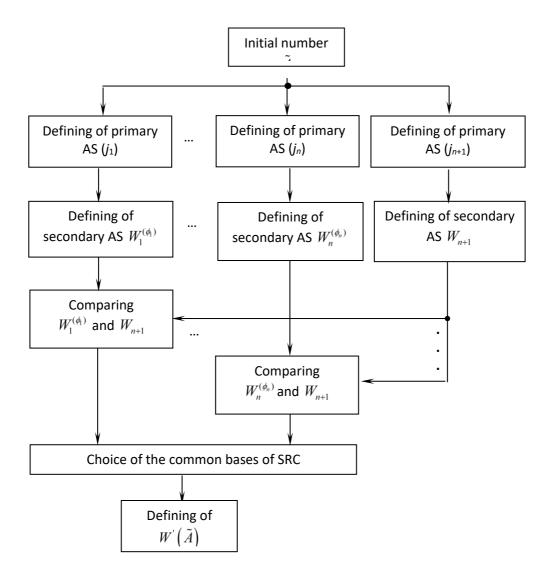


Figure 1: Scheme of choice of bases in the alternative set of numbers in SRC

3. Geometrical Model of the Procedure of the Increasing AS Informativeness

The geometrical model of the suggested method should be considered. The defining of the number (j+1) of the interval of distorted number \tilde{A} occurrence, which is influenced by error Δa_i , is equivalent to the shift of this number in the interval $\left[j\frac{M_i}{m_i},(j+1)\frac{M_1}{m_1}\right]$ to the left to value $j\frac{M_1}{m_1}$. Decompose numerical sequence to corresponding intervals with length: $\frac{M_1}{m_1}, \frac{M_1}{m_2}, \dots, \frac{M_1}{m_{n+1}}$. Define the numbers of intervals (j+1), in which there is an operand \tilde{A} on each of the numerical segments as

$$T_{j_{1}} = \left[j_{1} \frac{M_{1}}{m_{1}}, (j_{1}+1) \frac{M_{1}}{m_{1}} \right],$$

$$T_{j_{n+1}} = \left[j_{n+1} \frac{M_{1}}{m_{n+1}}, (j_{n+1}+1) \frac{M_{1}}{m_{n+1}} \right].$$
(5)

Defining the primary ASs (1) corresponds to defining the intervals numbers (5). Defining the secondary AS $W'(\tilde{A})$ geometrically correspond to defining the interval $[z_1, z_2]$, where

$$z_1 = \max \in j_i \frac{M_1}{m_i};$$

$$z_2 = \min \in (j_i + 1) \frac{M_1}{m_i}$$

i.e. sought interval is being defined as intersecting of intervals sets (5)

$$T_{W'(\tilde{A})} = T_{j_1} \wedge T_{j_2} \wedge \dots \wedge T_{j_{n+1}}.$$

It is obvious, that

$$z_2 - z_1 = \frac{M_1}{M_{n+1}} \,. \tag{6}$$

Condition (6) is equivalent to condition (3). If error moves operand \tilde{A} to the interval $\left[\left(m_{n+1}-1\right), M, M_1\right]$, then

$$z_2 - z_1 = \frac{M_1}{m_{n+1}} = M .$$
⁽⁷⁾

Condition (7) is equivalent to condition (4).

The suggested geometrical model confirms the correctness of the method's mathematical description, and also more clearly demonstrates the idea of the procedure of informativeness AS increasing or the reduction of the numerical interval of distorted number \tilde{A} occurrence.

Consider an example of defining AS of number \tilde{A} according to the developed method. There is SRC with bases $m_1 = 2$, $m_2 = 3$, $m_3 = 5$. The code words of this SRC are presented in Table 1.

Thus,

$$M = 2 \cdot 3 = 6$$
, $M_1 = M \cdot 5 = 30$, $m_{n+1} = m_3 = 5$, $A = (0, 2, 2)$, $\Delta A = (0, 2, 0)$

Table 1.
The set of codewords

The set of codew	orus						
A in PN	m_1	m_2	m_3	A in PN	m_1	m_2	m_3
0	0	0	0	15	1	0	0
1	1	1	1	16	0	1	1
2	0	2	2	17	1	2	2
3	1	0	3	18	0	0	3
4	0	1	4	19	1	1	4
5	1	2	0	20	0	2	0
6	0	0	1	21	1	0	1
7	1	1	2	22	0	1	2
8	0	2	3	23	1	2	3
9	1	0	4	24	0	0	4
10	0	1	0	25	1	1	0
11	1	2	1	26	0	2	1
12	0	0	2	27	1	0	2
13	1	1	3	28	0	1	3
14	0	2	4	29	1	2	4

Assume, that after the influence of a single error

$$\Delta A = (0, 0, \dots, \Delta a_i, \dots, 0)$$

by i^{th} basis ($\Delta a_2 = 2$) there is a number

$$\tilde{A} = A + \Delta A = (0, 2, 2)$$

In order to define the set of primary ASs it is needed to tentatively define values j_k .

For this, the nuvelization of a number \tilde{A} accordingly to the tables of nuvelization constants (Tables 2–4) is performed.

After this there are

$$\gamma_1 = 1, \ \gamma_2 = 1, \ \gamma_3 = 2.$$

The set of primary ASs is defined as

$$\begin{split} & W_{1\rho_{1}}\left(\tilde{A}\right) = \left\{m_{2}, m_{3}\right\}, \\ & W_{2\rho_{2}}\left(\tilde{A}\right) = \left\{m_{1}, m_{3}\right\}, \\ & W_{3\rho_{3}}\left(\tilde{A}\right) = W\left(\tilde{A}\right)^{-}\left\{m_{1}, m_{2}, m_{3}\right\} \end{split}$$

Table 2

<i>m</i> ₁	<i>m</i> ₂
(1, 1, 1)	(0, 1, 4)
	(0, 2, 2)
Table 3	
<i>m</i> 1	<i>m</i> ₃
(1, 1, 1)	(0, 0, 1)
	(0, 2, 2)
	(0, 0, 3)
	(0, 1, 4)
Table 4	
m ₂	<i>m</i> ₃
(0, 1, 0)	(1, 0, 3)
(1, 2, 0)	(0, 2, 2)
	(1, 1, 1)

The set of secondary ASs is defined by Tables 5–7, which are formed by values j_n :

• for $\gamma_3 = 2$,

• for $\gamma_2 = 1$,

$$W_2^{(1)}(\tilde{A}) = \{1, 0, 2\};$$
$$W_2^{(2)}(\tilde{A}) = \{0, 0, 3\};$$

 $W_3(\tilde{A}) = \{1, 1, 2\};$

• for $\gamma_1 = 1$,

$W_1^{(1)}(\tilde{A}) = \{0, 2, 3\};$
$W_1^{(2)}(\tilde{A}) = \{0,0,4\}.$

Table 5

γ ₃	Possible values of errors	$W_i^{(\Psi i)}$
0	none	-
1	$\Delta a_2 = 1, \\ \Delta a_3 = 1,$	$W_3^{(1)}(\tilde{A}) = \{0, 1, 1\},\$

	$\Delta a_1 = 1$,	
2	$\Delta a_2 = 1$,	$W_{3}^{(1)}(\widetilde{A}) = \{1, 1, 2\},\$
	$\Delta a_3 = 2,$	
	$\Delta a_1 = 1,$	
3	$\Delta a_2 = 2,$	$W_3^{(1)}(\tilde{A}) = \{1, 2, 3\},\$
	$\Delta a_3 = 3$,	
4	$\Delta a_2 = 2,$	$W_{3}^{(1)}(\tilde{A}) = \{0, 2, 4\}$
4	$\Delta a_3 = 4$	W3*(A) = {0, 2, 4}

Table 6

γ2	Possible values of errors	$\mathcal{W}_i^{(arphi_i)}$
0	$\Delta a_3 = 1,$	$W_2^{(1)}(\tilde{A}) = \{0, 0, 1\},\$
1	$\Delta a_1 = 1,$ $\Delta a_3 = 1,$ $\Delta a_2 = 1,$	$W_2^{(1)}(\tilde{A}) = \{1, 0, 2\},\ W_2^{(2)}(\tilde{A}) = \{0, 0, 3\},$
2	$\Delta a_3 = 4$	$W_2^{(1)}(\tilde{A}) = \{0, 0, 4\}$
Table 7 γ ₁	Errors	$W_i^{(w_i)}$
0	$\Delta a_2 = 1,$ $\Delta a_3 = 2,$ $\Delta a_3 = 1,$	$W_1^{(1)}(\tilde{A}) = \{0, 1, 1\},\ W_1^{(2)}(\tilde{A}) = \{0, 0, 2\},$
1	$\Delta a_2 = 2,$ $\Delta a_3 = 3,$ $\Delta a_3 = 3$	$W_1^{(1)}(\tilde{A}) = \{0, 2, 3\},\ W_1^{(2)}(\tilde{A}) = \{0, 0, 4\}$

Implementation of choice of common SRC bases is suitable in the form of tables (Tables 8–11), where sign "+" means match of the components of secondary ASs, and sign "-" means mismatch. Those tables show, that vectors components match in the bases m_1 , m_3 , i.e. the sought AS is as $W_3(\tilde{A}) = \{m_1, m_3\}$ (table 8). Therefore, $W(\tilde{A}) > W'(\tilde{A})$. Thus, the increase of the informativeness about error placement in the distorted number \tilde{A} is guaranteed by the described method.

Table 8

<i>m</i> ₁	m_2	m ₃
1	1	2
1	0	2
+	-	+
Table 9		
<i>m</i> ₁	<i>m</i> ₂	m ₃
1	1	2
0	0	3
-	-	-
Table 10		
<i>m</i> 1	<i>m</i> ₂	m3
1	1	2
0	2	3
-	-	-

Tabl	e	11
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<i>m</i> ₁	<i>m</i> ₂	m ₃
1	1	2
0	0	4
-	-	-

The geometrical interpretation example for the given SRC is represented in the following way (Fig. 2). The segment [0,30) is decomposed according to numerical intervals [15,30), [10,15) and [12,18)

. Define numbers of intervals, in which an operand $\tilde{A} = (1, 2, 2)$ placed.

$$T_{j_1} = [15, 30), T_{j_2} = [10, 20), T_{j_3} = [12, 18).$$

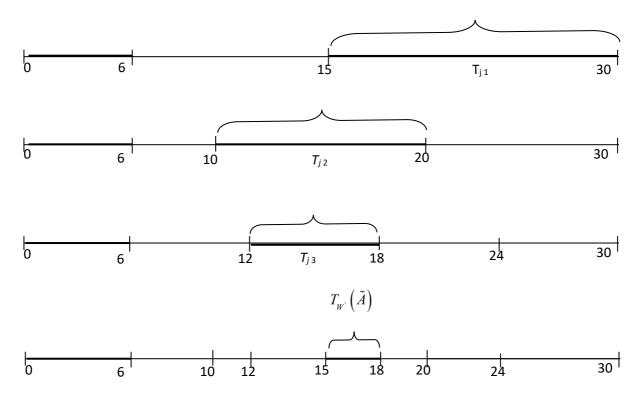


Figure 2: Scheme of defining a sought interval

A sought interval is defined by the expression

$$T_{W'(\tilde{A})} = [15, 18].$$

It is obvious, that interval $T_{W'(\tilde{A})}$ is being reduced, compared to T_{j_3} , by three units (by 50%), and that leads to a decrease of the number of options of possible errors.

Geometrical interpretation confirms the effectiveness of the considered method of data in SRC diagnostic.

4. Conclusion

Thus, the suggested method allows decreasing the time of diagnostic of errors of data, represented in SRC, which increases diagnostic operability. A reduction of the number of bases in AS increases informativeness AS about error placement and measure. It decreases the time of AS reduction to incorrect bases (the number of steps of tentatively AS defining is decreasing). The usage of the suggested method of operative diagnostic of data increases the total effectiveness and feasibility of using non-positional code structures in SRC in computing systems.

The time of data diagnostic, compared to known methods, is decreasing firstly due to excluding the procedure of transforming numbers in SRC to positional notation as in known methods, i. e. eliminating a positional operation of numbers comparing. Secondly, the time of data diagnostic is decreased by reducing the quantity of SRC bases, which are giving the possibility of mistakes. Thirdly, the time of data diagnostic is decreased due to the usage of tabular sample value of an alternative set (AS) of numbers in SRC in one beat.

Therefore, the suggested method allows reducing the time of diagnosis of data errors in NCS, represented in SRC, which is increasing the diagnostic operability while entering minimal informational redundancy. The geometric model of the procedure of AS informativeness increasing and specific examples of usage of the suggested method of diagnostic of data in SRC confirms its practical feasibility.

The most effective way of the method used is in the computational chain, which does not allow perform all planned procedures to AS reduction to the incorrect basis, i.e. in a quite long chain of calculations of CS.

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