Bi-periodically correlated random processes as a model for gear pair vibration

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Abstract

The model of gear pair vibration in the form of bi-periodically correlated random processes (BPCRP) that describes its stochastic recurrence with two periods is proposed. Particular cases of this model are considered. It is shown that BPCRP model allows one to analyses unequally the mean and the covariance function of the additive and multiplicative components. There are considered technologies for the estimation of the Fourier coefficients of the mean and the covariance functions.

Keywords

Bi-periodically correlated random processes, gear pair, vibration.

1. Introduction

The vibrations signals of the rotating elements can be characterized by their timely deviations whose features are cyclic repetition and stochasticity. When faults arise in machinery, some non-linear effects occur, and the interaction of different harmonics can be detected in vibration signal. This interaction can be detected by the analysis of the parameters of the periodical (about periodical) variation of the of the first and the second order moment functions of the random processes [1–4] (they also are called periodically or about periodically correlated random processes [5–9]). Therefore, it is preferable to select their parameters as the indicators for fault detection [10–17]. Gear pair excite vibration signal because of two main reasons: the periodic deviation of teeth stiffness because of the meshing phase and manufacturing inaccuracy. The manufacturing inaccuracy includes constant and variable step errors of the teeth. The periodic deviation of the mesh stiffness results in the appearance of the periodic components of the mesh frequency $f_m = rf_1 = nf_2$ and its multiples. Here f_1 and f_2 - the rotation frequencies of the gear wheels and r and n are some natural numbers. The error of the meshing step and the misalignment of axes and shafts are developed by the appearence of harmonics with base frequencies equal to kf_1 and lf_2 and combination frequencies $pf_m + kf_1$, $pf_m + lf_2$, where p, k, l are an integer numbers. In addition, the direct spectra of vibration can include the components that belong to some frequency band around the resonance frequency of the gear pair in the case of a vibro-impact regime occurring.

2. Modeling of gear pair vibration

The methods offered in [12, 18] for analysis of vibration of gear pair grounded on the transmission error model considered in [19]:

$$x(\theta) = x_e(\theta)\left[W + x_m(\theta) + x_1(\theta) + x_2(\theta)\right], \tag{1}$$

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where W is a some load and $\theta=\theta(t)$ is an angular position of the gear. The terms $x_m(\theta)$ and $x_e(\theta)$ describe the contact properties of the gears, while terms $x_1(\theta)$ and $x_2(\theta)$ are caused by manufacturing error. It is supposed that each term $x_i(\theta)$, $i=\overline{1,2}$ is periodic with a rotation period $P_i=1/f_i$ of the corresponding gear. There are three periodic terms in (1), namely $x_e(\theta)\left[W+x_m(\theta)\right]$, $x_e(\theta)x_1(\theta)$ and $x_e(\theta)x_2(\theta)$, which are periodic functions with period $P_m=1/f_m$, P_1 and P_2. The model of the cyclostationarity offered in [12, 18] was obtained by introducing the fluctuations of the angular position of the gears as a some random variable. The mean function of this random process includes the harmonic with frequencies f_m, f_1 and f_2. The covariance function consists of three different kinds of harmonics, in that, the harmonics with frequencies that are a linear combination of the rotation frequencies kf_1+lf_2, the harmonics of the mesh frequency nf_m and the harmonics with frequencies that are a linear combination of the mesh frequency and the rotation frequencies, i.e. nf_m+kf_i. The first and the second order non-stationarities have been substantiated by the processing of vibration signals measured on the gear systems [12, 18], and the quantities that describe the structure of the cyclostationarity estimated by means of synchronous averaging were proposed to be used for fault detection.

In [20–22], after applying the synchronous averaging with the period P_1 or P_2, the vibration signal is expressed as

$$g(t)=\sum_{l=1}^{M}A_l\left[1+a_l(t)\right]\cos\left(2\pi f_l t+b_l(t)+\varphi_l\right), \tag{2}$$

here M is the number of gear mesh harmonics, A_l and φ_l are the amplitude and the phase of the l^{th} harmonic respectively. The modulation effects are described by the functions $1+a_l(t)$ and $b_l(t)$, which are periodic with the considered rotation period. These functions are closely approximate to the signal's deterministic component corresponding to one revolution of the selected gear.

In [19, 23] the gear vibration signal is modeled as

$$x(\theta)=x_1(\theta)+x_2(\theta)+x_{1,2}(\theta)+x_c(\theta)+n(\theta), \tag{3}$$

where $x_1(\theta)$ and $x_2(\theta)$ describe the deterministic periodic oscillations generated by the rotation of the output and input wheels respectively, $x_{1,2}(\theta)$ is a component with period $P_{12}=r_1P_1+r_2P_2$, $x_c(\theta)$ is the second order cyclostationary process with period P_{12} and $n(\theta)$ is a fluctuation component. The deterministic part of the signal (3) can be extracted by means of synchronous averaging with common period P_{12} of the shafts as far as it is possible [19]. Paper is dedicated to the development of the cyclostationary models of gear vibrations considered in the literature, their concretization, and the elaboration on this basis of other estimation techniques for the analysis of the modulation effects occurring in the vibration signals as the faults originate.

3. BPCRP representation

The efficiency of vibration signal processing for machinery condition monitoring can be explained mostly by their possibility to develop modulations caused by the appearance of faults. The modulation effects in the vibration model as a periodically correlated random processes (PCRP), which describe the stochastic recurrence with one period can be explained by the jointly stationary random processes $\xi_k(t)$ in their harmonic representation [8, 9, 24]:

$$\xi(t)=\sum_{k\in Z}\xi_k(t)e^{ik\frac{2\pi}{P_1}t},$$

where Z is a set of integer numbers and P_1 is a period of the rotations for one of the wheels. Following this equation, we concludes that the modulation of the signals of two stochastic rhythms provided by the rotation of two wheels can be explained as

$$\xi(t)=\sum_{k\in Z}\xi_k^{(P_2)}e^{ik\frac{2\pi}{P_1}t}, \tag{4}$$

where the harmonic of frequency $2\pi/P_1$ and its multiples are modulated for this once by PCRP with period P_2: $\xi_k^{(P_2)}(t)=\sum_{l\in Z}\xi_{kl}(t)e^{il\frac{2\pi}{P_2}t}$.

Then, for the random process (4), we have:

$$\xi(t)=\sum_{k,l\in Z}\xi_{kl}(t)e^{i\Lambda_{kl}t}, \tag{5}$$

where $\xi_{kl}(t)$ are jointly stationary random processes and $\Lambda_{kl}=k(2\pi/P_1)+l(2\pi/P_2)$. The random processes presented by series (5) are bi-periodically correlated random processes (BPCRP) [9, 25, 26]. As we can see from (5), a BPCRP is a sum of the amplitude and phase modulated harmonics. Here frequencies Λ_{kl} are the linear combination of the two main frequencies $\Lambda_{10}=k(2\pi/P_1)$ and $\Lambda_{01}=l(2\pi/P_2)$. The modulating processes have the mathematical expectations $m_{kl}=E\xi_{kl}(t)$ which are the Fourier coefficients of the mean:

$$m(t)=E\xi(t)=\sum_{k,l\in Z}m_{kl}e^{i\Lambda_{kl}t}. \tag{6}$$

For the covariance function $R(t,\tau)=E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+\tau)$, $\overset{\circ}{\xi}(t)=\xi(t)-m(t)$, we have

$$R(t,\tau)=E\xi(t)=\sum_{k,l\in Z}R_{kl}(\tau)e^{i\Lambda_{kl}t}, \tag{7}$$

where

$$R_{kl}(\tau)=\sum_{p,q\in Z}r_{p-k,q-l,p,q}e^{i\Lambda_{pq}\tau}, \tag{8}$$

and $r_{pqkl}(\tau)=E\overline{\overset{\circ}{\xi}_{pq}(t)}\overset{\circ}{\xi}_{kl}(t+\tau)$, $\overset{\circ}{\xi}_{pq}(t)=\xi_{pq}(t)-m_{pq}$ are the cross-covariance functions of the PCRP processes, and the "¯" signifies complex conjugation. Thus, the cross-covariance functions of the modulating processes defines the Fourier coefficients of the covariance function (7) in which the numbers are shifted by k and l. It follows from (8) those cross-correlations of modulating processes $\overset{\circ}{\xi}_{kl}(t)$ with different numbers k and l lead to bi-periodical non-stationarity of the second order. As the result of these correlations, it is appear the correlation of the spectral components, which can be characterized by the appropriate Fourier transformation of (8):

$$f_{kl}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_{kl}(\tau)e^{-i\omega\tau}d\tau. \tag{9}$$

It follows from (8) that

$$f_{kl}(\omega)=\sum_{p,q\in Z}f_{p-k,q-l,p,q}(\omega-\Lambda_{pq}),$$

where

$$f_{pqkl}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}r_{pqkl}(\tau)e^{-i\omega\tau}d\tau,$$

are the cross-spectral densities of the modulating processes $\xi_{pq}(t)$. The functions (8) and (9) are respectively the covariance and spectral components [9, 25, 26].

The zeroth covariance component $R_{00}(\tau)$ is determined by auto-covariance functions $r_{pq}(\tau)=E\overline{\overset{\circ}{\xi}_{pq}(t)}\overset{\circ}{\xi}_{pq}(t+\tau)$: $R_{00}(\tau)=\sum_{p,q\in Z} r_{pq}(\tau)e^{-i\Lambda_{pq}\tau}$.

This covariance function of the stationary approximation for the BPCRP is averaged BPCRP covariance function.

The zeroth spectral component

$$f_{00}(\omega)=\sum_{p,q\in Z} f_{pq}(\omega-\Lambda_{pq}), \quad (10)$$

is a power spectral density of the stationary approximation for the BPCRP. It defines the spectral decomposition of the averaged in time instantaneous power $R(0,t)$ for the oscillations.

We should note that the covariance and the spectral components are the total characteristics of the amplitude and the phase modulation of the BPCRP carrier harmonics. The zeroth spectral component, as can be seen from (10), is a sum of the power spectral densities of the modulating processes $\xi_{pq}(t)$ shifted by Λ_{pq}. The components $f_{kl}(\omega)$ explained in (9) are a sum of the shifted cross-spectral densities for modulating processes. Their numbers differs by k and l. Proceeding from the above-mentioned, it is possible to conclude that the zeroth spectral function $f_{00}(\omega)$ describes the spectral composition of the oscillations and the non-zeroth functions $f_{kl}(\omega)$. It explains the correlations of the harmonics of this composition for the components with frequencies shifted by $\Lambda_{kl}=k(2\pi/P_1)+l(2\pi/P_2)$. When modulating processes of the corresponding numbers are mutually correlated, than these correlations are not equal to zero.

4. Method for statistical analysis

The time synchronous averaging (TSA) method was one of the early techniques used for the analysis of hidden periodicities [27, 28]. If the hidden periodicity is presented and modeled as a PCRP, then such technology was used for evaluation of its mean and covariance function [9, 25, 26]. It is so-called the coherent method [29, 30]. Synchronous averaging was also used for analysis of the vibration signals, which are characterized by the recurrence of two or more periods [3, 7, 9, 13, 18, 22]. We consider below its application for the estimation of BPCRP characteristics.

The coherent statistics of the BPCRP mean function have the form

$$\hat{m}_l=\frac{1}{N_l}\sum_{n=0}^{N_l-1}\xi(t+nP_l), \quad (11)$$

where P_l is one of the non-stationarity periods and N_l is the number of averaged periods. The mathematical expression of (11) for $l=1$ is equal to

$$E\hat{m}_1(t)=\frac{1}{N_1}\sum_{n=0}^{N_1-1}m_1(t+nP_1)=\sum_{k\in Z}m_{0k}e^{ik\frac{2\pi}{P_2}t}+\sum_{k\in Z}\sum_{l\in Z}m_{kl}e^{ik\Lambda_{kl}t}s_{N_1}\left(l\frac{P_1}{P_2}\right),$$

where

$$s_{N_1}\left(l\frac{P_1}{P_2}\right)=e^{i(N_1-1)\frac{\pi}{P_2}lP_1}\sin\left(N_1\pi\frac{P_1}{P_2}\right)/N_1\sin\left(\pi\frac{P_1}{P_2}\right).$$

If $P_1=nP_2$ and n is a natural number, then $s_{N_1}(l(P_1/P_2))=1$ and $E\hat{m}_1(t)=m(t)$, i.e. formula (11) is the unbiased estimator of the BPCRP mean function. In other cases, formula (11) is a biased estimator of the mean additive component with period P_1. The bias value depends on the ratio P_1/P_2 and tends to zero as $N_1\to\infty$.

Using (11), we can form the formulae

$$\hat{m}_{k0}=\frac{1}{P_1}\int_0^{P_1}\hat{m}_1(t)e^{-ik\frac{2\pi}{P_1}t}dt\,,\ \hat{m}_{0l}=\frac{1}{P_2}\int_0^{P_2}\hat{m}_2(t)e^{-ik\frac{2\pi}{P_2}t}dt\,,$$

which, in the general case, are the asymptotically unbiased estimators of the Fourier coefficients of the mean additive components.

It is easily see that unbiased estimators of the BPCRP mean function and its Fourier coefficients can be obtained using synchronous averaging with common period P :

$$\hat{m}(t)=\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nP)\,,\ \hat{m}_{kl}=\frac{1}{P}\int_0^{P}\hat{m}(t)e^{-i\Lambda_{kl}t}dt\,. \tag{12}$$

Here N is the number of realization periods P which are averaged.

Taking into account (11), we can form the coherent estimators of the covariance function and its Fourier coefficients:

$$\hat{R}(t,\tau)=\frac{1}{N}\sum_{n=0}^{N}\left[\xi(t+nP)-\hat{m}(t+nP)\right]\left[\xi(t+\tau+nP)-\hat{m}(t+\tau+nP)\right], \tag{13}$$

$$\hat{R}_{kl}(\tau)=\frac{1}{P}\int_0^{P}\hat{R}(t,\tau)e^{-i\Lambda_{kl}t}dt\,. \tag{14}$$

Using the synchronous averaging of the BPCRP samples over one of the periods P_2 in the form

$$\hat{R}_l(t,\tau)=\frac{1}{N_2}\sum_{n=0}^{N_2}\left[\xi(t+nP_2)-\hat{m}(t+nP_2)\right]\left[\xi(t+\tau+nP_2)-\hat{m}(t+\tau+nP_2)\right],$$

we can detect only the additive covariance components and determine their Fourier coefficients:

$$\hat{R}_{k0}(\tau)=\frac{1}{P_1}\int_0^{P_1}\hat{R}_1(t,\tau)e^{-ik\frac{2\pi}{P_1}t}dt\,,\ \hat{R}_{0l}(\tau)=\frac{1}{P_2}\int_0^{P_2}\hat{R}_2(t,\tau)e^{-il\frac{2\pi}{P_2}t}dt\,.$$

Note that we must use in (13) the unbiased or asymptotically unbiased estimator of the mean function.

Component estimators are represented by trigonometric polynomials:

$$\hat{m}(t)=\sum_{k,l=-L_1}^{L_1}\hat{m}_{kl}e^{i\Lambda_{kl}t}\,, \tag{15}$$

$$\hat{R}(t,\tau)=\sum_{k,l=-L_2}^{L_2}\hat{R}_{kl}(\tau)e^{i\Lambda_{kl}t}\,, \tag{16}$$

where L_r, $r=\overline{1,2}$ are the numbers of the highest harmonics. The coefficients of the polynomials are determined by the formulae

$$\hat{m}_{kl}=\frac{1}{T}\int_{-T}^{T}\xi(t)e^{-\Lambda_{kl}t}dt\,, \tag{17}$$

$$\hat{R}_{kl}(\tau)=\frac{1}{T}\int_{-T}^{T}\left[\xi(t)-\hat{m}(t)\right]\left[\xi(t+\tau)-\hat{m}(t+\tau)\right]e^{-\Lambda_{kl}t}dt\,, \tag{18}$$

where T is the length of signal realization. The number of harmonics to be taken into account in (15) and (16) can be obtained on the basis of the results of experimental data processing by means of the coherent method or stationary spectral estimation.

In the general case, employing formulae (17) and (18) leads to an increase of the additional errors caused by leakage effects. These effects are absent as $T=NP$. Formulae (17) and (18) can then be re-written in the form of (11) and (13). Indeed,

$$\hat{m}_{kl}=\frac{1}{NP}\sum_{k=0}^{N-1}\int_{kP}^{(k+1)P}\xi(t)e^{-i\Lambda_{kl}t}dt=\frac{1}{P}\int_0^{P}e^{-i\Lambda_{kl}t}\left[\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nP)\right]dt\,.$$

Similarly,

$$\hat{R}_{kl}(\tau)=\frac{1}{P}\int_0^P e^{-i\Lambda_{kl}t}\left[\frac{1}{N}\sum_{n=0}^{N-1}\left[\xi(t+nP)-\hat{m}(t+nP)\right]\left[\xi(t+\tau+nP)-\hat{m}(t+\tau+nP)\right]\right]dt\ .$$

The discrete estimators for the Fourier coefficients of the mean and covariance functions can be formed by substituting the integral transformations (17) and (18) by integral sums:

$$\hat{m}_{kl}=\frac{1}{K}\sum_{n=0}^{K-1}\xi(t)e^{-i\Lambda_{kl}nh}\ ,$$

$$\hat{R}_{kl}(rh)=\frac{1}{K}\sum_{n=0}^{K-1}\left[\xi(nh)-\hat{m}(nh)\right]\left[\xi((n+r)h)-\hat{m}((n+r)h)\right]e^{-i\Lambda_{kl}nh}\ .$$

Here $P_1=(M_1+1)h$, $P_2=(M_2+1)h$ and $T=Kh$, where $K=rN(M_1+1)=nN(M_2+1)$.

To avoid the aliasing effects of the first and the second kinds [32], it is recommended to choose the sampling interval h in accordance with the inequalities

$$h\le\frac{P_i}{2L_1+1},\ h\le\frac{P_i}{4L_2+1},\ i=\overline{1,2}$$

If these inequalities are satisfied, the expressions (15) and (16) can be considered as the interpolation formulae for the estimators. We should note that in the case of $T=NP$ the component estimators coincide with the estimators determined by the least squares (LS) method [9, 31, 32]. However, using the LS method allows one to avoid the leakage errors in general case. These errors can be significant in cases when the values of the rotation frequency and/or their combinations are close. To construct the LS estimators for the mean and the covariance function we rewrite the series (6) and (7) in the form

$$m(t)=m_0+\sum_{l=1}^{M_1}\left(m_l^c\cos\omega_l t+m_l^s\sin\omega_l t\right),$$

$$R(t,\tau)=R_0(\tau)+\sum_{l=1}^{M_2}\left[R_l^c(\tau)\cos\omega_l t+R_l^s(\tau)\sin\omega_l t\right]$$

,

where $m_l=m_{l_1l_2}=\frac{1}{2}\left(m_l^c-im_l^s\right)$, $R_l(\tau)=R_{l_1l_2}(\tau)=\frac{1}{2}\left[R_l^c(\tau)-iR_l^s(\tau)\right]$,

$$m_0=m_{00},\ R_0(\tau)=R_{00}(\tau),\ \omega_l=\sum_{j=1}^{2}l_j\frac{2\pi}{P_j},\ l_1=\overline{1,L_1},\ l_2=\overline{1,L_2}\ .$$

and $N_1=2L_1(L_1+1)$, $N_2=2L_2(L_2+1)$. The LS estimators for the Fourier coefficients of mean and covariance function are defined as the quantities which provide the minimum values of the quadratic functions

$$F_1\left(\hat{m}_0,\hat{m}_1^c,...,\hat{m}_{M_1}^c,\hat{m}_1^s,...,\hat{m}_{M_1}^s\right)=\int_0^T\left[\xi(t)-\left[\hat{m}_0+\sum_{l=1}^{M_1}\left(\hat{m}_l^c\cos\omega_l t+\hat{m}_l^s\sin\omega_l t\right)\right]\right]^2dt,\ , \tag{19}$$

$$F_2\left[\hat{R}_0(\tau),\hat{R}_1^c(\tau),...,\hat{R}_{M_2}^c(\tau),\hat{R}_1^s(\tau),...,\hat{R}_{M_2}^s(\tau)\right]=$$
$$=\int_0^T\left[\xi(t,\tau)-\left[\hat{R}_0(\tau)+\sum_{l=1}^{M_2}\left[\hat{R}_l^c(\tau)\cos\omega_l t+\hat{R}_l^s(\tau)\sin\omega_l t\right]\right]\right]^2dt\ , \tag{20}$$

where $\xi(t,\tau)=\left[\xi(t)-\hat{m}(t)\right]\left[\xi(t+\tau)-\hat{m}(t+\tau)\right]$. They are the solutions of the system equations which represent the necessary conditions for the existence of the minimum of functionals (19) and (20):

$$\frac{\partial F_1}{\partial\hat{m}_0}=0,\ \frac{\partial F_1}{\partial\hat{m}_r^c}=0,\ \frac{\partial F_1}{\partial\hat{m}_r^s}=0,\ r=\overline{1,M_1}\ , \tag{21}$$

$$\frac{\partial F_2}{\partial \hat{B}_0}=0\ \frac{\partial F_2}{\partial \hat{B}_r^c}=0\ \frac{\partial F_2}{\partial \hat{B}_r^s}=0\,,\ r=\overline{1,M_2}\,. \tag{22}$$

The lag-dependent vanishing of the covariance function is the enough sufficient condition of the mean square consistency of the Fourier coefficients for the mean function. It also can be indicator of the asymptotic unbiasness of the estimators of covariance component. This condition is also sufficient for the consistency of mean square of the covariance component estimators for Gaussian BPCRP. For similar purposes, the series procedures were introduced, the latest of them are self-adaptive noise cancellation [33] and spectral method [34]. The best result are obtained using time synchronous averaging, however it requires a separate operations including individual resampling in each considered case.

5. Conclusions

The advantage of the LS estimators is the absence of the leakage effect. The possible bias of the LS estimators can be caused only by the previous inexact estimation of the mean function. When the realization length increases, values of the component estimators and the variances for the LS are quickly drawing together. So, the LS estimation can be rated as the preferable technique for statistical processing of the PCRP experimental time series.

6. References

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