# Characteristic Function of Conditional Linear Random Process

Mykhailo Fryz [a], Leonid Scherbak [b], Mikolaj Karpinski [c], and Bogdana Mlynko [a]

[a] *Ternopil Ivan Puluj National Technical University, Ruska st. 56, Ternopil, 46001, Ukraine*
[b] *Kyiv International University, Lvivska st. 49, Kyiv, 03179, Ukraine*
[c] *University of Bielsko-Biala, Willowa st. 2, Bielsko-Biala, 43309, Poland*

**Abstract**

A continuous-time conditional linear random process (CLRP) is studied as a mathematical model of the stochastic signal generated in the form of a sum of a large quantity of stochastically dependent random impulses occurring at the Poisson times with applications in different areas of computer science and information technology. The model has been defined as a special stochastic integral driven by the process with independent increments. The characteristic function of CLRP has been obtained, the examples of finding the moment functions have been considered. The conditions for CLRP to be strict sense stationary have been justified.

**Keywords 1**

Mathematical model, digital signal processing, conditional linear random process, stochastic integral, conditional characteristic function, moment functions, strict sense stationary process

## 1. Introduction

A justification of mathematical and computer simulation models of informative stochastic signals, images, processes and interferences is one of the most important steps of creating information and measuring systems, controlling systems in radio engineering and communications, computer-aided systems of analysis and prediction of electricity, gas, water consumption, information technology of medical diagnostics, etc. A mathematical model is theoretical foundation for the structural, program and technical implementation of the developed information systems and technologies, the basis of algorithms for digital signal processing, decision-making methods. Therefore, the model should be adequate to the measuring signal or process, to reflect the physical mechanism of its generation, and also be suitable for its theoretical analysis and solving the problems of identification and estimation of informative characteristics by the results of experiments, computer simulation.

The above requirements are satisfied by signal models in the form of linear random processes (LRP) and sequences [1, 2]. Most often LRP with infinitely divisible distributions is used in the problems of mathematical modelling of signals, which are formed by additive interaction of infinity number of stochastically independent random impulses occurring at the jump points of a Poisson counting process [1, 2, 3]. However, if these impulses are stochastically dependent random functions and/or occur at non-Poisson times (e.g. in the problems of the electricity consumption forecasting [3], analysis of radar clutter, dynamic loads of mechanical systems [4], electrophysiological signals of the brain [5], etc.), then an adequate mathematical model is conditional linear random process (CLRP). Thus, studying the properties of CLRP is an important problem for mathematical modelling in the above applications.

The idea of a conditional linear random process definition is to generalize a linear random process by replacing the deterministic kernel of LRP with a random function, which obviously has to satisfy the certain conditions to ensure the convergence of the corresponding stochastic integral, and has properties that allow the practical application of such kind of model.

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EMAIL: mykh.fryz@gmail.com (A. 1); prof_scherbak@ukr.net (A. 2); mpkarpinski@gmail.com (A. 3); mlynko@ukr.net (A. 4)
ORCID: 0000-0002-8720-6479 (A. 1); 0000-0002-1536-4806 (A. 2); 0000-0002-8846-332X (A. 3); 0000-0003-0780-5365 (A. 4)

Very often a linear random process is also interpreted as the output signal of some linear dynamic system where the input signal is the white noise. From this point of view, the idea of constructing a conditional linear random process is to represent it as a response of a linear dynamic system with random parameters to the input signal in the form of white noise.

Analysis of literature shows that the concept of "conditional linear random process" has been defined by P.A. Pierre [6] in the framework of research conducted by RAND Corporation (USA) on the problems of radar digital signal processing and interference. A physically reasonable model of the signal has been presented as a random process $\xi(t)=\sum_{k=-\infty}^{\infty}\varphi_k(t-\tau_k)$, that is as a sum of large amount of random impulses $\varphi_k(t)$ occuring at the random time moments $...<\tau_{k-1}<\tau_k<\tau_{k+1}<...$. P.A. Pierre has been called the process $\xi(t)$ linear if $\varphi_k(t)$ are stochastically independent and identically distributed random impulses, and time moments of their occurrence form the Poisson flow. If $\varphi_k(t)$ are stochastically dependent or/and $...<\tau_{k-1}<\tau_k<\tau_{k+1}<...$ is not Poisson flow then $\xi(t)$ is "conditionally linear random process".

The main goal of the paper is to justify the expressions of univariate and multivariate characteristic functions of a conditional linear random process and prove then conditions for CLRP to be strict sense stationary. That is, the paper extends some results presented in [7].

## 2. Conditional Linear Random Process Definition

***Definition.*** A real-valued CLRP $\xi(\omega,t),\omega\in\Omega,t\in(-\infty,\infty)$ (where $\{\Omega,\mathfrak{F},\mathbf{P}\}$ is some probability space) is defined in [7] as:

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)d\eta(\omega,\tau),\ \omega\in\Omega,t\in\mathbb{R}, \tag{1}$$

where $\Omega$ is a sample space; $\varphi(\omega,\tau,t),\tau,t\in\mathbb{R}$ is a real-valued *stochastic* kernel of CLRP; $\eta(\omega,\tau),\tau\in(-\infty,\infty)$ is a mean square continuous Hilbert process with independent increments, that satisfies the following conditions: $\mathbf{E}\eta(\omega,\tau)=a(\tau)<\infty$ and $\mathrm{Var}\left[\eta(\omega,\tau)\right]=b(\tau)<\infty$, $\forall\tau$; random functions $\varphi(\omega,\tau,t)$ and $\eta(\omega,\tau)$ are *stochastically independent*.

For the simplicity, the argument $\omega$ is usually further omitted. Also stochastic integral (1) is assumed to be exist in the mean-square convergence sense [7].

If kernel $\varphi(\tau,t)$ is nonrandom function then $\xi(t)$ is LRP [1]. The properties of LRP have been studied comprehensively [1, 2] using the method of characteristic functions. The relationships between linear and conditional linear random processes have been represented in [7].

P. Pierre [6] has been defined the CLRP as the stochastic integral (1), but the process with independent increments is homogeneous and centered. It should also be noted that in contrast to the constructions of stochastic integrals of Itô and Stratonovich [8] where the kernel function is measurable with respect to the filtration of sigma algebras generated by the process $\eta(\tau)$, in the expression (1) random functions $\varphi(\omega,\tau,t)$ and $\eta(\omega,\tau)$ are stochastically independent, which on the one hand, makes it easier to analyze the probabilistic characteristics of CLRP, and on the other hand is adequate to the above mentioned applied problems of mathematical modeling of stochastic signals in computer science and information technology. The stochastic integral (1) is constructed as follows.

Let $\left[a,b\right)\in\mathbb{R}$ be the interval which is divided on $n$ subintervals by $\tau_0,\tau_1,\tau_2,...,\tau_n$, such that $a=\tau_0<\tau_1<\tau_2<...<\tau_n=b$, that is the intervals we consider have the following form: $\left[\tau_0,\tau_1\right),\left[\tau_1,\tau_2\right),\left[\tau_{n-1},\tau_n\right)$. Let us construct the following integral sums for each time moment $t$:

$$I_n(t)=\sum_{i=1}^{n}\varphi(\tau_{i-1},t)\Delta\eta(\tau_i), \tag{2}$$

where $\Delta\eta(\tau_i)=\eta(\tau_i)-\eta(\tau_{i-1})$, $i=\overline{1,n}$.

Note that, the values of the function $\varphi(\tau,t)$ of the each item of integral sum (2) are taken at the points $\tau_{i-1}$, that is at the left endpoint of the each $i$-th subinterval $[\tau_{i-1},\tau_i), i=\overline{1,n}$. The same way the stochastic Itô integral is constructed [8]. But in our case any value $\varphi(\tilde{\tau}_i,t)$ can be taken, where $\tilde{\tau}_i$ is any point from the interval $[\tau_{i-1},\tau_i), i=\overline{1,n}$, the corresponding limit of the sequence of integral sums will not change, because the random functions $\varphi(\tau,t)$ and $\eta(\tau)$ are stochastically independent by definition.

Thus, if when $n\to\infty$ we have $\max\limits_{i=\overline{1,n}}(\tau_i-\tau_{i-1})\to 0$ and limit in the mean square exists of the sequence of the integral sums (2), then we write $\underset{n\to\infty}{l.i.m.}\sum\limits_{i=1}^{n}\varphi(\tau_{i-1},t)\Delta\eta(\tau_i)=\int\limits_a^b\varphi(\tau,t)d\eta(\tau)$.

Improper integral (1) is constructed then as a following limit (if it exists): $\xi(t)=\int\limits_{-\infty}^{\infty}\varphi(\tau,t)d\eta(\tau)=\underset{\substack{a\to-\infty\\ b\to\infty}}{l.i.m.}\int\limits_a^b\varphi(\tau,t)d\eta(\tau)$.

## 3. Conditional and Unconditional Characteristic Functions

Let $\mathfrak{F}_\varphi\subset\mathfrak{F}$ is a $\sigma$-subalgebra generated by the random function $\varphi(\omega,\tau,t)$ satisfying the following conditions:

$$\int\limits_{-\infty}^{\infty}|\varphi(\omega,\tau,t)||da(\tau)|<\infty,\quad \int\limits_{-\infty}^{\infty}|\varphi(\omega,\tau,t)|^2\,db(\tau)<\infty,\ \forall t \text{ with probability 1.}$$

Consider $I_n(\omega,t)=\sum\limits_{i=1}^{n}\varphi(\omega,\tau_{i-1},t)\Delta\eta(\omega,\tau_i)$ (this is the sequence of integral sums (2), but here we just add $\omega\in\Omega$ to improve the clarity of representation).

Let $\mathbf{E}\Delta\eta(\omega,\tau_i)=\Delta a(\tau_i)$, $\mathrm{Var}[\Delta\eta(\omega,\tau_i)]=\Delta b(\tau_i)$ be the mathematical expectation and variance of the increment $\Delta\eta(\tau_i)=\eta(\omega,\tau_i)-\eta(\omega,\tau_{i-1})$, and $\Delta K(x;\tau_i)$ be its Poisson jump spectrum in Kolmogorov's form, $i=\overline{1,n}$.

To obtain the expression for the characteristic function of CLRP we use the mathematical theory of conditional characteristic functions. Definitions and properties of conditional characteristic function, and also concept of conditionally independent random variables have been developed in [9, 10, 11].

Elements of the sum (2) are conditionally independent ($\mathfrak{F}_\varphi$-independent) infinitely divisible random variables. According to [12], conditional (with respect to $\mathfrak{F}_\varphi$) characteristic function ($\mathfrak{F}_\varphi$-characteristic function) of the sum of $\mathfrak{F}_\varphi$-independent random variables is equal to the product of their $\mathfrak{F}_\varphi$-characteristic functions. Thus, $\mathfrak{F}_\varphi$- characteristic function of random variable $I_n(\omega,t)$ (at the certain $t$) is equal:

$$f_n^{\mathfrak{F}_\varphi}(u;t)=\mathbf{E}\left(e^{iuI_n(\omega,t)}\middle|\mathfrak{F}_\varphi\right)=$$

$$=\exp\left[iu\sum_{i=1}^{n}\varphi(\omega,\tau_{i-1},t)\Delta a(\tau_i)+\sum_{i=1}^{n}\int\limits_{-\infty}^{\infty}\left(e^{iux\varphi(\omega,\tau_{i-1},t)}-1-iux\varphi(\omega,\tau_{i-1},t)\right)\frac{d_x\Delta K(x;\tau_i)}{x^2}\right].$$

Taking into account the above expression and also the results of [10], we can state that

$$\lim_{\substack{a\to\infty\\ b\to-\infty}}\lim_{n\to\infty}f_n^{\mathfrak{F}_\varphi}(u;t)=f_\xi^{\mathfrak{F}_\varphi}(u;t)$$

with probability 1, where $f_\xi^{\mathfrak{F}_\varphi}(u;t)$ is characteristic function of linear random process [1, 2]. That is, with probability 1 the following holds:

$$f_{\xi}^{\Im_{\varphi}}(u;t)=\mathbf{E}\left(e^{iu\xi(\omega,t)}\middle|\Im_{\varphi}\right)=$$

$$=\exp\left[iu\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)da(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\omega,\tau,t)}-1-iux\varphi(\omega,\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right].$$

Let $f_n(u;t)=\mathbf{E}e^{iuI_n(\omega,t)}=\mathbf{E}\left[\mathbf{E}\left(e^{iuI_n(\omega,t)}\middle|\Im_{\varphi}\right)\right]=\mathbf{E}f_n^{\Im_{\varphi}}(u;t)$ be unconditional characteristic function of the integral sum $I_n(\omega,t)$. Taking into account the results of [12], we can obtain the following expression (this holds with probabiity 1):

$$\lim_{\substack{a\to\infty\\ b\to-\infty}}\lim_{n\to\infty}f_n(u;t)=\lim_{\substack{a\to\infty\\ b\to-\infty}}\lim_{n\to\infty}\mathbf{E}f_n^{\Im_{\varphi}}(u;t)=\mathbf{E}\lim_{\substack{a\to\infty\\ b\to-\infty}}\lim_{n\to\infty}f_n^{\Im_{\varphi}}(u;t)=\mathbf{E}f_{\xi}^{\Im_{\varphi}}(u;t)=f_{\xi}(u;t).$$

Thus, one-dimensional characteristic function of CLRP has the following form:

$$f_{\xi}(u;t)=\mathbf{E}\left[\mathbf{E}\left(e^{iu\xi(\omega,t)}\middle|\Im_{\varphi}\right)\right]=$$

$$=\mathbf{E}\exp\left[iu\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)da(\tau)+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{iux\varphi(\omega,\tau,t)}-1-iux\varphi(\omega,\tau,t)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right] \quad (3)$$

We can easily find that mathematical expectation of CLRP is equal to

$$\mathbf{E}\xi(t)=\frac{1}{i}\frac{\partial f_{\xi}(u;t)}{\partial u}\bigg|_{u=0}=\int_{-\infty}^{\infty}\mathbf{E}\varphi(\tau,t)da(\tau).$$

Similarly, the moment function of second order is

$$\mathbf{E}\left(\xi(t)\right)^2=\frac{1}{i^2}\frac{\partial^2 f_{\xi}(u;t)}{\partial u^2}\bigg|_{u=0}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau_1,t)\varphi(\tau_2,t)\right)da(\tau_1)da(\tau_2)+\int_{-\infty}^{\infty}\mathbf{E}\varphi^2(\tau,t)db(\tau).$$

Thinking the same way we can easily find the expression for multivariate ($m$-dimensional) characteristic function of CLRP, which has the following form:

$$f_{\xi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=\mathbf{E}e^{i\sum_{k=1}^{m}u_k\xi(\omega,t_k)}=\mathbf{E}\left[\mathbf{E}\left(e^{i\sum_{k=1}^{m}u_k\xi(\omega,t_k)}\middle|\Im_{\varphi}\right)\right]=$$

$$=\mathbf{E}\exp\left[i\sum_{k=1}^{m}u_k\int_{-\infty}^{\infty}\varphi(\omega,\tau,t_k)da(\tau)+\right. \quad (4)$$

$$\left.+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi(\omega,\tau,t_k)}-1-ix\sum_{k=1}^{m}u_k\varphi(\omega,\tau,t_k)\right)\frac{d_x d_\tau K(x;\tau)}{x^2}\right],$$

$$u_k,t_k\in(-\infty,\infty),\ k=\overline{1,m}.$$

Moment function of third order of CLRP can be obtained (for example) using the expression (4) (we write $\varphi(\tau,t)$ here, omitting $\omega$ to simplify the expression):

$$\mathbf{E}\left(\xi(t_1)\xi(t_2)\xi(t_3)\right)=\frac{1}{i^3}\frac{\partial^3 f_{\xi}(u_1,u_2,u_3;t_1,t_2,t_3)}{\partial u_1\partial u_2\partial u_3}\bigg|_{u_1=u_2=u_3=0}=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau_1,t_1)\varphi(\tau_2,t_2)\varphi(\tau_3,t_3)\right)da(\tau_1)da(\tau_2)da(\tau_3)+$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau_1,t_1)\varphi(\tau_2,t_2)\varphi(\tau_2,t_3)\right)da(\tau_1)db(\tau_2)+$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau_1,t_2)\varphi(\tau_2,t_1)\varphi(\tau_2,t_3)\right)da(\tau_1)db(\tau_2)+$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau_1,t_3)\varphi(\tau_2,t_1)\varphi(\tau_2,t_2)\right)da(\tau_1)db(\tau_2)+ +\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}\left(\varphi(\tau,t_1)\varphi(\tau,t_2)\varphi(\tau,t_3)\right)x d_x d_\tau K(x;\tau).$$

As we already mentioned before, a linear random process has an infinitely divisible distribution function. From the above we can conclude that $\mathfrak{F}_\varphi$ - characteristic function $f_\xi^{\mathfrak{F}_\varphi}(u;t)=\mathbf{E}\left(e^{iu\xi(\omega,t)}\middle|\mathfrak{F}_\varphi\right)$ is infinitely divisible with probability 1, moreover

$$f_\xi^{\mathfrak{F}_\varphi}(u;t)=\mathbf{E}\left(e^{iu\xi(\omega,t)}\middle|\mathfrak{F}_\varphi\right)=\exp\left[ium^{\mathfrak{F}_\varphi}(\omega,t)+\int_{-\infty}^{\infty}\left(e^{iux}-1-iux\right)\frac{d_x K_\xi^{\mathfrak{F}_\varphi}(\omega,x;t)}{x^2}\right],$$

where $m^{\mathfrak{F}_\varphi}(\omega,t)=\mathbf{E}\left(\xi(t)\middle|\mathfrak{F}_\varphi\right)=\int_{-\infty}^{\infty}\varphi(\omega,\tau,t)da(\tau)$;

$$K_\xi^{\mathfrak{F}_\varphi}(\omega,x;t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\varphi^2(\omega,\tau,t)U(x-z\varphi(\omega,\tau,t))d_z d_\tau K(z,\tau).$$

Unconditional characteristic function of CLRP then can be represented as:

$$f_\xi(u;t)=\mathbf{E}f_\xi^{\mathfrak{F}_\varphi}(u;t)=\mathbf{E}\exp\left[ium^{\mathfrak{F}_\varphi}(\omega,t)+\int_{-\infty}^{\infty}\left(e^{iux}-1-iux\right)\frac{d_x K_\xi^{\mathfrak{F}_\varphi}(\omega,x;t)}{x^2}\right]. \quad (5)$$

M. Loeve [9] called this type of distribution as “weighted infinitely divisible distribution” (“weighting function” here is the distribution function of random characteristics $m^{\mathfrak{F}_\varphi}(\omega,t)$ and $K_\xi^{\mathfrak{F}_\varphi}(\omega,x;t)$), authors of [13] and others called this kind of distribution as «mixture of infinitely divisible distribution».

## 4. Strict Sense Stationarity of Conditional Linear Random Process

Using the obtained expressions for the characteristic functions of the conditional linear random process the properties of strict sense stationarity and cyclostationarity [7, 14] can be studied. We justify the conditions of CLRP to be strict sense stationary in this paper.

***Statement.*** Let for any $s\in\mathbb{R}$ the following conditions holds:

- random functions (fields) $\varphi(\tau,t)$ and $\varphi(\tau+s,t+s)$ are stochastically equivalent in the wide sense, that is, their finite-dimensional distributions are equal:

$$\mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\{\omega:\varphi(\tau_i,t_j)<x_{ij}\}\right)=\mathbf{P}\left(\bigcap_{i=1}^{n}\bigcap_{j=1}^{m}\{\omega:\varphi(\tau_i+s,t_j+s)<x_{ij}\}\right),\ x_{ij}\in\mathbb{R}; \quad (6)$$

- $\eta(\tau)$ is Levy process, that is:

$$da(\tau)=da(\tau+s)=a,$$
$$d_x d_\tau K(x;\tau)=d_x d_\tau K(x;\tau+s)=d_x K(x)d\tau.$$

Then $m$-dimensional characteristic function (4) of CLRP satisfies the following condition

$$f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=f_\xi(u_1,u_2,...,u_m;t_1+s,t_2+s,...,t_m+s),\ \forall s\in\mathbb{R}. \quad (7)$$

that is, CLRP is a strict sense stationary.

***Remark.*** We use some special notations (for example, from [15]) below to simplify the expressions. If random variables $\xi$ and $\eta$ have the same distribution functions (distribution laws), then we denote it as $Law(\xi)=Law(\eta)$, the same, if random vectors $(\xi_1,\xi_2,...,\xi_n)$ and $(\eta_1,\eta_2,...,\eta_n)$ have equal $n$-dimensional joint distribution functions, then we write $Law(\xi_1,\xi_2,...,\xi_n)=Law(\eta_1,\eta_2,...,\eta_n)$.

Indeed, from (6) it follows that

$$Law\left(a\sum_{k=1}^{m}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k)d\tau\right)=Law\left(a\sum_{k=1}^{m}u_k\int_{-\infty}^{\infty}\varphi(\tau+s,t_k+s)d(\tau+s)\right)=$$

$$=Law\left(a\sum_{k=1}^{m}u_k\int_{-\infty}^{\infty}\varphi(\tau,t_k+s)d\tau\right),$$

$$Law\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi(\tau,t_k)}-1-ix\sum_{k=1}^{m}u_k\varphi(\tau,t_k)\right)\frac{d_xK(x)d\tau}{x^2}\right)=$$

$$=Law\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi(\tau+s,t_k+s)}-1-ix\sum_{k=1}^{m}u_k\varphi(\tau+s,t_k+s)\right)\frac{d_xK(x)d(\tau+s)}{x^2}\right)=$$

$$Law\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(e^{ix\sum_{k=1}^{m}u_k\varphi(\tau,t_k+s)}-1-ix\sum_{k=1}^{m}u_k\varphi(\tau,t_k+s)\right)\frac{d_xK(x)d\tau}{x^2}\right).$$

Thus, we obtain that distribution of *m*-dimensional $\mathfrak{F}_\varphi$-characteristic function of CLRP satisfy the following equation:

$$Law(f_\xi^{\mathfrak{F}_\varphi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m))=Law(f_\xi^{\mathfrak{F}_\varphi}(u_1,u_2,...,u_m;t_1+s,t_2+s,...,t_m+s)).$$

Because of $f_\xi(u_1,u_2,...,u_m;t_1,t_2,...,t_m)=\mathbf{E}f_\xi^{\mathfrak{F}_\varphi}(u_1,u_2,...,u_m;t_1,t_2,...,t_m)$, we conclude that (7) holds. That is, CLRP under the above conditions is a strict sense stationary.

The methodology of applied mathematical modelling and digital stochastic signal processing using the theoretical basis of conditional linear random processes has been represented in [7].

Figure 1 represents the relationships between the most important mathematical models that are close by structure to the class of *stationary* conditional linear random processes and have a wide application area in computer science and information technology.

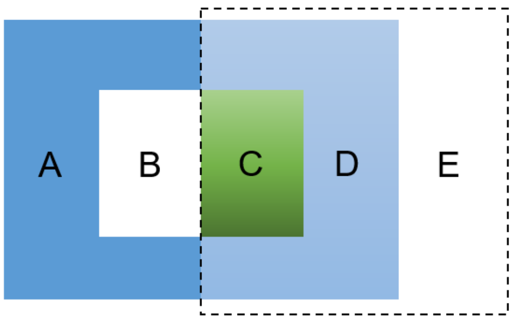


**Figure 1**: Venn diagram of classes of *stationary* random processes: AUBUCUD is CLRP (1) driven by Levy process; BUC is LRP driven by Levy process; A is CLRP driven by Wiener process; B is LRP driven by Wiener process; C is LRP driven by compound Poisson process; D is CLRP driven by compound Poisson process; E is a process of type (1), driven by renewal-reward process.

## 5. Conclusion

The construction of stochastic integral with the random kernel, driven by the process with independent increments has been considered, as a result the continuous-time real-valued conditional linear random process has been defined.

The expressions for univariate and multivariate characteristic functions of CLRP have been proven which belong to the class of the mixture of infinitely divisible distributions. The moment functions of first, second, and third order have been obtained as the examples of using the characteristic function method for probability analysis of stochastic signal using the mathematical model in the form of CLRP.

Using the conditional characteristic function approach the conditions for CLRP to be stationary in the strict sense have been proven which can be used to justify and study the corresponding properties of mathematical models of signals taking into consideration the natural features of their generation.

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