Construction of a Mathematical Model of the Heart Muscle

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Abstract

According to statistics, the problem of cardiovascular disease ranks first among diseases [1-3]. The purpose of this work is to build a mathematical model of the heart muscle, which will more accurately reflect the behavior of the heart, for example in pathological contractions, in which the reduction of arbitrary areas of arbitrary fibers or external mechanical impact. The main idea of the model of the ventricle and the outer shell of the heart is the division of the surface into longitudinal flexible fibers connected by elastic ligaments. The model is presented in the form of an elastic system, which is described in terms of a system of differential equations [4-8]. They simulate the contraction forces of the heart muscle. A static mathematical model was also used, which representing the left ventricle and the outer surface of the heart as a paraboloid of rotation and an elliptical paraboloid, respectively, taking into account the spiral course of muscle fibers. Euler's method was used to numerically solve the system of differential equations. Based on the above models, software was created that reflects the dynamic behavior of the heart muscle. In the course of the work it was possible to select such parameters at which the dynamics of the model became close to the dynamics of the real heart muscle and other models of heart work.

Keywords [1]

Mathematical model, differential equation system, software, medicine, heart muscle

1. Introduction

In modern life, the problem of cardiovascular disease is quite acute - according to statistics, they have the first place by cause of death. Therefore, there are constant attempts to improve the situation in this direction. One method of improvement is to build a mathematical model of heart contraction. Solving this problem will help to more accurately predict the work of the heart, which is important in crisis situations (eg, pre-infarction), in post-crisis rehabilitation, with medication, will detect problems in the early stages and more [9-12]. The main purpose of this work is to build a mathematical model of the heart muscle, which will more accurately reflect the behaviour of the heart, for example, in pathological contractions, in which the reduction of arbitrary areas of arbitrary fibers or external mechanical impact. Of course, scientists has tried to solve this problem many times, but there are still no exact mathematical models of heart muscle contraction [13-16]. The fact is that it has always been believed that the heart contracts like an enema, that is, just pushes blood. In fact, recent studies have shown that in this case, the aneurysm occurs rapidly, and the contraction is spiral, and the blood already enters the vessels in a twisted form. Thus, the task of modeling the behavior of the heart muscle under the condition of spiral contraction and the ability to simulate the disconnection of parts of the heart from the contraction process (in the future).

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2. Formulation of the problem

Using the results of studies of the structure of the heart muscle and the basics of the geometry of contraction of the left ventricular muscle and mathematical models based on them, a new mathematical model is built to reproduce the physical behavior of the heart muscle in spiral contraction. The possibility of exclusion from reduction of some sites of a myocardium is considered.

The paper uses the results of research by the Institute of Cardiology N.D. Strazhesko Academy of Medical Sciences of Ukraine and scientific works of Taras Shevchenko National University of Kyiv.

2.1. Model of the ventricle and the outer shell of the heart as an elastic surface

The main idea of the model of the ventricle and the outer shell of the heart is to break the surface into longitudinal flexible fibers, which are connected by elastic ligaments. This most fully reflects the physical nature of the muscles. Based on the results of previous research and mathematical models based on them, it is possible to build a new model, which is based on the "fibrous" nature and more fully reflects the behavior of the muscle in different situations.

2.2. Cardiac muscle fiber model as an elastic chain

The fiber model is as follows. Let the fiber be part of some curve in space. On this curve, points are sequentially selected (Fig. 1). They are connected in series by segments (ribs), which will act as connections between them. These connections themselves have spring-like properties. That is, when you change the length of the rib, the force that tries to return to the original length of the connection arose. Also, every two consecutive connections create an elastic system. When you change the position of one rib relative to another, forces that try to return them to their original position arose.

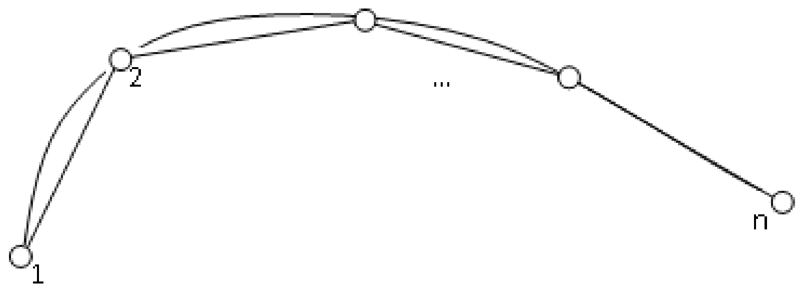

Figure: 1 Geometric model of cardiac muscle fiber

Each point in this chain is a material point with mass m_i. Suppose that at the moment of time t, the i-th point of the chain has a velocity $\overrightarrow{v_i(t)}$, and it is acted upon by the forces of elasticity of the bonds $\overrightarrow{F_i^u(t)}$ and $\overrightarrow{F_i^d(t)}$ co-directed with the corresponding edges connected with the given point. Also at this point acts the force of preservation of the relative position $\overrightarrow{R_i(t)}$ (Fig. 2). Let the equivalent of these forces is

$$\overrightarrow{F_i(t)} = \overrightarrow{F_i^u(t)} + \overrightarrow{F_i^d(t)} + \overrightarrow{R_i(t)} \quad (1).$$

Then you can write the following

$$\frac{dv_i}{dt} = \frac{F_i(t)}{m_i} \quad (2)$$

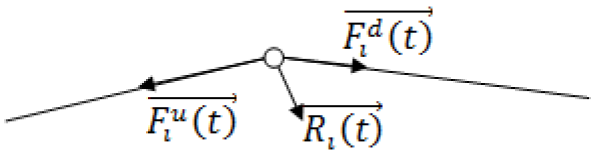

Figure: 2 Distribution of elastic and preservation forces acting on the point of the heart muscle fiber

The problem is that the value $\overrightarrow{F_i\ (t)}$ becomes known only at a time t that corresponds to the condition of "autonomy" of the system, the possibility of changing parameters for the operating system over time, including the sudden exclusion (included) of muscle areas, and so on. Therefore, we find the numerical solution of equation (2) for each i-th point by Euler's method.

We can construct the system of equations for all points:

$$\begin{cases} \dfrac{dv_1}{dt} = \dfrac{F_1(t)}{m_1} \\ \dfrac{dv_2}{dt} = \dfrac{F_2(t)}{m_2} \\ \dots\dots\dots\dots\dots \\ \dfrac{dv_n}{dt} = \dfrac{F_n(t)}{m_n} \end{cases} \tag{3}$$

We will show how to search $\overrightarrow{F_i^j(t)}$ and $\overrightarrow{R_i\ (t)}$

$$\overrightarrow{F_i^j(t)} = k^{ij}(t)(l^{ij}(t) - l_0^{ij}(t)), \tag{4}$$

where $k^{ij}(t)$ - the coefficient of elasticity of this edge, $l^{ij}(t)$ - the length between the i-th and j-th point at the time t; $l_0{}^{ij}(t)$ - the length of the rib at rest between the i-th and j-th point.

For example, $\overrightarrow{F_i^d(t)} = \overrightarrow{F_i^{i-1}(t)}$, $\overrightarrow{F_i^u(t)} = \overrightarrow{F_i^{i+1}(t)}$, if there exist, else $\overrightarrow{F_i^j(t)} = 0$

$$\overrightarrow{R_i\ (t)} = k_r^i(t)((\overrightarrow{p_i p_u}, \overrightarrow{p_i p_d}) - \varphi_0^i(t))) \tag{5}$$

where $k_r^i(t)$ - the coefficient maintaining the relative position of the edges associated with the i-th point; $\varphi_0^i(t)$ - the output angle between the edges connected to the i-th point, the angle to which the relative position of the edges goes.

Let is denote chain length as

$$L(t) = \sum_{i=1}^{n-1} l^{i,i+1}(t) \tag{6}$$

flexible chain as a triple $G(t) = (P(t), V(t), M)$, where $P(t)$ - position of the points of the chain $p_i(t) \in P(t)$, $p_i(t) = (x_i(t), y_i(t), z_i(t))$; $V(t)$ - speed of chain points $\overrightarrow{v_i(t)} \in V(t)$; M - mass of points.

Thus, we obtained a model of dynamic structure that tries to reach equilibrium and can change both its own parameters (stiffness, length, etc.) and respond to external factors. We use it to build a more complex structure that simulates the heart muscle.

2.3. The model of the heart muscle as a set of fibers

We represent a muscle as a set of longitudinal fibers connected by elastic ligaments. Let the surface be s chains ($G_1, G_{2,\dots,} G_s$). Then to each i-th point of the j-th chain, p_{ij}, you need to add "external" connections connecting the points of adjacent chains. Let each point be connected to only one point of the adjacent chain. Let two such edges be formed.

Then additional forces will appear $F_{ij}^{l}(t)$ and $F_{ij}^{r}(t)$, even those that will hold the connected points of adjacent chains, and will change the force $\overrightarrow{R_i(t)}$ responsible for maintaining the position of the edges relative to each other will change (Fig. 3).

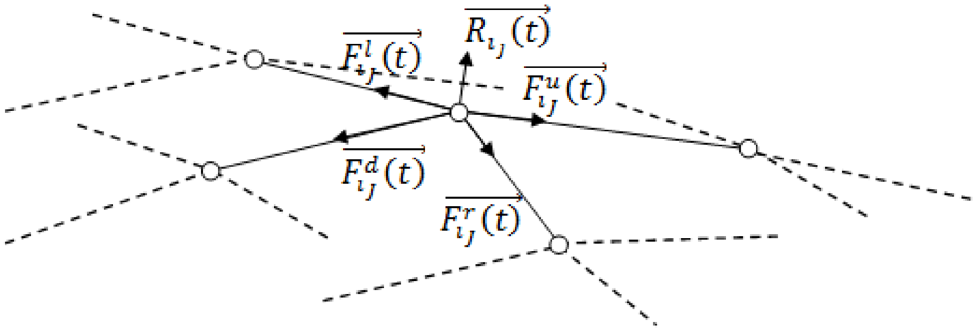

Figure: 3 Distribution of forces acting on the point of the heart muscle fiber, taking into account the set of longitudinal fibers

To this force must be added the force responsible for maintaining the relative position between the edges connecting adjacent chains and belonging to the i-th point of the j-th chain (similarly to (5)). Then, the equilibrium force will change

$$\overrightarrow{F_{ij}(t)} = \overrightarrow{F_{ij}^{u}(t)} + \overrightarrow{F_{ij}^{d}(t)} + \overrightarrow{F_{ij}^{l}(t)} + \overrightarrow{F_{ij}^{r}(t)} + \overrightarrow{R_{ij}(t)} \tag{7}$$

In general, the scheme will look like this

$$\begin{cases} \dfrac{dv_{1_1}}{dt} = \dfrac{F_{1_1}(t)}{m_{1_1}} \\ \dfrac{dv_{1_2}}{dt} = \dfrac{F_{1_2}(t)}{m_{1_2}} \\ \dots\dots\dots\dots\dots \\ \dfrac{dv_{1_s}}{dt} = \dfrac{F_{1_s}(t)}{m_{1_s}} \end{cases} \tag{8}$$

2.4. Euler's method for solving the problem

We show how to find $\overrightarrow{v_{ij}}$ by the system (8). Let i-th point of the j-th chain have an initial velocity $\overrightarrow{v_{ij}(t_0)}$. Then we get the Cauchy problem, which can be solved by Euler's method.

$$\overrightarrow{v_{ij}(t+h)} = \overrightarrow{v_{ij}(t)} + \frac{h}{m_{ij}}\overrightarrow{F_{ij}(t)}, \tag{9}$$

where h - step of method.

We modify the method, taking into account the distribution (scattering) of energy and, as a consequence, the attenuation of velocity over time.

$$\overrightarrow{v_{ij}(t+h)} = r\overrightarrow{v_{ij}(t)} + \frac{h}{m_{ij}}\overrightarrow{F_{ij}(t)}, \tag{10}$$

where $r \le 1$ - attenuation coefficient.

Also by Euler's method we find the position of the node at the time t

$$\overrightarrow{p_{ij}(t+h)} = \overrightarrow{p_{ij}(t)} + h\overrightarrow{v_{ij}(t)} \tag{11}$$

2.5. The structure of a muscle as an elastic system of nodes

It is important to define the structure of the muscle, ie all the connections in the fibers and between them. To do this, we use a static mathematical model that represents the left ventricle and the outer surface of the heart as a paraboloid of rotation and an elliptical paraboloid, respectively, taking into account the spiral course of muscle fibers. Based on the results of the study of the geometry of the contraction of the left ventricle, the basis of his mathematical model was based on the paraboloid of rotation $y = k(x^2 + z^2)$ or in the coordinate system (ρ, φ, y), $y = k\rho^2, k > 0$, which is bounded by the plane $y = H$ on axis Oy. The fibers, according to the chosen model, lie completely on the surface of the paraboloid and are spiral-shaped spatial curves, the beginning of which coincides with the center $O(0,0,0)$, and the end belongs to the bounding plane $y = H$.

A family of logarithmic spirals having equations in polar coordinates of the form $\rho = \alpha \exp(l(\varphi - \varphi_0))$, $l = \ln \alpha$ was taken as projections of fibers on a plane $O\rho\varphi$. The fact that they often occur in various natural phenomena speaks in favor of the choice of logarithmic spirals. From the whole family we will choose a curve with a phase $\varphi_0 = 0$. Based on these data, we can show that, $\frac{tg\alpha}{4k} = l\rho$, or $tg\alpha = 4kl\rho$ - the tangent of the angle is directly proportional to the current radius. The logarithmic spiral has only one drawback for this case - asymptoticity at a point $O(0,0,0)$. But this problem is easy to solve, assuming that ρ_0-around the point $O(0,0,0)$ of the fibers go into the vortex and go to the opposite wall of the paraboloid.

Based on research, the outer shell of the heart can also be modeled as a paraboloid, but no longer circular, but elliptical: $y = k_x x^2 + k_z z^2$, ($k_x > 0, k_z > 0$), for simplicity, we believe that $k_x < k_z$. The fibers of the outer shell can no longer be projected by logarithmic spirals and at the same time rise monotonically upwards. Therefore, the model is based on the calculation that before all calculations, an affine transformation of space is performed - scaling along the axis Ox (multiplication by a factor $\sqrt{\frac{k_z}{k_x}}$), and after them - the inverse transformation of scaling (multiplication by a factor $\sqrt{\frac{k_x}{k_z}}$). Thus, after scaling, the paraboloid becomes a body of rotation, and the fibers in the projection will have the form of logarithmic spirals, after which it will be possible to apply the above modeling methods.

The right ventricle is modeled as a cavity between the outer surface of the heart and the left ventricle. This is because it is much weaker than the left ventricle.

Based on the above model, it is possible to construct curves on the surface of paraboloids that will correspond to the fibers, and on their basis to construct the structure of chains, and link them together. On the surface of the paraboloid we choose the s curves that come out of the point O and give logarithmic spirals in the projection. Let these curves divide the slice of the paraboloid $y = h$ ($0 < h \le H$) into equal s parts. We will consider these curves as fibers of a heart muscle. Draw n slices $y = h_i$, ($0 < h_i \le H$, $i = \overline{1,n}$).The points p_{ij} of intersection of the j-th curve and this slice - h_i, choose as nodes on the j-th curve. Since the connections between nodes (points) in the chain do not differ from the connections between nodes from different chains, so we represent all connections between nodes as a connected graph, the vertices of which are points p_{ij}, and the edges are connections between them. We also add a point O to the set of vertices of the graph and denote it p_{O_0}. Add links (p_{O_0}, p_{1_j}), $j = \overline{1,s}$ to the set of edges. For everyone $j = \overline{1,s}$ add the edges that connect the points in the j-th fiber (p_{i_j}, p_{i+1_j}), $i = \overline{1,(n-1)}$. Also, add the ribs that connect the nodes of adjacent fibers (in a circle) – to $i = \overline{1,n}$ add the ribs $(p_{i_j}, p_{i_{j+1}})$, $j = \overline{1,s}$ and the rib (p_{i_s}, p_{i_1}) that "loops" the cross section. The resulting graph defines the structure of the muscle, and formulas (6), (8), (9) define the behavior of this structure over time

2.6. Muscle contraction control

One of the important aspects of the system is the control of muscle contraction. Namely, setting rules that will allow this system of nodes to reduce or increase the volume of the internal cavity. This control is performed by setting the functions $k^{ij}(t)$ - the stiffness coefficients of the rib and $l_0{}^{ij}(t)$ - the length of the rib, which it tries to achieve $(i, j \in \{1..n\} \times \{1..s\} \cup \{(0,0)\})$. As you increase $k^{ij}(t)$ and decrease $l_0{}^{ij}(t)$, there is a force that tries to reduce the length of the rib. Thus, if this behavior is set for all edges, it will reduce the linear size of the system, and thus reduce the volume of the cavity, which is limited by this set of edges in space. The above steps can be used to simulate muscle contraction. To simulate its relaxation, it is necessary to increase $l_0{}^{ij}(t)$ and gradually decrease $k^{ij}(t)$ at the right time, which will correspond to a slower relaxation than reduction, as it actually happens.

We need to find this point in time when the heart needs to relax. Studies have shown that heart fibers reduce their linear size by about 15%. That is, when, for example, the average fiber length reaches 85%, the heart will begin to relax. That is $\frac{\sum_{j=1}^{s} L_j(t)}{s} \approx 0.85 L_j(t_0)$, where $L_j(t)$ is the length of the j-th chain (fiber). When the length of the fibers again reaches approximately the original size, there is a reduction. In fact, this process is much more complicated, but for simulation so far this approach has been chosen for simplification. In the future, this process can be complicated by the choice of others $l_0{}^{ij}(t)$ and $k^{ij}(t)$

2.7. Realization

Based on the above-described mathematical model of the elastic system, software was created that reflects this model and its behavior.

For the internal representation of the graph of connections between nodes, an incidence list was used (ie a list in which for each vertex the vertices incident to it are stored), which allowed to calculate each step of Euler's method for time $O(wns)$, where w is the number of edges leaving the node, and, since in this case w not large (except for the node p_0 does not exceed 4), we can assume that the time is proportional $O(ns)$. This is achieved by calculating the next step of the Euler method to go through all $ns+1$ nodes, and calculate the forces arising on the edges and the force of maintaining the relative position. The ***C#*** programming language and the Microsoft.NET platform were used to write this software application, which significantly accelerated the design and development.

2.8. Selection of model parameters

One of the most important parts of modeling is the correct selection of parameters and functions, so that the simulated process as closely as possible reflects the actual behavior of the muscle.

The parameters of the static model of the left ventricle and the outer shell of the heart are taken from previous work on the model with paraboloids [1].The coefficients in the equation of the outer shell of the heart are equal to $k_x = 0.5, \quad k_z = 1.0$. The coefficient in the equation of the left ventricle is $k = 1.5$. The center of the left ventricle is displaced relative to the center of the outer shell by a magnitude $(-0.9;0.5;0.0)$. The height of the outer shell $H_{out} = 9.0$, the height of the left ventricle - $H_{in} = 8.5$ respectively. The initial coefficients for the equation of logarithmic spirals are: for the outer shell - 2.08, for the left ventricle - 1.45. At such values, the fibers of the outer shell make one curl, and the left ventricle - two (as it really is).

In the course of many experiments with system parameters, it was empirically found that the following values and functions are the most suitable:

$$n = 20,\ s = 20,\ y = h_i = H(\frac{i}{n})^2,\ m_{ij} = 0.00012\sqrt{\frac{i}{n}},\ k^{ij}(t) = 25,\ k^{i}{}_{r}(t) = 5,\ h = 0.001,\ r = 0.6,$$

$$l_0{}^{ij}(t) = \begin{cases} 0.7 l_0{}^{ij}(t_0), \text{when decreasing} \\ 1.2 l_0{}^{ij}(t_0), \text{when relaxing} \end{cases}$$

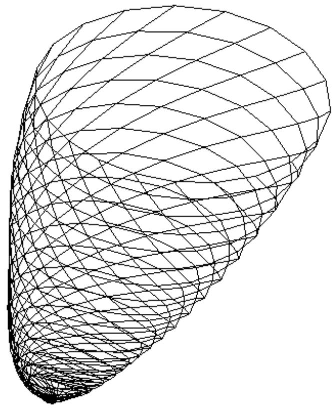

Figure: 4. The surface of the heart is built by the program**.**

3. Results and comparisons

This model was compared with the model of the ventricle and the surface of the heart, which represent them as paraboloids, in which the coefficients at the coordinates and angles of inclination of the fibers change over time. Visually, the work of both models is very similar [17-21].

The following method was used to calculate the volume of the cavity bounding this system of rib. We draw an axis through a point p_{O_0} and a parallel axis Oy. Select two points on this axis $A_0 = p_{O_0}$ and $A_n = \frac{\sum_{j=1}^{s} p_{nj}}{s}$. Actions on points mean actions on the corresponding coordinates. The segment $A_0 A_n$ is divided into n successive parts. Let the points be chosen A_i, $i = \overline{1, n-1}$. Then the volume of this figure can be divided into n volumes bounded by the following sets:

$$V_{gi}(t) = V(\{p_{i_j} \mid j = 1..s\} \cup A_i \cup \{p_{i-1_j} \mid j = 1..s\} \cup A_{i-1}) \qquad (12)$$

where $p_{Oj} = p_{O_0}$, $j = 1..s$.

The total volume is as follows

$$V_g(t) = \sum_{i=1}^{n} V_{gi}(t) \qquad (13)$$

According to the given formulas and parameters found in the previous section, the volume dynamics is found, which is presented in Fig.5 and Fig.6.

4. Conclusion

In the course of this work, a mathematical model of the heart muscle was built, which is quite close in physical content to its actual structure. Muscle is represented as a set of elastic fibers that are interconnected by elastic ligaments and together form an elastic surface, the behavior of which can be controlled to some extent. The surface consists of nodes with a certain mass and elastic connections between them. The system seeks to achieve a position of equilibrium relative to the internal forces that arise when changing control parameters or external action on the surface.

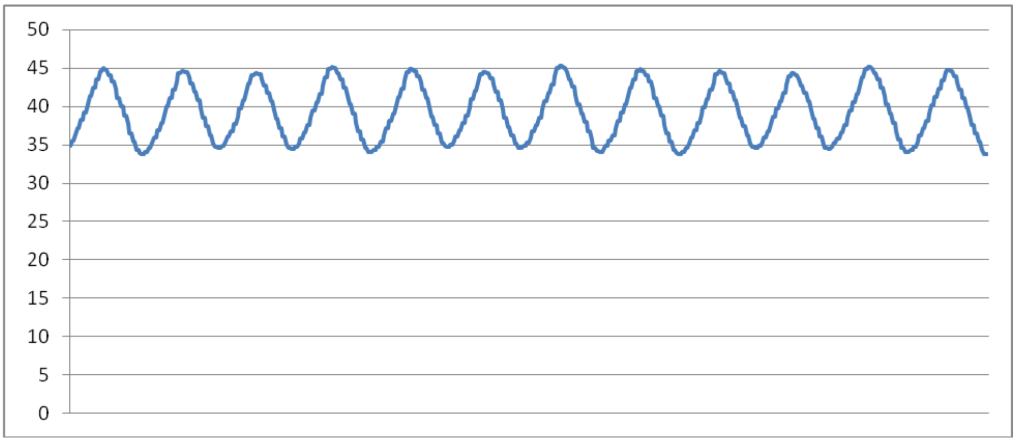

Figure 5: Graph of the inner volume of the outer shell of the heart

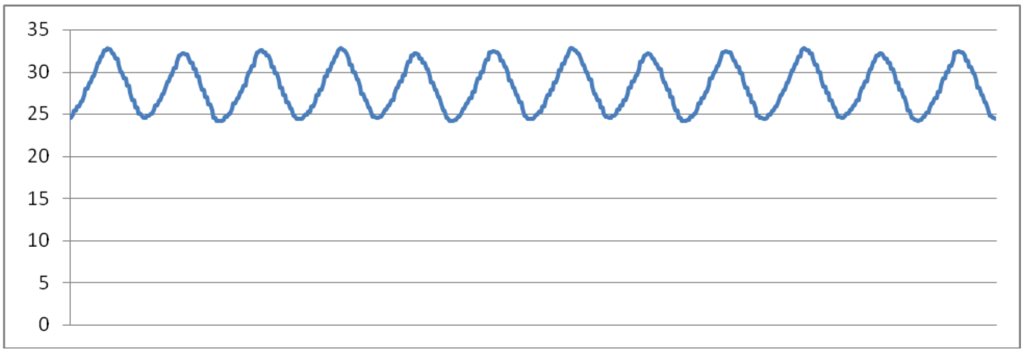

Figure 6: Graph of the internal volume of the left ventricle of the heart

In the course of work it was possible to select such parameters that the dynamics of the model is close to the dynamics of the real heart muscle and other heart models.

Building a mathematical model of the heart muscle will be widely used in cardiology. The results of the work can be used to build a model of pathological contraction, in which arbitrary areas of arbitrary fibers fall out of the contraction. Also, one of the ways to improve the model is to set the initial structure based on the real geometric parameters of the heart, rather than paraboloids.

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